ARITHMETIC

AND ITS APPLICATIONS;

DESIGNED AS A

TEXT BOOK FOR COMMON SCHOOLS,

HIGH SCHOOLS, AND ACADEMIES.

BY

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PREFACE.

The principles involved in Arithmetic are few, the methods of applying them many. To be a perfect master of the subject, a person must possess, —

1. A knowledge of the nature and use of numbers, with the methods of representing and expressing them:

2. A knowledge of the nature and use of the various numerical operations, with the methods of indicating and of performing them.

3. Such mental training and cultivation of the reasoning powers as shall enable him to understand the conditions of any given problem, and to determine from them what operations are necessary to its solution.

The first of these includes every thing belonging to Notation and Numeration.

The second includes the operations of Addition, Subtraction, Multiplication, and Division; to which some would add, as a separate operation, the Comparison of Numbers, as in Fractions and Ratio.

The third requires a power of grasping the various conditions of a problem, of tracing their relations to each other, and of finding from them what operations must be performed, and what new relations determined, to obtain the result required.

They are, however, mutually dependent, so that no person

can master one without learning much of the others. Counting is but addition; and to understand the nature and use of the number *two*, we must know that it equals 1 and 1, two ones, or two times 1; that it is 1 more than 1; that if 1 be taken from it 1 will be left, and so on with the other numbers — things which require a knowledge of numerical operations, and also a power of tracing and appreciating relations.

The operations and exercises included in the first two points are eminently adapted to give quickness of thought and rapidity of mental action. They are to Arithmetic what a knowledge of the nature and power of letters, and of their combination into words, is to reading. The processes included in them may be called the mechanical processes of Arithmetic, and by practice may and should be made so familiar that the moment a number or a combination is suggested, the mind can appreciate it, and determine the result.

The third requires and imparts a power of investigation, of tracing out the relations of cause and effect, and habits of accuracy both in thought and expression.

To secure these results, it is necessary that the pupil should be taught in the simplest as well as in the most complicated problems to reason for himself; to trace fully and clearly the connection between the conditions of a problem and the steps taken in its solution; to state not only what he does, but why he does it, and indicate the precise character of the result obtained by each step. Finally, he must learn to grasp the whole mechanical process before performing any part of it, so that he may know before writing a figure just what additions, subtractions, multiplications, divisions, and comparisons he has to make, and be assured that if made correctly they will lead to the true result.

Such a course as this is usually taken in works on Oral Arithmetic. In studying them the scholar is thrown on his

own resources; is compelled to learn principles; to follow out rigid reasoning processes and connected trains of thought; to examine and know for himself the necessity and the reason for each step taken, and for each operation performed. The result is, that the study gives strength, vigor, and healthful discipline to the mind, and becomes an almost invaluable part of the educating process.

Why should not the same result follow a similar course in Written Arithmetic? Aside from the writing of numbers, there is no difference in the principles involved, in the reasoning processes demanded, or in the operations required.

In the preparation of this work, the author has kept these things in view. He has endeavored to present the subject of Arithmetic as it lies in his own mind, and without any effort either to follow or to deviate from the course pursued by other writers. He has aimed to arrange the work in such a way as to lead those who may study it to understand the principles which lie at the foundation of the science, to learn to reason upon them, apply them, and to trace out their connections, relations, and combinations. He has given very full explanations and illustrations, especially of the fundamental operations; he has endeavored every where to state principles rather than rules; to throw the pupil constantly on his own resources, and force him to investigate and think for himself.

He has omitted some subjects usually found in school arithmetics, because they do not belong legitimately to the subject of Arithmetic, because they are of theoretical rather than of practical importance, or because they require neither special explanation nor peculiar exercise of the mind.

He has differed from other authors of school arithmetics in giving algebraic rather than geometrical explanations of the principles involved in Square and Cube Roots. In this way he has been able to give more rigid demonstrations, and more

full explanations, and at the same time (as he conceives) to simplify the subject. Moreover the processes are in their nature so essentially algebraic that by the use of squares and cubical blocks we can do nothing more than illustrate some of their applications.

He has given no answers to his problems, because he believes that to place them within reach of the pupil is always injurious.

In the first place, such tests are unpractical, for they can never be resorted to in the problems of real life. What merchant ever thinks of looking in a text book or a key, or of relying on his neighbor, to learn whether he has added a column correctly, drawn a correct balance between the debit and credit sides of an account, or made a mistake in finding the amount of a bill?

When a pupil, having left the school room, performs a problem of real life, how anxious is he to know whether his result be correct! Neither text book nor key can aid him now, and he is forced to rely on himself and his own investigations to determine the truth or the falsity of his work. If he must always do this in real life, and if his school course is to be a preparation for the duties of real life, ought he not to do it as a learner in school? Is it right to lead him to rely on such false tests?

Besides, the labor of proving an operation is usually as valuable arithmetical work as was the labor of performing it, and will oftentimes make a process or solution appear perfectly simple and clear, when it would otherwise have seemed obscure and complicated.

Again: the science of Mathematics, of which Arithmetic is a branch, is an exact science; it deals in no uncertainties; its reasonings are always accurate, and, if based on true premises, must always lead to true results. In Arithmetic the

pupil may always *know* that a certain step is a true one, and one which he has a right to take. He may *know* whether he has taken it correctly, and thus be *certain* of the truth of his first result. He may be as sure of the truth of his second step and second result, and of his third and his fourth; and when he reaches the end, and obtains his final result, he may be as sure of the truth of that as of any preceding — so sure that he will be willing to abide by it, and stake his reputation upon it. (See page 49.)

Why, then, should not the subject be so presented as to require the pupil to apply such tests, to determine for himself the truth and accuracy of his processes, and thus to form a habit of patient investigation and just self-reliance? Why should he not be from the first thrown on his own resources, and held strictly responsible for the accuracy of his work? Would not such a course, if faithfully followed, almost entirely prevent the formation of those careless habits which scholars so often acquire?

The articles on business forms and transactions have been carefully prepared, with a hope of so presenting them as to give the student true ideas of their use, and of the relations and obligations of the parties to them.

The materials were drawn from various sources — from legal works, from intercourse with business men, and from an article in Mann and Chase's Arithmetic, published originally in the Common School Journal. To insure accuracy they were submitted to the inspection of Abraham Payne, Esq., an eminent lawyer of this city, to whom I am indebted for some important suggestions.

The work as a whole resembles all other text books (good or bad) in this — that it requires a good teacher to teach it well; as also in this — that it does not contain exactly the right kind and amount of exercises to meet the wants of every

school or of every class of scholars. The judicious teacher will of course extend the exercises which are too meagre, abridge those which are too full, and omit those which are not adapted to the wants of his class. We earnestly beg of him, however, to notice their arrangement, their gradual character, and their dependence on each other; and not to pass any till he has convinced himself that they are inappropriate, or that the scholar is master of the operations which they involve.

To my former teacher, N. Tillinghast, Esq., for many years principal of the State Normal School at Bridgewater, Massachusetts, I am more deeply indebted than to any other, or all others, for the ideas embodied in this work. Many of the processes were learned under his tuition; and the training which laid the foundation for whatever real mathematical knowledge I may possess, was, in a great measure, received from him. Only those who have been his pupils can appreciate the value of his instructions, and the justice of this acknowledgment.

The work in its plan and arrangement is entirely my own, and for its defects I alone must be held responsible. Such as it is I present it to the public, with a hope that it may be found useful.

DANA P. COLBURN.

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ARITHMETIC.

SECTION I.

1. *Definitions.*

1. QUANTITY is a term applied to whatever may be increased, diminished, or measured.

2. A CONCRETE UNIT is any quantity which may be considered by itself, and made the measure of other similar quantities.

(*a.*) It may be a single thing, as *an apple, a book, a pound, a foot, a dollar;* or it may be a collection of single things, as *a dozen of apples, a score of sheep.*

3. An ABSTRACT UNIT is the idea or conception of one. or of unity, without reference to any particular thing or quantity. Any number of abstract units, considered as forming a single collection or whole, may also be regarded as an abstract unit; as, *one ten, one hundred, one thousand,* &c.

(*a.*) The term *unit,* when used without any limitation, always refers to a single thing or quantity, or to the abstraction, one.

4. A CONCRETE NUMBER expresses a single thing or quantity, considered as a unit of measure; or it expresses how many such units there are in any given quantity; as, *one yard, three shillings, eight barrels, ten birds.*

5. An ABSTRACT NUMBER expresses how many times a unit is taken or repeated, without any reference to the nature of the unit; as, *one, eight, twelve.*

6. The unit which any number expresses is called the UNIT OF MEASURE, or the UNIT OF COMPARISON.

(*a.*) In an abstract number it is the abstract unit, and in a concrete

number it is the unit which the number measures. Thus, in *seven dollars* it is *a dollar*, in *twelve bushels* it is *a bushel*, in *forty-three dozen of eggs* it is *a dozen of eggs*, in *six tens* it is *one ten*, &c.

7. ARITHMETIC is the SCIENCE OF NUMBERS and ART OF NUMERICAL COMPUTATION.

(*a.*) It treats of numbers with reference to their nature and use, their properties and relations; explains the various methods of representing them; and includes the theory of all numerical operations, as well as the practical methods of performing them.

8. The operations of which numbers are susceptible are four in number; viz., *Addition*, *Subtraction*, *Multiplication*, and *Division*.

9. Among the characters used to indicate numerical operations or relations are the following:—

(*a.*) $=$ The sign of *equality*, called *equal*, or *equal to*, signifies that the quantities between which it is placed are equal to each other.

(*b.*) $+$ The sign of *addition*, called *plus* or *and*, signifies that the quantities between which it is placed are to be added together.

Thus, $7 + 4 = 11$, is read, *seven plus four equal eleven;* or, *seven and four equal eleven;* and means that seven added to four equal eleven.

(*c.*) $-$ The sign of *subtraction*, called *minus* or *less*, signifies that the number following it is to be subtracted.

Illustration. $9 - 6 = 3$, is read, *nine minus six*, or *nine less six*, *equal three*, and means that nine diminished by six equal three.

(*d.*) $\times$ The sign of *multiplication*, called *times* or *multiplied by*, signifies that the numbers between which it is placed are to be multiplied together.

Illustration. $7 \times 5 = 35$, is read, *seven times five equal thirty-five;* or, *seven multiplied by five equal thirty-five.*

(*e.*) $\div$ The sign of *division*, called *divided by*, means that the number before it is to be divided by that following it.

Illustration. $12 \div 3 = 4$, is read, *twelve divided by three equal four*, and means that twelve divided by three gives four for a quotient.

(*f.*) Division may also be expressed by writing the num-

ber to be divided above the number by which it is to be divided, with a line between them.

Illustration. $\frac{12}{3} = 4$, means the same as $12 \div 3 = 4$; i. e., that the quotient of twelve divided by three equals four.

(*g.*) Such expressions as $\frac{12}{3}$, $\frac{16}{4}$, $\frac{7}{8}$, are usually called *fractions*, and read thus: *twelve thirds*, *sixteen fourths*, *seven eighths;* though they may, with equal correctness, be read as *twelve divided by three*, *sixteen divided by four*, *seven divided by eight*. When read as fractions, the number above the line is called the *numerator*, and the number below it the *denominator*. See Section X.

(*h.*) The more common use of fractions is *to express the value of one or more such parts as are obtained by dividing a unit into a given number of equal parts*, or, which is the same thing, *to express the value of one or more equal parts of such kind that a given number of them will equal a unit.* Thus, $\frac{3}{4}$ (read *three fourths*) is used to express the value of three such parts as would be obtained by dividing a unit into four equal parts; or, in other words, the value of three equal parts of such kind that it would take four of them to equal a unit.

(*i.*) The numerical value of a fraction is the same, whether we consider that it expresses a division to be performed, or a certain number of equal parts, and, in either case, it is obvious that a fraction must equal unity whenever its numerator equals its denominator. Thus, $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6}$, &c.

SECTION II.

NOTATION AND NUMERATION.

2. *Definition of Terms.*

NOTATION and NUMERATION treat of the various methods of representing and expressing numbers.

(*a.*) The distinction usually made between notation and numeration is, that the former treats of the methods of representing numbers by written characters, while the latter treats of the methods of reading them, or of expressing them in words.

3. *Methods of representing Numbers, and Origin of the Decimal System.*

Numbers may be represented by material objects or visible marks, by words, and by figures.

(*a.*) As our ideas of number are derived primarily from material objects, so the most natural and obvious method of communicating them to others is by exhibiting as many such objects as there are units in the number considered.

(*b.*) It is probable that in the earlier stages of society numbers were represented only in this way, the fingers being, as a general thing, made use of as counters. Thus, three fingers would be shown as a symbol for the number three, five fingers for the number five, and the fingers of both hands for the number ten.

(*c.*) Such a method would naturally lead a people using it to represent large numbers by exhibiting the fingers of both hands as many times as there are tens in the numbers considered, and by thus leading them to reckon by tens, would lay the foundation for a system of numbers similar to the one in general use, which is known as the DECIMAL* SYSTEM.

4. *Nature of the Decimal System of Numbers.*

(*a.*) The fundamental idea of the Decimal System is, that ten single things may be regarded as forming a single collection or group; ten of these groups as forming a larger group; and so on, ten groups of one size forming a new group of a larger size, each capable of being regarded and dealt with as a single thing or unit. This idea renders it easy to represent the largest numbers, by having names for each of the first ten numbers, and for each group formed by combining ten of the smaller ones.

(*b.*) In conformity with it, we might count thus: *one, two, three, four, five, six, seven, eight, nine, ten, one and ten, two and ten, three and ten, four and ten, five and ten, six and ten, seven and ten, eight and ten, nine and ten, two tens, two tens and one, two tens and two*, &c., to *nine tens and eight, nine tens and nine, ten tens* or *one hundred, one hundred and one*, &c., to *nine hundreds nine tens and eight, nine hundreds nine tens and nine, ten hundreds* or *one thousand*, &c.

(*c.*) Adopting this method, and forming compound words by dropping the conjunction, we should count from ten thus: *one-ten, two-ten, three-ten, four-ten, five-ten, six-ten, seven-ten, eight-ten, nine-ten, two-tens, two-tens-one, two-tens two*, &c.

---

* The word *decimal* is derived from the Latin word *decem*, which signifies ten.

(*d.*) Changing the word *ten* to *teen*, and dropping the hyphen in counting from ten to two-tens, we should have *oneteen*, *twoteen*, *threeteen*, *fourteen*, *fiveteen*, *sixteen*, *seventeen*, *eighteen*, and *nineteen*.

(*e.*) By now changing *five* to *fif*, *three* to *thir*, and substituting for *one-teen* and *two-teen* the words *eleven* and *twelve*, signifying respectively *one left* and *two left*, (i. e., ten and one left, ten and two left,) we should have the familiar names, *eleven*, *twelve*, *thirteen*. *fourteen*, *fifteen*, *sixteen*, *seventeen*, *eighteen*, and *nineteen*.

(*f.*) Substituting the syllable *ty* for *tens* in the words *two-tens*, *three-tens*, &c., would give the words *twoty*, *threety*, *fourty*, *fivety*, *sixty*, *seventy*, *eighty*, and *ninety;* and again changing the *three* to *thir*, *four* to *for*, and the *five* to *fif*, and substituting *twen*, derived from *twain*, for *two*, would give the familiar names *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, and *ninety*.

(*g.*) These changes would enable us to count from two-tens or twenty thus: *twenty-one*, *twenty-two*, &c., to *twenty-nine*, *thirty*, *thirty-one*, &c., to *one hundred*, *one hundred and one*, &c.

(*h.*) This method of expressing numbers is the one now in general use.

5. *Arabic and Roman Methods of Notation.*

(*a.*) Numbers are usually represented to the eye by characters called *figures*, though sometimes by letters of the alphabet.

(*b.*) The method by figures is called the *Arabic Method*, because it was introduced into Europe by the Arabs.

NOTE. — The Arabs probably obtained it from the Persians, who had obtained it from the Hindoos. Its origin has never been satisfactorily determined.

(*c.*) The method by letters is called the *Roman Method*, because it was used by the ancient Romans.

6. *The Figures.*

The figures are ten in number, viz.: —

1 or *1*, called *one.*	6 or *6*, called *six.*
2 or *2*, called *two.*	7 or *7*, called *seven.*
3 or *3*, called *three.*	8 or *8*, called *eight.*
4 or *4*, called *four.*	9 or *9*, called *nine.*
5 or *5* called *five.*	0 or *0*, called *zero, cipher, nothing*, or *nought*

7. *The Place of a Figure determines its Denomination.*

Each of these figures represents as many units as its name indicates; but the size or denomination of those units is determined by the place or position of the figure with reference to a period or dot, called the *decimal point.*

8. *Names and Position of the Decimal Places.*

(*a.*) The figure immediately at the left of the point represents ones, or simple units; the second figure at the left represents tens, (i. e., units of the denomination or value of ten ones, or ten simple units;) the third figure represents hundreds; the fourth represents thousands, and so on; the figure in any place always representing ten times the value it would represent if it stood one place farther towards the right.

(*b.*) Hence each place has its peculiar name, the first, second, third, and fourth places being called, respectively, the units' place, the tens' place, the hundreds' place, and the thousands' place. Moreover, the position of these places is marked by the figures occupying them. Hence each figure performs a double office, viz.: it marks a place, and indicates as many of the denomination of that place as its name indicates.

(*c.*) The following will illustrate this: —

Thousands.	Hundreds.	Tens.	Units.	Point.
0	0	0	0	·

(*d.*) In the above example each zero marks a place, and shows that there are none of the denomination of the place it occupies expressed in that place.

(*e.*) As another illustration, take the expression 2503. Here each figure marks a place, and denotes as many of the denomination of that place as its name implies; i. e., the 3 marks the units' place, and shows that there are 3 units; the 0 marks the tens' place, and shows that there are no tens; the 5 marks the hundreds' place, and shows that there are 5 hundreds; and the 2 marks the thousands' place, and shows that there are 2 thousands. The number is read *two thousand five hundred and three.*

NOTE. — The zero is often called an insignificant figure, and the other nine digits significant figures; but there is no foundation for the distinction. The zero performs an office precisely similar to that of any other figure, as the above explanation of the use of the figures used in writing 2503 clearly shows. Even when standing by itself it is as expressive as any other figure.

(*f.*) The decimal point is often omitted in writing numbers; but in all such cases it is *understood* to belong at the right of the given number, thus making the right hand figure represent units.

9. *Method of reading Numbers.*

In reading numbers expressed by figures we begin at the left hand, i. e., with the highest denomination.

(*a.*) 546 = five hundreds, four tens, and six units, and is read *five hundred and forty-six.*

(*b.*) 398 = three hundreds, nine tens, and eight units, and is read *three hundred and ninety-eight.*

(*c.*) 407 = four hundreds, no tens, and seven units, and is read *four hundred and seven.*

(*d.*) 180 = one hundred, eight tens, and no units, and is read *one hundred and eighty.*

(*e.*) 64 or 064 = six tens and four units, and is read *sixty-four.*

10. *Exercises in reading and writing Numbers.*

Read the following numbers, and also give the value of each in hundreds, tens, and units: —

1. 507.	4. 379.	7. 031.
2. 409.	5. 211.	8. 37.
3. 528.	6. 403.	9. 200.

10. Explain the use of each figure used in the above numbers, as in the following model: —

MODEL. — In the first number, 507, the 7 marks the units' place, and shows that there are 7 units; the 0 marks the tens' place, and shows that there are 0 tens; the 5 marks the hundreds' place, and shows that there are 5 hundreds.

11. How will you write four hundred and seven?

Ans. By writing 4 in the hundreds' place, 0 in the tens', and 7 in the units'; thus, 407.

12. How will you write two hundred and seventeen?
13. How will you write eight hundred and forty-one?
14. How will you write eight hundred and twelve?
15. How will you write seven hundred and forty-six?
16. How will you write six hundred and ninety-four?
17. How will you write nine hundred and sixty-four?
18. How will you write four hundred and sixty-nine?
19. How will you write nine hundred?
20. How will you write seven hundred and eighty?

11. *Number of Decimal Places unlimited.*

Extending these principles, we can take as many places as we please, *by giving to the figure in each ten times the value it would have if written one place farther to the right.* The names of the places as far as the twenty-fourth are given in the following example.

Hundred-sextillions.	Ten-sextillions.	Sextillions.	Hundred-quintillions.	Ten-quintillions.	Quintillions.	Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Point.
0	0	0,	0	0	0,	0	0	0,	0	0	0,	0	0	0,	0	0	0,	0	0	0,	0	0	0	.
24th place,	23d place,	22d place,	21st place,	20th place,	19th place,	18th place,	17th place,	16th place,	15th place,	14th place,	13th place,	12th place,	11th place,	10th place,	9th place,	8th place,	7th place,	6th place,	5th place,	4th place,	3d place,	2d place,	1st place,	

12. *Division into Periods.*

(*a.*) By inspecting the above example it will be seen that the first three places are occupied by units, tens, and hundreds,— the second three by thousands, tens of thousands, and hundreds of thousands,— the third three by millions, tens of millions, and hundreds of millions, and so on. If the first three places were called, as they might be with perfect propriety, units, tens of units, and hundreds of units, and we should divide the number into periods of three figures each by commas, the first period would be the period of units, the second the period of thousands, the third of millions, the fourth of billions, &c.

(*b.*) The right hand figure in each period expresses units or ones of the denomination of that period, while the second figure expresses tens, and the third, or left hand figure, expresses hundreds of that denomination.

(*c.*) This is exhibited in the following table:—

000,000,000,000,000,000,000,000.

Period	Figures	Places
8th, or Sextillions' Period.	000	24th, or hundred-sextillions' place,... 23d, or ten-sextillions' place,........ 22d, or sextillions' place,...........
7th, or Quintillions' Period.	000	21st, or hundred-quintillions' place,.. 20th, or ten-quintillions' place,...... 19th, or quintillions' place,.........
6th, or Quadrillions' Period.	000	18th, or hundred-quadrillions' place, 17th, or ten-quadrillions' place,..... 16th, or quadrillions' place,.........
5th, or Trillions' Period.	000	15th, or hundred-trillions' place,.... 14th, or ten-trillions' place,........ 13th, or trillions' place,............
4th, or Billions' Period.	000	12th, or hundred-billions' place,..... 11th, or ten-billions' place,......... 10th, or billions' place,.............
3d, or Millions' Period.	000	9th, or hundred-millions' place,..... 8th, or ten-millions' place,......... 7th, or millions' place,.............
2d, or Thousands' Period.	000	6th, or hundred-thousands' place,... 5th, or ten-thousands' place,........ 4th, or thousands' place,............
1st, or Units' Period.	000	3d, or hundreds' place,............. 2d, or tens' place,................. 1st, or units' place,................
Point.	.	

13. *Exercises to secure Familiarity with the Places and Periods.*

1. What is the name of the first period at the left of the point? of the second period? of the third? of the fourth? of the fifth? of the sixth? of the seventh? of the eighth?

2. What is the name of the period occupying the first, second, and third places at the left of the point?

3. Occupying the fourth, fifth, and sixth places?

4. Occupying the seventh, eighth, and ninth?

5. Occupying the tenth, eleventh, and twelfth?

6. Occupying the thirteenth, fourteenth, and fifteenth?

7. Occupying the sixteenth, seventeenth, and eighteenth?

8. Occupying the nineteenth, twentieth, and twenty-first?

9. Occupying the twenty-second, twenty-third, and twenty fourth?

10. What is the number of the millions' period?

Ans. The third period at the left of the point.

11. What is the number of the thousands' period?
12. What is the number of the quintillions' period?
13. What is the number of the trillions' period?
14. What is the number of the units' period?
15. What is the number of the sextillions' period?
16. What is the number of the billions' period?
17. What is the number of the quadrillions' period?

How many places are there between the point and the

18. Millions' period?	22. Thousands' period?
19. Quintillions' period?	23. Billions' period?
20. Units' period?	24. Sextillions' period?
21. Quadrillions' period?	25. Trillions' period?

26. In which place of what period would the fourth figure at the left of the point be?

Ans. In the first place of the second or thousands' period.

27. In which place of what period would the seventh figure at the left of the point be?

28. Would the tenth?	38. Would the thirteenth?
29. Would the sixteenth?	39. Would the seventeenth?
30. Would the second?	40. Would the fifth?
31. Would the eleventh?	41. Would the fourteenth?
32. Would the twentieth?	42. Would the twenty-third?
33. Would the twenty-second?	43. Would the eighth?
34. Would the third?	44. Would the seventeenth?
35. Would the twelfth?	45. Would the twenty-fourth?
36. Would the twenty-first?	46. Would the eighteenth?
37. Would the sixth?	47. Would the fifteenth?

48. What would be the denomination of a figure in each of the above-named places?

Ans. The denomination of a figure in the fourth place at the left of the point would be thousands, that of a figure in the seventh place at the left would be millions, that of the tenth would be billions, &c.

49. Where must a figure be placed to represent trillions?

Ans In the first place of the fifth period, which is the thirteenth place at the left of the point.

Where must a figure be placed to represent

50. Millions?
51. Ten-millions?
52. Units?
53. Ten-thousands?
54. Quadrillions?
55. Thousands?
56. Hundred-trillions?
57. Hundreds?
58. Hundred-billions?
59. Ten billions?

14. *Method of reading Numbers.*

EXERCISES.

(*a.*) In reading a number represented by figures, we ordinarily commence at the left hand, and read each period as though its figures stood alone, giving afterwards the name of the period.

For instance, the number 42,000,070,294,600,706 would be read in the same way, and would express the same value, as if written 42 quadrillions, 70 billions, 294 millions, 600 thousands, and 706.

NOTE. — The scholar should regard a mistake in reading numbers as one of the most dangerous which can be made, for he will not only be likely to copy the numbers in the same manner as he reads them, but he will give those to whom he reads a false idea, which, unless they have the figures before them, they cannot correct.

(*b.*) Read each of the following numbers: —

1. 43,271.
2. 500,207.
3. 24,000,217.
4. 53,279,412.
5. 432,160,023.
6. 70,000,000.
7. 86,102,102.
8. 150,437,986,216.
9. 20,020,020,020.
10. 200,200,200,200.
11. 70,000,007,700,077.
12. 2,008,002,008,002,008.
13. 6,000,070,000,600,007.
14. 3,200,020,006,307,004.
15. 73,052,700,060,007.
16. 607,429.
17. 1,579,432.
18. 914,307,426.
19. 53,729,415.
20. 21,437,986,512.
21. 42,000,042,042.
22. 547,547,547,547.
23. 101,101,101,101.
24. 2,002,002,002,002.
25. 130,201,040,999,999.
26. 73,006,200,474.
27. 900,000,726,000.
28. 74,206,372,704.
29. 407,000,000,030,002.
30. 703,700,000,000,006.

(*c.*) Explain the use of the figures used in writing the above numbers. (*See model* following the 10th example in **10.**)

15. *All intervening Places to be filled.*

(*a.*) When, as is usually the case, the places are only marked by the figures occupying them, every place between any given figure and the point must be occupied by some one of the digits, for otherwise it will be difficult or impossible to tell the place or denomination of the given figure.

Illustration. The 3 of the number 3 24 may mean 3 thousands, or 3 ten-thousands; but when the space between the 3 and 2 is filled, the denomination of the 3 is at once apparent. Thus, in 3024, or 3124, or 3724, the 3 represents 3 thousands; but in 30024, 31024, 32324, or 37824, it represents 3 ten-thousands.

(*b.*) The digit to be used in any intervening place is determined by the number of units to be represented of the denomination of that place. If there are none, then zero should be used; if one, then 1; if two, then 2; &c.

Illustration. In order that 9 may represent 9 ten-millions, it must be written in the eighth place, at the left of the point, and hence there must be seven figures to the right of it. If we wish to express only 9 ten-millions, or 90 millions, the intervening places must be filled by zeroes, thus, 90,000,000; but if we wish to express 90 millions, 3 thousand, 8 hundred, and 7, we write 9 in the eighth place, as before, 3 in the fourth, 8 in the third, 7 in the first, and zeros in all the intermediate places; thus, 90,003,807.

16. *Method of writing Numbers.*

EXAMPLES.

(*a.*) In writing numbers by figures we may begin at the left, and write in each successive period the figures necessary to express the required number of units of the denomination of that period; or we may begin at the right, and write in each successive place the figures expressing the required number of units of the denomination of that place, taking care to write a zero in each place otherwise unoccupied.

(*b.*) In writing numbers be careful to distinguish the decimal point from the commas used to separate the periods. The former should be a dot, so carefully made that it cannot be mistaken for a comma, while the latter should be made with equal care. No number has more than one decimal point.

(c.) Accuracy in reading and writing numbers is of the greatest importance. If the numbers used in solving a problem are copied incorrectly, the results obtained will of necessity be wrong; and if the book from which the problem was obtained is not at hand, or if the facts and transactions on which the problem was based are forgotten, it will be very difficult, and usually impossible, to make the necessary correction.

1. Represent by figures five hundred and twenty-seven thousand, four hundred and eighty-nine.

2. Eight thousand, four hundred and seven.

3. Eighty-five thousand.

4. Eighty-five thousand and one.

5. Eighty-five thousand and thirty-one.

6. Nine million, eight hundred and fifty-six thousand, seven hundred and twenty-one.

7. Twelve million, twelve thousand, and twelve.

8. Four billion, eight hundred seventy-six million, five hundred and four thousand, three hundred and one.

9. Four billion, eight hundred and four million, eight hundred and four thousand, eight hundred and four.

10. Thirty-seven million, eight hundred and fifty-nine thousand.

11. Thirty-seven billion, eight hundred and fifty-nine million.

12. Thirty-seven billion, eight hundred and fifty-nine thousand.

13. Thirty-seven million, eight hundred and fifty-nine.

14. Forty billion, three hundred and forty million, four hundred and eighty-seven thousand, five hundred and nine.

15. Five billion, eight hundred and seventy-six thousand, seven hundred and forty-six.

16. Seventy-five trillion, eight hundred and seventy-six billion, four hundred and eighty-two million, four hundred and seventy-six thousand, three hundred and twenty-seven.

17. Four trillion, seven hundred and sixty-four billion, eight hundred and twenty-one million, six hundred and seventeen thousand, four hundred and fifty-one.

18. Seven hundred and twenty-five trillion, eight hundred and seventy-six billion, four hundred and three million, eight hundred and fifty thousand, four hundred.

19. Three hundred and six trillion, eighteen billion, four hundred million, three thousand, four hundred and seventy-five.

20. Three trillion, three hundred and ninety-nine billion, three hundred and ninety-nine million, three hundred thousand, four hundred and three.

21. Eighty-seven trillion, five hundred and four billion, three hundred million, seven thousand, six hundred and seventy-five.

22. Seventy quadrillion, seven hundred and seven billion, seven thousand and seven.

23. Eighty-seven quintillion, eight trillion, seven hundred billion, eight hundred and seventy thousand, and eighty-seven.

24. Three hundred and fifty-four sextillion, four hundred and seven quintillion, two hundred and nine quadrillion, nine hundred and seventeen trillion, seven hundred billion, eighty-six million, seven thousand, eight hundred and fifty-two.

25. Seven hundred and seven quintillion, two hundred and six thousand.

26. Five hundred and seventy quadrillion, five hundred and seventy.

27. Eight sextillion, eight trillion, eight thousand, and eight.

17. *Any Combination of Figures may be read as though alone.*

(*a.*) Combinations of figures, wherever placed, can be read as though they stood alone, if the name of the place of the right hand figure be given after reading the figures.

For instance, 347 always stands for, and may be read as, three hundred and forty-seven of the denomination of the place occupied by the 7.

(*b.*) To illustrate this still further, we have written opposite each of the following numbers the value expressed by 347 in that number.

1.	347,241, . . .	347 thousands.
2.	347,100, . . .	347 thousands.
3.	4,134,721, . . .	347 hundreds.
4.	23,476,258,675, . .	347 ten-millions.

(*c.*) Let the pupil give the value expressed by the 409 in each of the following numbers, and also the value expressed by the 27: —

1. 40,927.	6. 568,340,927.
2. 274,090.	7. 40,927,534.
3. 8,172,764,096,134.	8. 27,409.
4. 52,706,409,273.	9. 4,092,768,375,674.
5. 134,090,062,746.	10. 1,973,409,327,547.

18. *Analysis of Numbers.*

1. What is the greatest number of tens that can be taken from 546,372?

Ans. 54637 tens.

2. What is the greatest number of ten-thousands that can be taken from 53,075,423,697?

Ans. 5307542 ten-thousands.

What is the greatest number of hundreds that can be taken

3. From 8,643,792?	7. From 79,762?
4. From 27,948,673?	8. From 279,876,372?
5. From 2,876?	9. From 25,986,137,953?
6. From 487,962?	10. From 542,763?

11. What is the greatest number of thousands that can be taken from each of the above numbers? of tens? of millions? * of billions? of hundred thousands? of ten millions?

12. Express the value of 457869 in hundreds and units.

Ans. 457869 equals 4578 hundreds and 69 units.

Express the value of each of the following in hundreds and units: —

13. 8,796,784.	17. 6,972.
14. 86,724.	18. 79,843.
15. 7,807,375.	19. 987,509,875.
16. 46,739,725,876.	20. 570,072,307,679.

* If any number is less than a million, as 2876, the answer may be, 2876 *is less than a million, and therefore no millions can be taken from it.*

21. Express the value of each of the above in tens and units. In ten-thousands and units.

22. Express as much of the value of each of the above as you can in ten-thousands; as much of the remainder as you can in hundreds, and the rest in units.

Model of Answer. 8796784 = 879 ten-thousands, 67 hundreds, and 84 units.

23. Express as much of the value of each of the above as you can in ten-millions, as much of the remainder as you can in thousands, and the rest in units.

19. *Comparison of the Values represented by the same Figure in different Places.*

(*a.*) Since, as we have seen, the figure in any place represents ten times the value it would represent if written one place farther towards the right, one hundred times the value it would represent if written two places farther towards the right, &c., it follows that it must represent one * tenth of the value it would represent if written one place farther towards the left, one one-* hundredth of the value it would represent if written two places farther towards the left, &c.

NOTE. — The expression "Numbers increase from right to left in a tenfold ratio," is not a true statement of the fact.

A *unit* of one decimal denomination always bears the same ratio to a *unit* of the next higher that 1 does to 10; but the ratio which the value of a *figure* of one decimal denomination bears to the value of a *figure* of the next higher is as 1 to 10 only when the figures are alike. For instance, in 22 the value of the left hand figure is ten times that of the right hand figure, while in 25 it is only four times, and in 91 it is ninety times.

Even when the figures are alike the ratio of *increase* is not *ten*fold. We increase a number by adding to it. To increase a number once, one addition must be made to it. To increase it twice, two additions must be made, &c. A number increased by once itself will produce twice it-

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* If the explanations on page 3d are not sufficient to enable the pupil to understand the meaning and use of the fractional expressions contained in this section, he should study the first part of the section on fractions before going farther.

self, increased by twice itself, will produce three times itself; increased by nine times itself, will produce ten times itself; increased by ten times itself, will produce eleven times itself, &c. Twenty is ten times two, or ten times as large as two, but is only nine times larger; for if we make two larger by nine times itself, that is, if we add nine twos to two, the result will be twenty, equal to ten times two.

1. Compare the values expressed by the 4's in the number 444.

Ans. The first 4 at the right represents one tenth of the value of the second 4, and one one-hundredth the value of the third 4; the second 4 represents ten times the value of the first or right hand, and one tenth of the value of the third or left hand 4; the third 4 expresses ten times the value of the second, and one hundred times that of the first.

2. Compare the values expressed by the 6's in 66; in 666; in 606; in 660.

3. Compare the values expressed by the 8's in 88; in 880; in 808; in 8888; in 8008.

4. Compare the values expressed by the 3's in 333; in 3030; in 3303; in 333333.

20. *Places at Right of Decimal Point.*

(*a.*) In conformity with the same principle, a figure written in the first place at the right of the point would represent tenths of units, or simply tenths; one written in the second place at the right would represent hundredths of units, or simply hundredths, &c.

(*b.*) Hence, the places at the right of the point have been named as indicated below.

Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
.	0	0	0	0	0	0	0	0	0
	1st place,	2d place,	3d place,	4th place,	5th place,	6th place,	7th place,	8th place,	9th place,

1. Which place from the point is occupied by the millionths figure?

Ans. The sixth place at the right.

Which place from the point is occupied by the

2. Tenths' figure?
3. Hundredths'?
4. Ten-thousandths'?
5. Millionths'?
6. Thousandths'?
7. Billionths'?
8. Hundred-thousandths'?
9. Hundred-millionths'?

10. What would be the denomination of a figure in the second place at the right of the point?

11. In the fifth?
12. In the sixth?
13. In the fourth?
14. In the seventh?
15. In the first?
16. In the ninth?
17. In the eighth?
18. In the third?

21. *Method of reading Numbers expressed by Figures at Right of Decimal Point.*

(*a.*) Numbers expressed by figures written at the right of the point are read on exactly the same principles as those expressed by figures at the left.

Thus, if we wish to read .37, we observe that the right hand figure is in the hundredths' place. We read it then as though written thirty-seven one hundredths, or $\frac{37}{100}$. The following will furnish further illustrations of the principle involved: —

1. $.86 = \frac{86}{100}$.
2. $.0086 = \frac{86}{10000}$.
3. $.0349 = \frac{349}{10000}$.
4. $3.49 = \frac{349}{100} = 3\frac{49}{100}$.
5. $6.07 = \frac{607}{100} = 6\frac{7}{100}$.
6. $10.1 = \frac{101}{10} = 10\frac{1}{10}$.
7. $30.03 = \frac{3003}{100} = 30\frac{3}{100}$.
8. $7.008706 = \frac{7008706}{1000000} = 7\frac{8706}{1000000}$.
9. $.086 = \frac{86}{1000}$.
10. $.349 = \frac{349}{1000}$.
11. $.000349 = \frac{349}{1000000}$.
12. $34.9 = \frac{349}{10} = 34\frac{9}{10}$.
13. $1.01 = \frac{101}{100} = 1\frac{1}{100}$.
14. $300.3 = \frac{3003}{10} = 300\frac{3}{10}$.
15. $3.003 = \frac{3003}{1000} = 3\frac{3}{1000}$.

(*b.*) Read the following, expressing the value in all cases where it is greater than unity, both as an improper fraction and as a mixed number: —

16.	.73	26.	50.05
17.	.08	27.	0.0764
18.	.798	28.	37.9427
19.	6.4	29.	876.5874
20.	7.0037	30.	3299.4856328
21.	4.287	31.	1000000.0000001
22.	.940094	32.	87.203794
23.	3.06006	33.	87.000000079
24.	40.7000407	34.	.000000007
25.	21.3304206	35.	2006.000002006

22. *Decimal Fractions, Method of writing them.*

(*a.*) Any fraction whose denominator is a power* of ten may be expressed by writing the numerator so that its right hand figure shall occupy the place of the same name with its denominator, and omitting the denominator. Fractions thus written are called *Decimal Fractions,* to distinguish them from Vulgar Fractions, or those whose numerator and denominator are both written.

NOTE. — The first fifteen examples of **21** are written both as decimal and vulgar fractions, while the last twenty are only written as decimal fractions.

The greatest liability to error in writing decimal fractions is in placing the point. The learner should bear this in mind, and take pains to guard against it, remembering that if the point is not correctly placed, each figure of the number will express a wrong value.

(*b.*) Write the following in the form of decimal fractions: —

1. Eight tenths.
2. Fifty-four hundredths.
3. Eight hundred and seventy-five thousandths.
4. Fifty-four thousandths.
5. Eight hundred and sixty-four ten-thousandths.
6. Six, and eighty-seven hundredths.
7. Six, and eighty-seven ten-thousandths.
8. Four hundred and thirty-seven tenths.

* For definition of the word *power*, see article **105**, (d.)

9. Three hundred and four thousands, and three hundred and four thousandths.

10. Three hundred and four thousands, and three hundred and four millionths.

(*c.*) Write the following in the form of decimal fractions: —

11. $\frac{7}{10}$.	21. $\frac{27}{100}$.	31. $\frac{5276}{10000}$.
12. $\frac{46}{100}$.	22. $\frac{7}{1000}$.	32. $\frac{301}{10000}$.
13. $\frac{2}{100}$.	23. $\frac{47}{10}$.	33. $\frac{5}{10000}$.
14. $\frac{369}{1000}$.	24. $\frac{247}{100}$.	34. $\frac{27}{100000}$.
15 $\frac{24}{1000}$.	25. $\frac{83}{10000}$.	35. $\frac{400678}{100000}$.
16. $\frac{8437}{1000}$.	26. $\frac{62948}{10000}$.	36. $\frac{32}{1000000}$.
17. $3\frac{7}{10}$.	27. $97\frac{5}{100}$.	37. $500\frac{5}{100}$.
18. $40\frac{4}{100}$.	28. $80\frac{8}{10}$.	38. $50\frac{5}{1000}$.
19. $24\frac{7}{10}$.	29. $8\frac{8}{100}$.	39. $5\frac{5}{10000}$.
20. $4\frac{28}{100}$.	30. $17\frac{17}{100}$.	40. $637\frac{637}{100000}$

41. $46948\frac{6249}{10000}$.	48. $185476\frac{213}{100000}$.
42. $\frac{234579863}{1000000}$.	49. $342\frac{342}{10000000}$.
43. $8270000\frac{827}{10000000}$.	50. $4006\frac{4006}{10000000}$.
44. $\frac{213794521}{100000}$.	51. $\frac{2948673}{1000}$.
45. $1\frac{1}{100000}$.	52. $1000000\frac{1}{1000000}$.
46. $6000\frac{6}{1000}$.	53. $487\frac{326894}{1000000}$.
47. $72000000000\frac{72}{10000000000}$.	54. $851\frac{3794}{10000}$.

23. *Exercises in determining Position of Point.*

1. Where must the point be placed in order that the figures 746 may represent tenths?

Ans. Between the 4 and 6; thus, 74.6

2. Where must the point be placed in order that the figures 746 may represent millionths?

Ans. Six places to the left of the 6; which requires that three places be filled with zeros; thus, .000746

3. Where must the point be placed in order that the figures 573 may express 573 tenths? hundredths? units? tens? hundreds? thousands? thousandths? millions? millionths?

4. Where must the point be placed in order that the figures 87064 may express 87064 hundreds? hundredths? ten-thousands? ten-thousandths? hundred-millions? hundred-millionths? tens? tenths?

5. Where must the point be placed in order that the figures 497837 may express 497837 units? millions? billionths? hundredths? millionths? hundreds? hundred-thousandths?

24. *Effect of changing Place of Point.*

MULTIPLICATION AND DIVISION BY POWERS OF 10.

(*a.*) Since (**8, 19**) each figure of a number represents 10 times the value which it would represent if written one place farther towards the right, 100 times the value it would represent if written two places farther towards the right, &c., and $\frac{1}{10}$ of the value it would represent if written one place farther towards the left, $\frac{1}{100}$ of the value it would represent if written two places farther, &c., it follows that removing the *figures representing a number* one place farther towards the *left*, or, which is the same thing, removing the *point* one place to the *right*, *multiplies* the number by 10, while removing the *figures* one place to the *right*, or the *point* one place to the *left*, *divides* it by 10. A change of two places would in like manner multiply or divide by 100, of three places by 1000, &c.*

These principles generalized would be stated thus:—

(*b.*) *To express the product of any number multiplied by any power of* 10, *remove the decimal point as many places to the right as there are zeros used in writing the given multiplier.*

(*c.*) *To express the quotient of a number divided by any power of* 10, *remove the decimal point as many places to the left as there are zeros used in writing the given divisor.*

(*d.*) *When, by such change, any places between the number and point are left vacant, they must be filled by zeros.*

NOTE.—The rule usually given for multiplying by the powers of 10, is to "annex as many zeros to the multiplicand as there are zeros in the multiplier." It is, however, a very defective one, as it only applies when the decimal point is not written, and then only multiplies the number by

changing the place to which we refer the point. It is the more objectionable, as it tends to convey the false idea that the zero is essential to the multiplication. As a matter of fact, annexing any figure whatever to a number will, if the decimal point is omitted, multiply it by 10, for it will change the place to which the decimal point is referred; but if the annexed figure is other than zero, its value will be added to the product of the number by 10. Thus, annexing 7 to 43 gives 437, which equals 10 times 43, plus 7.

The principle involved is this: that every change which is made in the position of figures with reference to the decimal point, — whether it is made by changing the position of the point, or by writing other figures between the given figures and the point, — alters the value they represent, by multiplying or dividing them by 10 or some power of ten. A figure can only alter the value expressed by other figures when it i written between them and the point.

(*e.*) How will you express in figures the results of the following indicated operations?

1.	87×10 *	14.	4279×1000
2.	$.87 \times 10$	15.	$.6307 \times 100$
3.	$.0087 \times 10$	16.	$.00694 \times 10$
4.	$87 \div 10$	17.	$5429 \div 100$
5.	$.087 \div 10$	18.	$400.794 \div 10$
6.	$870 \div 10$	19.	$.0054279 \div 10000$
7.	$5.7 \div 100$	20.	$.0004 \times 1000000$
8.	$.057 \div 100$	21.	$.002 \times 10000$
9.	$.3278 \times 100$	22.	348.796×100
10.	4.5786×1000	23.	2×1000000
11.	$.4 \div 10$	24.	$2 \div 1000000$
12.	479.643×1000	25.	$.006 \times 1000000000$
13.	$479.643 \div 1000$	26.	$.006 \div 1000000000$

(*f.*) Let the student now tell by inspection, without changing the place of the point or re-writing the numbers, the result of the above indicated operations.

* The student should remember that when the decimal point is not marked, it is always understood to belong at the right of the given figures.

Thus. *eighty-seven multiplied by ten equals eight hundred and seventy; eighty-seven hundredths multiplied by ten equals eighty-seven tenths, or eight and seven tenths,* &c. He should learn to do this without the slightest hesitation.

25 *Change of Denomination.*

(*a.*) The value of a number may be expressed in terms of any other decimal denomination as well as in units, by making the requisite change in the place of the point. Thus: —

847 = 84.7 tens, = 8.47 hundreds, = .847 of a thousand, = .0847 of a ten-thousand, &c.

847 = 8470 tenths, = 84700 hundredths, = 847000 thousandths, &c.

642.06 = 6.4206 hundreds, = 64206 hundredths, &c.

(*b.*) Express the value of each of the following in tenths, then in tens; in hundredths, then in hundreds; in millionths, and then in millions: —

1. 4327	4. 2700	7. 4683.7642
2. 82794.6	5. .048	8. .00006
3. .437	6. 2.7	9. 8000000

26. *French Method of Numeration.*

The foregoing method of numeration is called the *French method.* It divides the figures expressing a number into periods of three figures each, making a unit of one period equal to one thousand units of the next lower period.

Thus one million equals one thousand thousands; one billion equals one thousand millions; one trillion equals one thousand billions, &c.

27. *English Method of Numeration.*

(*a.*) There is another method, called the *English method,* which divides the figures expressing a number into periods of six figures each, making a unit of one period equal to one million units of the next lower period.

(*b.*) By this method one billion equals one million millions; one trillion equals one million billions, &c. This is illustrated in the following example: —

Quadrillions.	Hundreds of Thousands of Trillions.	Tens of Thousands of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Thousands of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9,	7	5	2	7	3	0,	6	6	5	8	9	7,	2	5	3	0	6	0,	9	3	4	5	7	8.
	4th, or Trillions' Period.						3d, or Billions' Period.						2d, or Millions' Period.						1st, or Units' Period.					

(*c.*) In this method, as in the French, we read the figures in each period as though they stood alone, calling afterwards the name of the period.

(*d.*) The above number would be read, 9 quadrillions, 752730 trillions, 665897 billions, 253060 millions, 934578.

28. *Comparison of French and English Methods.*

(*a.*) It will be readily seen that while the English periods bear the same name as the French, and while one thousand and one million represent the same number in the two systems, one billion, one trillion, or a unit of any higher denomination is much greater in the English system than in the French.

Thus, an English billion equals a French trillion; an English trillion equals a French quintillion.

(*b.*) The French method is the one generally used in this country and on the continent of Europe, and being much more convenient than the English, has been adopted in part in England, and is likely to come into general use there.

29. *Numbers to be read according to the English Method.*

1. 426,794798,764387.
2. 86432,795876,942759.
3. 237000,568975,006723.
4. 6,456327,309670,800659.
5. 325,897563,475003,900065,654003.

30. *The Roman Method.*

(*a.*) The Roman method of notation represents numbers by letters of the alphabet.

(*b.*) It is now chiefly used in numbering the sections or chapters of a book, the pages of a preface or introduction, the year of the Christian era, or when it is necessary to distinguish one class of numbers from another.

(*c.*) The letters used are the following, viz.: —

I, which stands for One.
V, which stands for Five.
X, which stands for Ten.
L, which stands for Fifty.
C, which stands for One Hundred.
D, which stands for Five Hundred.
M, which stands for One Thousand.

(*d.*) Other numbers are represented by repetitions and combinations of these letters.

(*e.*) When a letter is repeated, it indicates that the number it represents is to be repeated.

Thus: II. = two; III. = three; XX. = twenty; XXX. = thirty, &c.

(*f.*) If a letter expressing one number be placed before a letter expressing a larger number, the former is to be subtracted from the latter; but if the letter expressing the larger value be placed first, the values of the two are to be added together.

Thus: IV. = four; IX. = nine; XL. = forty, &c.; while VI. = six; XI. = eleven; LX. = sixty, &c.

(*g.*) In the following columns, the letters stand for the numbers written against them: —

I.	One.	VII.	Seven.
II.	Two.	VIII	Eight.
III.	Three.	IX.	Nine.
IV.	Four.	X.	Ten.
V	Five.	XI.	Eleven.
VI.	Six.	XII.	Twelve.

XIII. . Thirteen.
XIV. . Fourteen.
XV. . Fifteen.
XVI. . Sixteen.
XVII. . Seventeen.
XVIII. . Eighteen.
XIX. . Nineteen.
XX. . Twenty.
XXI. . Twenty-one.
XXII. . Twenty-two.
XXIII. . Twenty-three.
XXIV. . Twenty-four.
XXV. . Twenty-five.
XXVI. . Twenty-six.
XXVII. . Twenty-seven.
XXVIII. . Twenty-eight.
XXIX. . Twenty-nine.
XXX. . Thirty.
XXXI. . Thirty-one.
XL. . Forty.
XLI. . Forty-one.
XLIX. . Forty-nine.
L. . Fifty.
LX. . Sixty.
LXX. . Seventy.
LXXX. . Eighty.
XC. . Ninety.
XCIX. . Ninety-nine.
C. . One Hundred.
CL. . One Hundred and Fifty.
CLXXXVIII. . One Hundred and Eighty-eight.
CC. . Two Hundred.
CCC. . Three Hundred.
CD. . Four Hundred.
D. . Five Hundred.
DC. . Six Hundred.
DCC. . Seven Hundred.
DCCC. . Eight Hundred.
CM. . Nine Hundred.
M. . One Thousand.
MDCCCLIV. . 1854.

(*h.*) A dash placed over a letter makes it express thousands instead of ones. Thus, $\overline{\text{V}}$. = 5000; $\overline{\text{VI}}$. = 6000; $\overline{\text{L}}$. = 50,000; $\overline{\text{XC}}$. = 90,000, &c.

(*i.*) Read the following numbers: —

XCVIII.
DCCXLII.
MDCCLXXVI.
$\overline{\text{XX}}$DCCCXCIX.
$\overline{\text{CCXLVIII}}$CCCL.
$\overline{\text{DCLI}}$DLXXI.
$\overline{\text{DCCCXCIX}}$CCCXXXIII.

SECTION III.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

31. *Introductory.*

(*a.*) All nations, excepting perhaps the most barbarous, have some kind of money; but each nation has a system peculiar to itself, and generally differing from every other in its denominations, coins, &c. The people of the United States reckon money in dollars and cents, the English reckon it in pounds, shillings, and pence, and the French in francs and centimes.

(*b.*) So too each nation has a peculiar system of weights and measures, some of the most important of which are illustrated in the following tables.

32. *United States, or Federal Money.*

(*a.*) The money of the United States is called Federal Money.

TABLE OF FEDERAL MONEY.

10 mills	=	1 cent.
10 cents	=	1 dime.
10 dimes	=	1 dollar.
10 dollars	=	1 eagle.

(*b.*) The coins of the United States are the dollar, the half dollar or fifty-cent piece, the quarter dollar or twenty-five-cent piece, the dime or ten-cent piece, the half dime or five-cent piece, the three-cent piece, the cent; the eagle or ten dollar piece, the double eagle or twenty-dollar piece, the half eagle or five-dollar piece, and the quarter eagle, worth two and a half dollars.

(*c.*) The dollar is coined both of gold and silver; the coins worth more than a dollar are of gold, and the others, with the exception of the *cent, are* of silver. The cent is of copper.

(*d.*) Values in federal money are usually expressed in dollars and cents, or in dollars, cents, and mills, the dollar being regarded as the unit, the cent as one hundredth of it, and the mill as one tenth of the cent, or one thousandth of the dollar. Indeed, dimes, cents, and mills being nothing more than tenths, hundredths, and thousandths of a dollar, the figures representing them may be, and usually are, written in the form of a decimal fraction, and may with perfect propriety be read as such.

(*e.*) The character $, placed at the left of figures, shows that they stand for dollars.

Illustrations. (*a.*) $3.75 = * 3 dollars and 75 cents, = 3 dollars and $\frac{75}{100}$ of a dollar, = 3 dollars, 7 dimes, and 5 cents, &c.

(*b.*) $237.264 = * 237 dollars, 26 cents, and 4 mills, = 237 dollars and $\frac{237}{1000}$ of a dollar, = $\frac{237264}{1000}$ of a dollar, = 23726.4 cents, = 237264 mills, &c.

(*c.*) $7.042 = * 7 dollars, 4 cents, and 2 mills, = 7 dollars, 42 mills, = 7 dollars and $\frac{42}{1000}$ of a dollar, = $\frac{7042}{1000}$ of a dollar, = 7042 mills, = 704.2 cents, &c.

EXERCISES.

(*f.*) Read each of the following, giving the value 1st in dollars and cents, or, where there are mills, in dollars, cents, and mills; 2d, in dollars and decimal parts of a dollar; 3d, in cents; 4th, in mills:—

1.	$ 6.79	7.	$ .073	13.	$ 7084.79
2.	$ 8.03	8.	$ 30.07	14.	$ 400.06
3.	$ 764.37	9.	$ 2587.00	15.	$ .125
4.	$ 28.00	10.	$ 25.00	16.	$ 97.886
5.	$ 5.976	11.	$ 19.875	17.	$ 20.07
6.	$ .073	12.	$ .625	18.	$ .06

33. *English Money.*

(*a.*) The money used in England is called English, or Sterling Money.

* The form marked with a star represents the usual method of reading these values.

TABLE OF STERLING MONEY.

4 farthings	=	1 penny.
12 pence	=	1 shilling.
20 shillings	=	1 pound.

(*b.*) When numbers expressing values in sterling money are used, the character £ marks the pounds, s. the shillings, d. the pence, and qr. the farthings.

Illustration. 27 pounds, 18 shillings, 5 pence, and 3 farthings, may be written thus: 28£, 18 s. 5 d. 3 qrs.; or thus: $\overset{£}{27}\ \overset{s.}{18}\ \overset{d.}{5}\ \overset{qrs.}{3}$. Farthings are also frequently expressed as fourths of a penny. Thus, 7 d. 3 qrs. = $7\frac{3}{4}$d., and both forms may be read as 7 pence and 3 farthings.

(*c.*) The coin representing the pound is called the sovereign. Its value in United States money varies from four dollars and eighty-three cents to four dollars and eighty-six cents, but is usually about four dollars and eighty-four cents. There are several other coins used in England, of which we will mention only two, viz., the guinea, or twenty-one shilling piece, and the crown, or five-shilling piece.

34. *Avoirdupois Weight.*

(*a.*) Almost all articles, except gold, silver, and jewels, are weighed by what is called Avoirdupois Weight.

TABLE OF AVOIRDUPOIS WEIGHT.

16 drams	=	1 ounce.
16 ounces	=	1 pound.
25 pounds	=	1 quarter.
4 quarters	=	1 hundred weight.
20 hundred weight	=	1 ton.

(*b.*) The abbreviations made use of in this weight are T. for ton, cwt. for hundred weight, qr. for quarter, lb. for pound, oz. for ounce, and dr. for dram.

For example, 5 T. 17 cwt. 3 qr. 13 lb. 8 oz. 7 dr. = $\overset{T.}{5}\ \overset{cwt.}{17}\ \overset{qr.}{3}\ \overset{lb.}{13}\ \overset{oz.}{8}\ \overset{dr.}{7}$ = 5 tons, 17 hundred weight, 3 quarters, 13 pounds, 8 ounces, and 7 drams.

(*c.*) Formerly the quarter was reckoned at 28 pounds, the hundred

weight at 112 pounds, and the ton at 2240 pounds, and they are so reckoned at the present time in Great Britain as well as in the standard of the United States government. Most of the states of the Union have, however, passed laws fixing the values as in the table, and they are almost always so reckoned by merchants in buying and selling.

35. *Troy Weight.*

(*a.*) The weight used in weighing gold, silver, and precious stones is called Troy Weight. This weight is also used in philosophical experiments.

TABLE OF TROY WEIGHT.

24 grains	=	1 pennyweight.
20 pennyweights	=	1 ounce.
12 ounces	=	1 pound.

(*b.*) In this weight, lb. stands for pound, oz. for ounce, dwt. for pennyweight, and gr. for grain.

Illustration. 13 pounds, 7 ounces, 18 pennyweights, 23 grains, may be expressed thus: 13 lb. 7 oz. 18 dwt. 23 gr.; or thus: $\overset{\text{lb.}}{13}\ \overset{\text{oz.}}{7}\ \overset{\text{dwt.}}{18}\ \overset{\text{gr.}}{23}$.

(*c.*) The "carat," which equals four grains, is used in weighing diamonds. The term *carat* is also used in stating the fineness of gold, and means the twenty-fourth part of any weight of gold or gold alloy. Pure gold is "24 carats fine." Gold is 22 carats fine when $\frac{22}{24}$ of it is pure gold and $\frac{2}{24}$ is alloy.

36. *Apothecaries' Weight.*

(*a.*) The weight used in compounding or mixing medicines is called Apothecaries' Weight. Physicians write their prescriptions in this weight, but medicines are bought and sold by Avoirdupois Weight.

TABLE OF APOTHECARIES' WEIGHT.

20 grains	=	1 scruple.
3 scruples	=	1 dram.
8 drams	=	1 ounce.
12 ounces	=	1 pound.

To mark the denominations of this weight, we use the following characters, viz.: ℔ for pound, ℥ for ounce, ʒ for dram, ℈ for scruple, and gr. for grain.

For example: 5 ℔ 6 ℥ 4 ʒ 2 ℈ 17 grs., would be read as 5 pounds, 6 ounces, 4 drams, 2 scruples, and 17 grains.

37. *Comparison of Troy, Avoirdupois, and Apothecaries' Weights.*

(*a.*) The only difference between Troy Weight and Apothecaries' Weight is, that in the former the ounce is divided into pennyweights and grains, while in the latter it is divided into drams, scruples, and grains. The pound, ounce, and grain are the same in both weights.

(*b.*) The value of denominations of the same name in Avoirdupois' and Troy Weights differs very materially, as may be seen from the following table, which shows the value in Troy grains of each denomination we have given in the preceding tables of weights.

TABLE OF COMPARISON.

1 lb. Av.	=	7000	gr.	Troy.
1 lb. Tr. = 1 ℔	=	5760	"	"
1 oz. Av.	=	$437\frac{1}{2}$	"	"
1 oz. Tr. = 1 ℥	=	480	"	"
1 dr. Av.	=	$27\frac{11}{32}$	"	"
1 ʒ	=	60	"	"
1 ℈	=	20	"	"
1 dwt.	=	24	"	"
1 gr. Ap.	=	1	"	"

(*c.*) From the above table we should find by calculation that

$$144 \text{ lb. Av.} = 175 \text{ lb. Tr.},$$
$$\text{and } 192 \text{ oz. Av.} = 175 \text{ oz. Tr.}$$
$$\text{Therefore, } 1 \text{ lb. Av.} = \tfrac{175}{144} \text{ or } 1\tfrac{31}{144} \text{ lb. Tr.},$$
$$\text{and } 1 \text{ oz. Av.} = \tfrac{175}{192} \text{ oz. Tr.}$$

(*d.*) Which is the heavier, and why, —

1. A pound of gold or a pound of feathers?
2. A pound of lead or a pound of feathers?
3. A pound of gold or a pound of lead?

4. An ounce of gold or an ounce of feathers?
5. An ounce of lead or an ounce of feathers?
6. An ounce of lead or an ounce of gold?

38. *Long Measure.*

(*a.*) Distances in any direction are measured by Long Measure.

TABLE OF LONG MEASURE.

12	lines	=	1 inch.
12	inches	=	1 foot.
3	feet	=	1 yard.
$5\frac{1}{2}$ / $16\frac{1}{2}$	yards, or / feet	=	1 rod or pole
40	rods	=	1 furlong.
8	furlongs	=	1 mile.
3	miles	=	1 league.

(*b.*) In this measure, le. stands for league, m. for mile, fur. for furlong, rd. for rod, yd. for yard, ft. for foot, and in. for inches.

(*c.*) Surveyors usually measure distances by means of a chain 4 rods in length, called Gunter's chain, or the Surveyor's chain. This chain contains 100 equal links; 25 links will, therefore, equal 1 rod, and 1 link will equal $7\frac{23}{25}$ inches.

39. *Cloth Measure.*

(*a.*) This measure is used for measuring cloth, ribbons, &c.

TABLE OF CLOTH MEASURE.

$2\frac{1}{4}$	inches	=	1 nail.
4	nails	=	1 quarter.
4	quarters	=	1 yard.

(*b*) In this measure, yd. stands for yard, qr. for quarter, and na. for nail.

(*c.*) The yard and inch are the same in length as the yard and inch in Long Measure

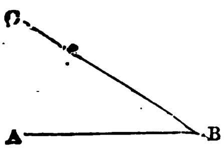
40. *Square Measure.*

(*a.*) Square Measure is used for measuring surfaces.

As true ideas of the nature of the right angle, rectangle, and square are essential to a just appreciation of this measure, we insert the following definitions and illustrations, leaving it with the teacher to give such others as he may deem necessary.

(*b.*) When two lines meet at a point, *their difference in direction* is called the *angle* of the two lines. The point where they meet is called the *vertex of the angle.*

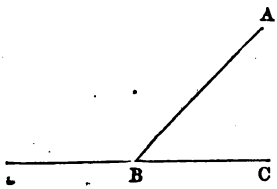
FIG. 1.



(*c.*) In this figure, the lines marked B A and B C form an angle whose vertex is at B.

In reading an angle, the letter at the vertex is always made the middle one. The angle in Fig. 1 may be read either as the angle A B C, or as the angle C B A.

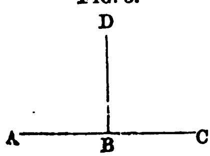
FIG. 2.



(*d.*) In this figure there are two angles, viz., D B A and A B C. The first is the difference in direction of the two lines A B and D B, and the second is the difference in direction of the two lines A B and B C. It is evident that the first angle is larger than the second.

(*e.*) When the two angles formed by one straight line meeting another are equal to each other, they are called *right angles,* and the two lines are said to be *perpendicular* to each other

FIG. 3.



(*f.*) Suppose that the straight line D B should so meet the straight line A C as to make the adjacent angles A B D and D B C equal to each other; then A B D and D B C will each of them be right angles, and D B and A C will be perpendicular to each other.

(*g.*) An angle greater than a right angle is called an *obtuse* (blunt) *angle,* and one less than a right angle is called an *acute* (sharp) *angle.*

In Fig. 2, D B A is an obtuse angle, and A B C is an acute angle. In Fig. 1, A B C is an acute angle.

(*h.*) A four-sided figure, having all its angles right angles, is called a *rectangle.*

(*i.*) A rectangle, having all its sides equal, is called a *square.* A square, then, has four equal sides and four equal angles.

Figs. 4 and 5 represent rectangles. Fig. 5 also represents a square.

FIG. 4.

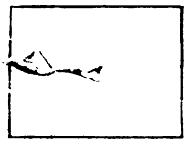
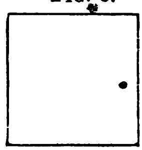


FIG. 5.



(*j.*) If a square measures a foot on each side, it is called a *square foot;* if it measures a yard on a side, it is called a *square yard,* &c. A *square unit,* or *superficial unit,* then, is any surface equivalent to a square 1 unit long and 1 unit wide.

(*k.*) All surfaces are measured by Square Measure, that is, by the number of squares of a given size to which they are equivalent. Thus, a surface contains 5 square feet when it is equivalent to 5 squares, each measuring 1 foot on a side.

TABLE OF SQUARE MEASURE.

144 square inches	=	1 square foot.
9 square feet	=	1 square yard.
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet,	=	1 square rod.
40 square rods	=	1 rood.
4 roods	=	1 acre.
640 acres	=	1 mile.

(*l.*) In this measure, Sq. M. stands for square mile, A. for acre, R. for rood, sq. rd. for square rod, sq. yd. for square yard, sq. ft. for square foot, and sq. in. for square inch.

NOTE.— Since a surface a unit long and a unit wide contains a square

unit, a surface two units long and one unit wide must contain two square units; a surface three units long and one unit wide must contain three square units; and generally, a surface one unit wide must contain as many square units as there are units in length.

A surface two units wide must contain twice as many square units as a surface one unit wide, i. e., twice as many as there are units in length; a surface three units wide must contain three times as many square units as a surface one unit wide, i. e., three times as many as there are units in length; and generally, any surface must contain as many square units as there are in the product obtained by multiplying its length by its breadth. The surfaces here spoken of are, in all cases, supposed to be rectangular ones.

41. *Cubic Measure.*

Cubic Measure is used in measuring solids.

(*a.*) A solid is a magnitude which has length, breadth, and thickness.

NOTE.—The term "solid," as used in mathematics, refers to space rather than to material substances.

(*b.*) A cube is a rectangular solid, whose length, breadth, and height are equal. It may also be defined as a solid which is bounded by six equal squares.

(*c.*) A cube 1 foot long, 1 foot wide, and 1 foot high would be a cubic foot. A cube 1 yard long, 1 yard wide, and 1 yard high would be a cubic yard. A cubic unit, then, is any solid equivalent to a cube 1 unit long, 1 unit wide, and 1 unit high.

(*d.*) The solid contents of bodies are measured by cubic measure, i. e., by the number of cubes of a given size which the bodies contain, or to which they are equivalent.

TABLE OF CUBIC MEASURE.

1728 cubic inches	=	1 cubic foot.
27 cubic feet	=	1 cubic yard.
16 cubic feet	=	1 cord foot.
8 cord feet, or 128 cubic feet,	=	1 cord of wood.

(*e.*) In this measure, C. stands for cord, Cd. ft. for cord foot, cu. ft. for cubic foot, cu. yd. for cubic yard, and cu. in. for cubic inch.

NOTE.— Since a solid one unit long, one unit wide, and one unit high contains a cubic unit, a solid one unit wide, one unit high, and two units long must contain two cubic units; one three units long must contain three cubic units; one four units long must contain four cubic units; and generally, a solid one unit wide and one unit high must contain as many cubic units as there are linear units in its length.

Again. Since a solid two units wide must contain twice as many cubic units as a solid one unit wide, and a solid three units wide must contain three times as many as a solid one unit wide, &c., it follows that a solid one unit high must contain as many cubic units as there are in the product of its length by its breadth, i. e., as many cubic units as there are superficial units in its base.

Again. Since a solid two units high contains twice as many cubic units as a solid one unit high, and a solid three units high contains three times as many cubic units as a solid one unit high, it follows that any solid must contain as many cubic units as there are in the product obtained by multiplying the number of superficial units in its base by the number of linear units in its height, i. e., as many cubic units as there are in the product of its length multiplied by its breadth, multiplied by its height.

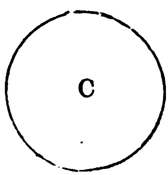
42. *Circular or Angular Measure.*

(*a.*) Circular or Angular Measure is used to measure angles, and the circumferences of circles.

(*b.*) A circle is a surface bounded by a curved line which is every where equally distant from a point within called the *centre.* The boundary line is called the *circumference* of the circle.

Fig. 6 represents a circle of which C is the centre.

FIG. 6.



(*c.*) The distance from the centre of a circle to the circumference is called the *radius* of the circle.

(*d.*) The distance from a point on one side of a circle through the centre to a point on the opposite side is called the *diameter* of the circle.

(*e.*) Any portion of the circumference is called an *arc.*

(*f.*) Every circumference of a circle, whether the circle be large or small, is supposed to be divided into 360 equal parts

called degrees. Each degree is divided into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds.

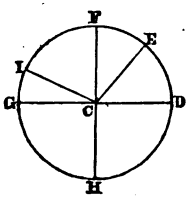
(*g.*) A degree is to be regarded simply as the 360th part of the circumference of the circle considered. Hence, it is obvious that its length, and that of its subdivisions, must vary with the size of the circle.

(*h.*) If from the vertex of an angle, as a centre, we should draw a circumference, some portion of that circumference would be included between the sides of the angle. The larger the angle is, the larger will be the arc included between its sides, and the smaller the angle is, the smaller will be the included arc.

(*i.*) Since the angle and the arc thus vary with each other, the arc is taken as the measure of the angle. If the arc contains 50 degrees, the angle is one of 50 degrees, &c.

Fig. 7 illustrates this.

FIG. 7.



(*k.*) In this figure let C be the centre of the circle and the vertex of the several angles, and let F H and G D be lines perpendicular to each other, and C E and C I be lines drawn at random. Then the arc E D is the measure of the angle E C D; the arc D F, of the angle D C F; the arc D I, of the angle D C I; the arc E F, of the angle E C F, &c.

(*l.*) Since F H and G D are perpendicular to each other, the angles F C D, D C H, H C G, and G C F are all right angles, and each of them must include $\frac{1}{4}$ of the angular space about the point C. Therefore the arc included between the sides of a right angle equals $\frac{1}{4}$ of the circumference, and the measure of a right angle is $\frac{1}{4}$ of 360 degrees, which equals 90 degrees. The measure of an acute angle is less than 90 degrees, and that of an obtuse angle more than 90 degrees.

TABLE OF CIRCULAR OR ANGULAR MEASURE.

60 seconds = 1 minute.
60 minutes = 1 degree.
360 degrees = 1 circumference.

(*m.*) Degrees are marked by the character °, minutes by ′,

seconds by ″: thus, 13° 27′ 49″ = 13 degrees, 27 minutes, and 49 seconds.

An arc of 90° is called a *quadrant.*

A *sign* is an astronomical measure of 30°.

43. *Dry Measure.*

(*a.*) Dry Measure is used for measuring all kinds of grain, beans, nuts, salt, &c.

TABLE OF DRY MEASURE.

2 pints	=	1 quart.
8 quarts	=	1 peck.
4 pecks	=	1 bushel.

(*b.*) The chaldron of 36 bushels is sometimes used in measuring coals. ch. stands for chaldron, bu. for bushel, pk. for peck, qt. for quart, and pt. for pint.

(*c.*) The bushel contains 2150$\frac{2}{5}$ cubic inches. The quart must, therefore, contain 67$\frac{1}{5}$ cubic inches.

44. *Liquid Measure.*

(*a.*) All kinds of liquids are measured by Liquid Measure.

LIQUID MEASURE.

4 gills	=	1 pint.
2 pints	=	1 quart.
4 quarts	=	1 gallon.

(*b.*) In this measure gal. stands for gallon, qt. for quart, pt. for pint, and gi. for gill.

(*c.*) The hogshead of 63 gallons is used in estimating the contents of reservoirs or other large bodies of water; but in all other cases, the term *hogshead* is not a definite measure. Casks containing from 50 or 60 to 100 or 200 gallons are called hogsheads.

(*d.*) A barrel of *cider* is usually reckoned at 31$\frac{1}{2}$ gallons.

(*e.*) The gallon contains 231 cubic inches.

(*f.*) The *beer gallon* is sometimes used for measuring milk, beer, and ale. It contains 282 cubic inches. The beer quart must therefore contain $70\frac{1}{2}$ cubic inches.

45. *Comparison of Dry, Liquid, and Beer Measures.*

The following table will be convenient for use in comparing Dry, Liquid, and Beer Measures: —

1 qt. Dry Measure	$= 67\frac{1}{5}$ cubic inches.
1 qt. Liquid Measure	$= 57\frac{3}{4}$ cubic inches.
1 qt. Beer Measure	$= 70\frac{1}{2}$ cubic inches.

46. TABLE OF TIME.

60 seconds	= 1 minute.
60 minutes	= 1 hour.
24 hours	= 1 day.
7 days	= 1 week.
$365\frac{1}{4}$ days, or 52 weeks and $1\frac{1}{4}$ days	= 1 year.

(*a.*) The year is divided into 12 months, which differ somewhat in length. In this measure yr. stands for year, mo. for month, wk. for week, da. for day, h. for hour, m. for minute, and sec. for second.

(*b.*) To avoid the inconvenience of reckoning $\frac{1}{4}$ of a day with each year, every fourth year (called *leap year*) is reckoned at 366 days, and the others are reckoned at 365 days. A leap year may always be known by this, viz.: Its number can be divided by 4. Thus we know that 1852 is a leap year, because 1852 can be divided by 4 without a remainder.

(*c.*) The year in reality contains but 365 days, 5 h. 48 m. 48 sec.; so that by reckoning $365\frac{1}{4}$ days we make a slight error each year, which in 100 years amounts to about 1 day. The centennial years are not, therefore, reckoned as leap years, unless the number of the year be divisible by 400. Thus the year 1900 will not be a leap year; but the year 2000 will be.

TABLE OF THE MONTHS.

January has 31 days.	July has 31 days.
February * has 28 days.	August has 31 days.
March has 31 days.	September has 30 days.
April has 30 days.	October has 31 days.
May has 31 days.	November has 30 days.
June has 30 days.	December has 31 days.

47. *Miscellaneous.*

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

A barrel of beef or pork weighs 200 lbs.
A barrel of flour weighs 196 lbs.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.

BOOKS.

A sheet	folded	into	2	leaves	is	called	a	folio.
" "	"	"	4	"	"	"	"	quarto.
" "	"	"	8	"	"	"	"	octavo.
" "	"	"	12	"	"	"	"	duodecimo or 12mo.
" "	"	"	18	"	"	"	"	18mo.

48. *French Measures and Weights.*

(*a.*) The following measures and weights are often referred to in this country, especially in scientific works.

FRENCH LONG MEASURE.

10 millimetres = 1 centimetre.
10 centimetres = 1 decimetre.
10 decimetres = 1 metre.
10 metres = 1 decametre.
10 decametres = 1 hectometre.

* In leap year February has 29 days.

10 hectometres = 1 kilometre.
10 kilometres = 1 myriametre.

The metre is regarded as the unit of measure, and equals 39.371 of our inches. It is the twenty-millionth part of the distance measured on a meridian, from one pole to the other.

(*b.*) FRENCH WEIGHTS.

10 milligrammes = 1 centigramme.
10 centigrammes = 1 decigramme.
10 decigrammes = 1 gramme.
10 grammes = 1 decagramme.
10 decagrammes = 1 hectogramme.
10 hectogrammes = 1 kilogramme.
10 kilogrammes = 1 myriagramme.

The gramme is regarded as the unit of this weight, and equals 15.434 Troy grains.

The kilogramme is the weight most frequently used in business transactions, and equals 15434 Troy grains, or very nearly 2.204857 pounds Avoirdupois.

(*c.*) FRENCH MONEY.

10 centimes = 1 decime.
10 decimes = 1 franc.

The franc = $.186; hence, the five-franc piece, often seen in the United States, is equal in value to 93 cents.

SECTION IV.

ADDITION.

49. *Definitions, Illustrations, and Explanations.*

(*a.*) ADDITION IS THE PROCESS BY WHICH, HAVING SEVERAL NUMBERS GIVEN, WE FIND A NUMBER EQUAL IN VALUE TO ALL OF THEM.

(*b.*) The number thus obtained is called the SUM OR AMOUNT.

Thus: How many are $7 + 3 + 5$, would be a problem in addition, and the answer, 15, would be the sum of 7, 3, and 5.

(*c.*) In order that numbers may be added, it is necessary that the things they represent shall be of the same name or denomination.

Illustrations.—2 books and 3 slates would be neither 5 books nor 5 slates; but since books and slates are both things, we can change the denomination of both by calling them things; when we shall have, 2 things and 3 things are 5 things.

In like manner 2 tens and 3 units would be neither 5 tens nor 5 units; but by reducing the tens to units, calling them 20 units, we shall have 2 tens $+$ 3 units $=$ 20 units $+$ 3 units $=$ 23 units.

2 shillings $+$ 3 pence are neither 5 shillings nor 5 pence; but since 2 shillings $=$ 24 pence, 2 shillings $+$ 3 pence must equal 24 pence $+$ 3 pence, or 27 pence.

(*d.*) For such reasons it will be found convenient, in writing large numbers for addition, to write those of the same denomination near each other. This can best be done by writing them in vertical columns, so that units shall come under units, tens under tens, &c., and pounds under pounds, shillings under shillings, pence under pence, &c.

(*e.*) In adding, we can begin with any denomination we choose; but it will usually be more convenient to begin with the lowest, or the one at the right hand, and to reduce the sum of each column to a higher denomination, when it can be done.

NOTE.—Addition is the most important of the four numerical operations, both because it is the foundation of all the others, and because it is the one most frequently used in all the departments of practical life. Moreover, it is the one in which there is the greatest liability to error. For these reasons, and many others which might be urged, the student should be very careful to master it fully.

(*f.*) The methods of applying these principles are illustrated in the following examples and solutions.

50. *Simple Addition.*

(*a.*) Abstract numbers, or concrete numbers, which represent values in terms of a single denomination, as *in pounds*, *in bushels*, or *in dollars*, are called SIMPLE NUMBERS; but *concrete* numbers, which represent values in terms of several

different denominations, as *in pounds, shillings, and pence,* or *in bushels, pecks, and quarts,* are called COMPOUND NUMBERS.

(*b.*) SIMPLE ADDITION is the addition of *simple numbers.*

What is the sum of 75798 + 24687 + 39764 + 86328 + 4395 + 283 + 86536?

Solution. — We first write the numbers, placing units under units, tens under tens, &c., in order that figures expressing the same denominations may be near together Thus: —

$$\begin{array}{r} 75798. \\ 24687. \\ 39764. \\ 86328. \\ 4395. \\ 283. \\ 86536. \\ \hline 317{,}791. \end{array}$$

* Beginning at the bottom of the units column, (because the lowest denomination mentioned is units,) and naming only results, we add thus: 6, 9, 14, 22, 26, 33, 41 units, which are equal to 4 tens and 1 unit.

Writing 1 as the units' figure of the amount, we add the 4 tens with the figures of the tens column; thus, 4, 7, 15, 24, 26, 32, 40, 49 tens, which are equal to 4 hundreds and 9 tens.

Writing 9 as the tens' figure of the amount, we add the 4 hundreds with the figures of the hundreds column, thus; 4, 9, 11, 14, 17, 24, 30, 37 hundreds, which are equal to 3 thousands and 7 hundreds.

Writing 7 as the hundreds' figure of the amount, we add the 3 thousands with the figures of the thousands column; thus, 3, 9, 13, 19, 28, 32, 37 thousands, which are equal to 3 ten-thousands and 7 thousands.

Writing 7 as the thousands' figure of the amount, we add 3 ten-thousands with the figure of the ten-thousands column; thus, 3, 11, 19, 22, 24, 31 ten-thousands, which are equal to 3 hundreds thousands and 1 ten-thousand; and as there are no higher denominations, we write the 3 and 1 in their appropriate places.

Having thus added all the denominations, we must have the sum, or amount of the numbers, which is 317,791.

NOTE. — Many call the names of the separate numbers added, as well as the results of the addition, and would add, thus: 6 and 3

* If the learner does not readily understand this method of addition, let him for a time *call the separate* numbers added, as explained in the note.

are 9, and 5 are 14, and 8 are 22, and 4 are 26, and 7 are 33, and 8 are 41; 41 units are equal, &c. This method is much less expeditious than the first one given, and therefore should not be adopted. Indeed, we may say, that calling the names of the separate numbers comprising a sum is to addition what spelling, i. e., calling the letters composing a word, is to reading.

51. *Compound Addition.*

(*a*.) Compound Addition is the addition of compound numbers.

What is the sum of £8 15 s. 11 d. 2 qr. + £3 12 s. 8 d. 3 qr. + £9 19 s. 9 d. 1 qr. + £7 18 s. 10 d. 3 qr. + £5 18 s. 2 qr. + £6 8 d. 1 qr. + £5 13 s. 3 d. + 16 s. 8 d. 1 qr.?

Solution.—We first write the numbers, placing pounds under pounds, shillings under shillings, &c., in order that figures expressing the same denomination may be near each other.

£	s.	d.	qr.	
8	15	11	2	
3	12	8	3	
9	19	9	1	
7	18	10	3	
5	18	0	2	
6	0	8	1	
5	13	3	0	
	16	8	1	
48	16	0	1	= Amount.

Beginning with the right hand column, as before, and reducing as we add, we proceed thus: 1 qr. and 1 qr. are 2 qr., and 2 qr. are 4 qr. = 1 d., and 3 qr. are 1 d. 3 qr., and 1 qr. are 1 d. 4 qr. = 2 d., and 3 qr. are 2 d. 3 qr., and 2 qr. are 2 d. 5 qr. = 3 d. 1 qr.

Writing 1 as the farthings' figure of the sum, we add the 3 d. with the numbers in the pence column, thus: 3 d. and 8 d. are 11 d., and 3 d. are 14 d. = 1 s. 2 d.,* and 8 d. are 1 s. 10 d., and 10 d. are 1 s. 20 d. = 2 s. 8 d.,* and 9 d. are 2 s. 17 d. = 3 s. 5 d.,* and 8 d. are 3 s. 13 d = 4 s. 1 d.,* and 11 d. are 4 s. 12 d.* = 5 s.

Writing 0 as the pence figure of the sum, we add the 5 s. with the numbers in the shillings column, thus: 5 s. and 16 s. are 21 s. = £1

* Since 12 d. = 1 s

1 s.,* and 13 s = £1 14 s., and 18 s. are £1 32 s.* = £2 12 s., and 18 s. are £2 30 s. = £3 10 s.,* and 19 s. are £3 29 s. = £4 9 s.,* and 12 s are £4 21 s. = £5 1 s.,* and 15 s. are £5 16 s.

Writing 16 as the shillings' figure of the sum, we add £5 with the numbers of the pounds column, thus: 5, 10, 16, 21, 28, 37, 40, 48.

As all the denominations of the given numbers have been added, the amount sought must be £48 16 s. 0 d. 1 qr.

(*b.*) In the above form of solution, the numbers to be added have been named merely to insure that the explanations should be understood, but in practical work the addition should be performed by naming only results.

Thus: 1 qr., 2 qr., 4 qr. = 1 d.; 1 d. 3 qr., 1 d. 4 qr. = 2 d.; 2 d. 3 qr., 2 d. 5 qr = 3 d. 1 qr. Write 1 qr. 3 d., 11 d., 14 d. = 1 s. 2 d.; 1 s. 10 d., 1 s. 20 d. = 2 s. 8 d.; 2 s. 17 d. = 3 s. 5 d., &c.

52. *Compound and Simple Addition compared.*

(*a.*) Compound Addition involves precisely the same principles that Simple Addition does. In both, numbers of the same denomination are placed under each other, in order that they may be more readily distinguished. In both, we commence to add at the lowest denomination, in order to avoid the necessity of erasing or altering figures which have been once written; in both, we reduce the sum of each column to units of the next higher denomination, in order that the answer may appear in its simplest form; and in both we add the units thus obtained with those written in the column of the next higher denomination. Moreover, the same methods of proof apply to both.

(*b*) The slight differences in the methods of applying these principles result from the fact, that in simple numbers 10 units of one denomination always equal one of the next higher, while in compound numbers there is no uniformity in this respect.

NOTE TO THE TEACHER. — The explanations and examples are so arranged that, should the teacher think it inexpedient to teach Compound Addition at the same time that Simple Addition is taught, he can

* Since 20 s. = £1.

defer it till in his opinion, the class are prepared for it. We would, however, recommend that whenever it is taught, it should be presented as a further application of the principles involved in Simple Addition.

The same thing may be said of Simple and Compound Subtraction, Multiplication, and Division.

53. *Methods of Proof.*

(*a.*) We can test the correctness of the work in many ways, a few of which we will mention.

First Method. — Go over the work carefully a second time in the same manner as at first.

Second Method. — Begin to add at a different part of the column from that at which the first addition was commenced; i. e., if the first addition was commenced at the top, begin the second at the bottom, and *vice versa.* This, by presenting the figures in a different order, renders it improbable that any mistake which may have been made in the first work will be repeated.

Third Method. — Separate the numbers to be added into two or more parts, add the parts separately, and then add their sums. This method is illustrated below.

PROOF OF EXAMPLE IN **50** BY THIRD METHOD.

$$\left.\begin{array}{r}75798\\24687\\39764\\86328\end{array}\right\}\text{1st part.}$$

$$\left.\begin{array}{r}4395\\283\\86536\end{array}\right\}\text{2d part.}$$

317791 = First Answer.

a = 226577 = Sum of first part.
b = 91214 = " " second part.
317791 = Sum of the two partial sums = first answer.
Thus showing that the work was correct.

PROOF OF EXAMPLE IN 51 BY THIRD METHOD.

£.	s.	d.	qr.	
8	15	11	2	1st part.
3	12	8	3	
9	19	9	1	
7	18	10	3	
5	18	0	2	2d part.
5	0	8	1	
5	13	3	0	
	16	8	1	
48	16	0	1	= First Answer.
30	7	4	1	= Sum of 1st part.
18	8	8	0	= Sum of 2d part.

48 16 0 1 = Sum of partial sums = first answer obtained. Thus showing the work to be correct.

NOTE. — When long columns are to be added, it may sometimes be convenient to divide them in this way in performing the first addition. The student should, however, accustom himself to adding the longest columns without any separation into parts.

Fourth Method. — Beginning either at the right or at the left hand to add, write the sum of each denomination separately, and then add these sums together.

Fifth Method. — Begin with the left hand column, and proceed as follows: —

PROOF OF THE EXAMPLE IN 50.

By adding the ten-thousands column we find that its sum is 28; but as there are 31 ten-thousands in the answer first obtained, we infer that 3 ten-thousands were brought from the lower denominations. 3 ten-thousands = 30 thousands, which, added to the 7 written in the thousands' place of the answer, gives 37 thousands to be accounted for. The sum of the thousands column is 34, which, taken from 37, leaves 3; thus showing, that if the work is correct, 3 thousands must have been brought from the lower denominations. 3 thousands = 30 hundreds, which, added to the 7 written in the hundreds' place of the answer

gives 37 hundreds to be accounted for. The sum of the hundreds column is 33, which, taken from 37, leaves 4; thus showing, that if the work is correct, 4 hundreds must have been brought from the lower denomination. 4 hundreds = 40 tens, which, added to the 9 tens written in the tens' place of the answer, gives 49 tens to be accounted for The sum of the tens column is 45, which, taken from 49, leaves 4; thus showing, that if the work is correct, 4 tens must have been brought from the units column. 4 tens = 40 units, which, added to the 1 unit written in the units' place of the answer, gives 41 units to be accounted for.

As the sum of the units column is 41, we infer that the work is correct.

PROOF OF THE EXAMPLE IN **51.**

By adding the pounds column, we find its sum is £43, which, taken from the £48 written in the answer, leaves £5; thus showing, that if the answer is correct, £5 must have been brought from the lower denominations. £5 = 100 s., which, added to the 16 s. written in the answer, gives 116 s. to be accounted for. The sum of the shillings column is 111 s., which, subtracted from the 116 s., leaves 5 s.; thus showing, that if the answer is correct, 5 s. must have been brought from the lower denominations. 5 s. = 60 d., which, as there are no pence written in the answer, gives 60 d. to be accounted for.

The sum of the pence column is 57 d., which, taken from 60 d., leaves 3 d.; thus showing, that if the answer is correct, 3 d. must have been brought from the column of farthings; 3 d. = 12 qr., which, added to the 1 qr. written in the answer, gives 13 qr. to be accounted for.

As the sum of the farthings column is 13, we infer that the answer is correct.

(*b.*) The first, second, and third methods of proof are the most practical, but as the fourth and fifth furnish valuable illustrations of the nature of the various changes and reductions, and call the reasoning faculties into healthful exercise, they should not be omitted by the student.

(*c.*) If, by any of these methods, we obtain a different result from the one we first obtained, we may be sure there is an error in one or both operations, and should examine both carefully to find it.

(*d.*) Some method of proof should always be resorted to, until the pupil acquires sufficient skill to be sure of the accuracy of his work without it.

54. *Importance of Accuracy and Certainty.*

(*a.*) No person who is willing to allow an error to pass undetected can be a good arithmetician. *Accuracy, absolute accuracy*, should be aimed at in every operation; and no labor is too great which is necessary to secure it. Not only should the results be accurate, but the *computer should know for himself that they are so.* If he has any doubt concerning a result, he should examine each and every step of his work, to see,

First. *That it was a proper one to take.*

Second. *That it was taken at the right time.*

Third. *That it was taken correctly.*

(*b.*) One problem thus solved and proved by a learner is of more real value to him than ten solved by him and proved by another, or tested by comparison with a printed answer. The accountant does not hesitate to spend hours, and even days, in looking over long and complicated accounts, to discover the cause of an error of a few cents in a trial balance sheet,* and surely the student ought not to shrink from the task of proving the correctness of his solutions of the much more simple problems contained in a school text book.

(c.) Rapidity in the performance of numerical operations is scarcely of secondary importance to accuracy and certainty. The most accurate computers are usually the most rapid in their work.

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* The trial balance sheet is used in keeping books by double entry, as a means of determining whether any errors exist in the entries which have been made in some given time, as a month, a quarter, (i. e., three months,) six months, or a year. By its aid, the *existence* of an error may be ascertained; but the error itself cannot be discovered without examining the separate entries and accounts.

An intelligent and highly accomplished accountant, who has charge of the books of a large manufacturing establishment, employing three hundred men, once spent nearly a week in examining his accounts, to discover the cause of an error of a few cents; and said he, "I never spent the same amount of time more profitably." Another gentleman, bearing also a high reputation, and receiving a good salary as an accountant, spent, to use his own language, "the greater part of four days in searching out the cause of an error of ten cents." Both these gentlemen say, that if they should adopt any other principle than that of *absolute accuracy*, they could not retain their situations. Every accountant, business man, and practical man bears similar testimony, and confirms these views. Indeed, most of them say, that the knowledge of arithmetic acquired in the school room has been of little practical value to them, because they did not learn to be accurate and rapid in performing their work, and to know for themselves that they had been accurate.

5

55. *Problems for Solution.*

Add the numbers in the following examples : —

1.	2.	3.
476392	832547	76486795
584273	482938	37639487
143253	548276	15327943
624415	398254	82753428

4.	5.	6.
206805795	5786.73	379.46
740376525	2974.37	82.375
814256324	8362.52	986.3
234567890	4593.67	59.486
325462813	5876.79	576.829
453269983	2394.16	403.586

7.	8.	9.
4273.75	43.057	786.73
28.9873	2.6875	2.7598
5794.0000	138.2654	.58678
38.5946	53.4867	6.82594
3954.2765	867.9583	36.95006
5.7986	15.8787	.00487
.385	587.483	30.2857
59.8678	865.9486	5.7642

10.	11.	12.
8732.175	8.7037	49.0064
6243.262	2.9675	2.206
3711.547	36.08972	69.0425
2952.364	5.86102	87.63214
428.249	2.407967	83.8006
1497.168	82.27313	70.3728
6557.438	25.7529	74.857632
4386.295	4.8063	428.4269

13. What is the sum of 679487 + 386754 + 329687 + 435429 + 276834 + 579487 ?

14. What is the sum of 324067 + 235143 + 543345 + 425341 + 876583 + 947869 ?

15 What is the sum of \$473.87 + \$526.94 + \$857.93 + \$297.16 + \$87.43 + \$528.60 + \$35.29 ?

16. What is the sum of 857436.57 + 25986.483 + 295463.867 + 297484.253 + 80672.005 ?

17. What is the sum of 258.647943 + 547.685329 + 27.84372 + 9765.4837 + 736.852066 + 542.063794 ?

18. What is the sum of 984137.612 + 257.00684 + 43687.5792 + 574869.23757 + 2068439.14238 + 1748.2 + 13.37 ?

19. What is the sum of 1864 + 437.29 + 58.697 + 12.86 + 7527.385 + 167.97 + 848.396 + 4.584 ?

20. What is the sum of 389.40067 + 2768.4372 + 5894.276 + 1385.7281 ?

21. What is the sum of 728 + 436 + 549 + 278 + 367 + 825 ?

22. What is the sum of 426764572681 + 894737629437 + 179428630006 + 576428670639 + 584967245876 ?

23. What is the sum of 3798643 + 5978642 + 5489379 + 675986 + 3768543 + 27864 + 3798742 + 8957387 + 9583796 + 8395989 + 3865372 ?

24. What is the sum of 83679 + 54873 + 72352 + 95873 + 8756 + 35906 + 87506 + 29764 + 38756 + 35742 ?

25. What is the sum of 57386 + 2864.3 + 379.86 + 28.697 + 5.4738 + .97986 + 7.5983 + 86.794 + 886.79 + 2937.6 + 70003 + 9764.2 + 859.86 + 48.375 ?

26. What is the sum of \$8.69 + \$13.48 + \$4.48 + \$8 64 + \$37.15 + \$47.13 + \$.86 + \$.25 + \$9.37 + \$6.08 + \$3.54 + \$7.06 + \$2.37 + \$4.68 + \$20.08 + \$7.57 + \$7.48 ?

27. What is the sum of \$4.175 + \$3.867 + \$5.384 + \$9.375 + \$5.78 + \$8.378 + \$2.635 + \$.875 + \$1.25 +

$4.28 + $3.19 + $8.625 + $5.846 + $9.738 + $5.96 + $7.50 + $3.25 ?

28. What is the sum of $.27 + $.63 + $1.04 + $.50 + $.375 + $1.50 + $.07 + $.42 + $.625 + $.375 + $3.27 + $5.94 + $.86 + $1.83 + $.06 + $.40 + $.125 + $1.33 ?

29. What is the sum of $85.76 + $77.25 + $.86 + $34.50 + $7.38 + $50.50 + $7.13 + $.47 + $.68 + $28.17 + $29.50 + $8.07 + $5.00 + $17.84 + $.03 + $5.28 ?

30. What is the sum of 58694 + 67867.9432 + 45879.8376 + 28697.4 + 38679.58432 + 27598.542 + 36789.754 + 58767.5437 + 86427.58697 + 98003.79 + 28547.3298 + 28475.9767 ?

31. What is the sum of 958679.4437 + 298673.925 + 586732 + 9678.4593 + 486.7923 + 5878.6532 + 185.3 + 28.6734 + 86.79635 + 28.76 + 59.836 + 45173.425 ?

32.

£.	s.	d.	qr.
17	13	11	1
18	15	8	3
29	19	6	1
47	8	10	3
25	13	11	2
87	18	9	3

33.

£.	s.	d.	qr.
164	13	8	1
231	0	6	3
485	19	11	2
738	18	2	3
487	16	10	0
833	19	11	3

34.

T.	cwt.	qr.	lb.	oz.
13	18	2	23	14
23	19	1	24	15
6	8	0	17	3
24	16	1	24	12
3	19	0	20	9
8	13	3	22	13

35.

T	cwt.	qr.	lb.	oz.	dr.
8	4	3	7	5	9
2	18	1	24	15	15
9	19	3	24	15	15
7	13	0	20	11	14
6	17	3	15	10	13
7	14	1	22	11	12

36.

lb.	oz.	dwt.	gr.
4	6	18	23
2	11	13	5
1	7	3	21
	8	16	20
1	10	9	17
5	9	18	20

37.

lb.	oz.	dwt.	gr.
5	9	19	21
2	11	13	23
1	8	6	12
2	5	9	6
3	10	19	19
8	7	17	22

38.

℔	℥	ʒ	℈	gr.
6	8	5	2	16
9	8	4	1	17
3	9	2	2	8
7	3	7	1	19
4	6	6	2	13
9	8	3	2	15

39.*

m.	fur.	rd.	yd.	ft.	in.
7	6	37	4	2	8
9	4	24	2	1	2
6	7	34	1	2	3
8	5	28	1	1	7
9	3	37	2	2	8
7	4	19	1	1	6

40.†

m.	fur.	rd.	yd.	ft.	in.
8	6	34	4	2	7
7	2	38	3	1	11
4	5	12	5	2	4
2	7	26	4	1	10
9	3	32	5	0	9
8	5	13	3	2	6

41.‡

rd.	yd.	ft.	in.
8	2	1	11
3	5	2	6
4	3	1	9
7	2	0	8
3	4	2	10
7	2	2	3

* In adding yards, it will usually be well to consider every eleven yards as two rods. Such a course will, to a very great extent, avoid the use of fractions. The pupil should, however, bear in mind that half of a yard = 1 ft. 6 in., and that when in any number there are 5 yards, and 1 ft. 6 in. besides, the value may be better expressed as 1 rod.

† Obtaining the answer to the 40th example in the usual method, we shall find it to be 41 m. 7 fur. 39 rd. 5 yd. 2 ft. 11 in. This is correct, but it is not in the best form, for although there are not units enough expressed of any denomination to make one of the next higher, it equals 42 m. 0 fur. 0 rd. 0 yd. 1 ft. 5 in. Show the truth of this statement, and show also *why* the answer does not *at first* appear in the best form.

‡ The answer to the 41st example will take the form at first of 35 rd

42.

m.	fur.	rd.	yd.	ft.	in.
3	4	19	3	1	6
8	2	16	5	0	5
9	4	39	4	2	9
2	1	25	3	2	11
5	2	38	5	1	3
8	7	18	4	2	10

43.*

A.	R.	sq. rd.	sq. yd.	sq. ft.	in.
16	2	28	19	7	132
14	3	37	8	2	47
66	1	19	17	3	142
18	2	39	16	4	27
17	3	21	28	8	99
25	2	31	30	6	100

56. *Another, and often a shorter, Method of reducing Compound Numbers.*

It will often be more convenient to make the reductions by adding enough of one number to another to give a sum equivalent to a unit of the next higher denomination. We will take, for illustration, the example given in **51.**

£.	s.	d.	qr.
8	15	11	2
3	12	8	3
9	19	9	1
7	18	10	3
5	18	0	2
6	0	8	1
5	13	3	0
	16	8	1
48	16	0	1

Explanation. — Having found the sum of the farthings column to be equal to 3 d., we add the 3 d. with the column of pence, thus: —

$4\frac{1}{2}$ yd. 2 ft. 11 in., which should be changed, for the sake of simplicity, to 35 rd. 5 yd. 1 ft. 5 in. Show the equality of the two expressions, and the method by which the reduction can be made.

* In adding square yards, it will be of service to notice that $60\frac{1}{2}$ sq. yd. = 2 sq. rd; that $90\frac{3}{4}$ sq. yd. = 3 sq. rd.; that 121 sq. yd. = 4 sq. rd.; and that $\frac{1}{4}$ of a sq. yd. = 2 sq. ft. 36 sq. in. This will avoid any difficulty in the use of fractions.

3 d. and 8 d. are 11 d., and 3 d. are 1 s. 2 d., (by adding 1 of the 3 d. with the 11 d.,) and 8 d. are 1 s. 10 d., and 10 d. are 2 s. 8 d., (by adding 2 of one 10 d. with the other 10 d.,) and 9 d. are 3 s. 5 d., (by adding 3 of the 8 d. with the 9 d.,) and 8 d. are 4 s. 1 d., (by adding 4 of the 5 d. with the 8 d.,) and 11 d. are 5 s. 0 d.

Adding the 5 s. with the shillings column, we have 5 s. and 16 s. are £1 1 s., (by adding 4 of the 5 s. with 16 s.,) and 13 s. are £1 14 s., and 18 s. are £2 12 s., (by adding 2 of the 14 s. with 18 s.,) and 18 s. are £3 10 s., (by adding 2 of the 12 s. with 18 s.,) and 19 s. are £4 9 s., (by adding 1 of the 10 s. with the 19 s.,) and 12 s. are £5 1 s., (by adding 8 of the 9 s. with 12 s.,) and 15 s. are £5 16 s. = sum of shillings column.

(*b.*) We have mentioned the numbers added in order to secure clearness of explanation, but in practical work the results alone should be named.

Thus, beginning with the farthings, we have, —

1 qr., 2 qr., 1 d., 1 d. 3 qr., 2 d., 2 d. 3 qr., 3 d. 1 qr. Write 1 qr.

3 d., 11 d., 1 s. 2 d., 1 s. 10 d., 2 s. 8 d., 3 s. 5 d., 4 s. 1 d., 5 s. 0 d. Write 0 d.

5 s., £1 1 s., 1 d. 14 s., £2 12 s., £3 10 s., £4 9 s., £5 1 s., £5 16 s. Write 16 s.

The pounds are added as before.

57. *Common Method of adding Compound Numbers.*

By the method of adding compound numbers which is commonly given, the entire sum of each column is found before reducing to higher denominations. This method, however, will, as a general thing, be found much less expeditious than either of the others.

It is illustrated in the following solution of the example given in the last article.

Explanation. — By adding the farthings column, we find that its sum is 13 qr., which, as 4 qr. = 1 d., must equal as many pence as there are times 4 in 13, which are three times, with a remainder of 1. Therefore, 13 qr. = 3 d. 1 qr.

Writing 1 as the farthings figure of the amount, we add the 3 d. with the figures of the pence column; this gives 60 d., which, as 12 d. = 1 s., are equal to as many shillings as there are times 12 in 60, which are 5 times. Therefore, 60 d. = 5 s.

Writing 0 as the pence figure of the amount, we add the 5 s. with the figures of the shillings column. This gives 116 s., which, as 20 s. = £1, are equal to as many pounds as there are times 20 in 116, which are 5 times, with a remainder of 16. Therefore, 116 s. = £5 16 s.

Writing 16 s., we add the £5 with the figures of the pounds column. This gives £48, which, being the highest denomination, we write.

As all the denominations have now been added, the sum or amount must be £48 16 s. 1 qr.

58. *Examples for Practice in the Methods of* 50, 51, *and* 56.

1. What is the sum of £4 17 s. 11 d. 2 qr. + £84 13 s. 3 d. + £7 19 s. 8 d. 3 qr. + £16 18 s. 9 d. 1 qr. + £7 15 s. 1 d. + £18 16 s. 11 d. 2 qr.?

2. What is the sum of £1386 15 s. 6 d. + £3576 18 s. 10 d. + £463 19 s. 4 d. + £23 5 s. 8 d. + £648 4 s. 6 d. + £100 10 s. 3 d.?

3. What is the sum of 40 lb. 7 oz. 5 dwt. 6 gr. + 9 lb. 8 oz. 19 dwt. 22 gr. + 2 lb. 11 oz. 19 dwt. 23 gr. + 7 lb. 8 dwt. 19 gr. + 11 oz. 6 dwt. + 3 lb. 1 oz. 15 gr. + 8 lb. 17 dwt. + 3 lb. 23 gr. + 18 dwt. 7 gr. + 9 oz. 15 gr. + 7 lb. 3 oz. 13 dwt. 15 gr.?

4. What is the sum of 18 w. 4 da. 21 h. 37 m. 5 sec. + 37 w. 5 da. 16 h. 43 m. 57 sec. + 19 w. 3 da. 14 h. 46 m. 38 sec. + 19 w. 6 da. 23 h. 56 m. 27 sec. + 43 w. 5 da. 2 h. 17 m. 38 sec. + 28 w. 1 da. 1 h. 5 m. 7 sec.?

5. What is the sum of 47 gal. 3 qt. 1 pt. 2 gi. + 37 gal. 1 qt. 1 pt. 1 gi. + 85 gal. 2 qt. 2 gi. + 25 gal. 2 qt. 1 pt. 3 gi. + 54 gal. 2 qt. 1 pt. 3 gi. + 18 gal. 2 qt. 1 pt. 2 gi. + 37 gal. 3 qt. 0 pt. 1 gi. + 19 gal. 3 qt. 1 pt. 2 gi. + 43 gal. 0 qt. 0 pt. 3 gi.?

6. What is the sum of 15 yd. 2 qr. 2 na. + 18 yd. 3 qr. 1 na. + 27 yd. 3 qr. 3 na. + 42 yd. 1 qr. + 87 yd. 3 na. + 3 qr. 3 na. + 26 yd. 1 qr. 1 na. + 57 yd. 3 qr. 2 na. + 42 yd. 0 qr. 0 na. + 64 yd. 3 qr. 3 na.?

7. What is the sum of 18 ℔ 6 ℥ 5 ʒ 2 ℈ 5 gr. + 7 ℔ 8 ℥ 7 ʒ 1 ℈ 18 gr. + 4 ℔ 11 ℥ 4 ʒ 2 ℈ 13 gr. + 25 ℔ 9 ℥ 1 ℈ 4 gr. + 11 ℥ 1 ℈ + 2 ℔ 5 ℥ 6 ʒ 0 ℈ 16 gr. + 5 ℔ 11 ℥ 4 ʒ 1 ℈ 14 gr.?

8. What is the sum of 2 m. 7 fur. 28 rd. 4 yd. 1 ft. 3 in. + 6 m. 5 fur. 19 rd. 2 yd. 2 ft. 11 in. + 25 m. 4 fur. 37 rd. 5 yd.

8 in. + 94 m. 1 fur. 24 rd. 4 yd. 2 ft. 8 in. + 14 m. 6 fur. 23 rd. 2 yd. 0 ft. 7 in. + 23 m. 5 fur. 37 rd. 4 yd. 1 ft. 10 in. + 57 m. 0 fur. 33 rd. 5 yd. 1 ft. 1 in.?

9. What is the sum of 19 m. 5 fur. 37 rd. 2 yd. 2 ft. 2 in. + 16 m. 4 fur. 18 rd. 5 yd. 1 ft. 7 in. + 37 m. 15 rd. 2 yd. 2 ft. 8 in. + 17 rd. 5 yd. 7 in. + 3 m. 7 fur. 18 rd. 4 yd. 2 ft. 9 in. + 46 m. 3 fur. 13 rd. 2 yd. 1 ft. 9 in. + 33 m. 4 fur. 27 rd. 5 yd. 1 ft. 4 in. + 19 m. 0 fur. 34 rd. 3 yd. 0 ft. 10 in.?

10. What is the sum of 14 A. 3 R. 28 sq. rd. 27 sq. yd. 8 sq. ft. 12 sq. in. + 27 A. 2 R. 31 sq. rd. 17 sq. yd. 5 sq. ft. 137 sq. in. + 35 A. 1 R. 31 sq. rd. 18 sq. yd. 5 sq. ft. 116 sq. in. + 21 A. 26 sq. rd. 25 sq. yd. 5 sq. ft. 107 sq. in. + 43 A. 2 R. 14 sq. rd. 19 sq. yd. + 1 R. 15 sq. rd. 37 sq. in.?

11. I bought some flour for $6.75; some cloth for $17.25; a hat for $3.37; a coat for $19.42; a vest for $3.87; some calico for $3.25; some flannel for $4.93; some silk for $23.99; a pair of boots for $5.33; an overcoat for $22.75; a shawl for $6.68; a pair of gloves for $1.46; an umbrella for $1.37; and a pair of overshoes for $1.17. What was the amount of my purchase?

12. A trader sold 17 cases of broadcloth; the first case contained 317 yards, the second 296, the third 319, the fourth 339, the fifth 259, the sixth 347, the seventh 329, the eighth 286, the ninth 321, the tenth 294, the eleventh 337, the twelfth 248, the thirteenth 324, the fourteenth 346, the fifteenth 299, the sixteenth 338, and the seventeenth 207. How many yards were there in all?

13. He received $984.36 for the first case, $849.23 for the second, $1097.28 for the third, $1342.94 for the fourth, $836.28 for the fifth, $1297.89 for the sixth, $1048.30 for the seventh, $857.82 for the eighth, $1004.28 for the ninth, $976.87 for the tenth, $1248.67 for the eleventh, $827.61 for the twelfth, $1176.04 for the thirteenth, $1327.98 for the fourteenth, $876.48 for the fifteenth, $1200.36 for the sixteenth, and $758.93 for the seventeenth. How much did he receive for all?

14. In the course of the year 1853, a flour dealer bought

649 barrels of flour for \$3798.75; 357 barrels for \$2039.25; 439 barrels for \$2679.00; 987 barrels for \$6198.42; 299 barrels for \$1925.37; 1168 barrels for \$7385.94; 627 barrels for \$4369.27; 1359 barrels for \$9967.84; 538 barrels for \$4279.63; 275 barrels for \$2383.50; 96 barrels for \$816.00; 948 barrels for \$8472.56; 358 barrels for \$3615.80; 796 barrels for \$8237.29; and 2962 barrels for \$25,851.00. How many barrels did he buy in all? How many dollars did he pay for the whole?

15. He gained \$324.50 on the first lot; \$178.50 on the second; \$109.75 on the third; \$740.25 on the fourth; \$29.90 on the fifth; \$584 on the sixth; \$600 on the seventh; \$1359 on the eighth; \$470.75 on the ninth; \$277.75 on the tenth; \$89.28 on the eleventh; nothing on the twelfth and thirteenth; \$398 on the fourteenth; and \$2154.25 on the fifteenth. What was the amount of his gains?

59. *Addition of several Columns at one Operation.*

(*a.*) Accountants often add two or three, and sometimes four or more, columns of figures at a single operation.

(*b.*) The following illustrates some of the methods of doing it:—

$$\begin{array}{r} 67 \\ 85 \\ 94 \\ 28 \\ 69 \\ \hline 343 \end{array}$$

Explanation. — 69 plus 20 = 89, plus 8 = 97, plus 90 = 187, plus 4 = 191, plus 80 = 271, plus 5 = 276, plus 60 = 336, plus 7 = 343.

(*c.*) By adding tens first, and then units, as before, and naming only results, we have 69, 89, 97, 187, 191, 271, 276, 336, 343.

(*d.*) A little practice will enable a person to add without separating each number into tens and units, thus: 69, 97, 191, 276, 343.

(*e.*) After the student has become familiar with the method of adding by single columns, he will find it a very valuable exercise to add as above explained. We recommend that he perform, at least, the first twenty examples under **55** by adding two or more columns at a time.

60. *Leger Columns.*

A great part of the work of an accountant consists in adding long leger columns, like the following. Let the pupil find the sum of the numbers in each, being as careful to obtain a correct result as he would be if he were to receive or pay the several amounts.

1.	2.	3.
8.37	.78	673.28
4.33	.47	597.84
7.62	.53	3426.87
.48	2.75	219.48
.97	1.20	8.37
2.50	4.37	167.84
6.19	8.29	5986.32
10.00	13.85	6749.31
4.28	2.00	4863.27
8.07	.62	7542.35
4.37	.25	2986.28
9.48	1.37	379.87
4.21	9.83	2.59
13.26	6.75	69.80
1.20	8.43	4060.75
.57	20.48	309.71
3.08	6.00	124.87
4.96	1.00	8520.06
.85	1.50	2493.28
4.00	7.69	48.75

4.	5.	6.
11427.81	2178.63	4178.38
13.87	748.29	5137.96
49.00	4.37	2000.00
674.00	59.48	1697.81
2.75	246.53	528.63
9483.25	58.47	5428.49
27.96	697.58	954.86
4.37	792.43	2797.78
1948.74	1246.58	934.67
647.25	642.17	528.39
34.98	428.00	776.95
297.26	1000.00	82.55
49.87	386.74	167.73
348.54	17.19	4127.48
9.78	4.26	298.49
6.25	269.73	5842.76
4327.69	49.47	378.35
514.38	583.28	49.27
693.27	679.59	189.01
43.96	2874.43	1101.48
279.84	897.61	698.41
5786.39	2854.55	64.81
284.62	7443.75	587.65
75.28	41.28	14.39

SECTION V.

SUBTRACTION.

61. *Definitions and Illustrations.*

(*a.*) SUBTRACTION IS THE PROCESS BY WHICH WE FIND THE DIFFERENCE OF TWO GIVEN NUMBERS, OR THE EXCESS OF ONE GIVEN NUMBER OVER ANOTHER.

(*b.*) The following are questions in subtraction: —

Joseph had 34 apples, and gave away 6 of them. How many did he have left?

Samuel had 16 cents and George had 9. How many more had Samuel than George?

8 from 16 leaves how many?

How many are 12 — 8?

(*c.*) The larger given number, or one from which we subtract, is called the *Minuend;* the smaller given number, or one subtracted, is called the *Subtrahend;* and the result obtained is called the *Difference* or *Remainder.*

Illustration. — In the first of the above examples 34 is the minuend, 6 is the subtrahend, and 28 is the difference or remainder.

(*d.*) The minuend and subtrahend must represent things of the same kind, otherwise the subtraction cannot be performed.

Illustrations. — 5 apples from 7 apples leave 2 apples, and 5 pears from 7 pears leave 2 pears; but it would be impossible to take 5 pears from 7 apples, or 5 apples from 7 pears. We cannot subtract 5 cents from 7 dimes; but if we should exchange one of the dimes for its value in cents, we should have 6 dimes and 10 cents, from which if we should subtract 5 cents, there would be 6 dimes and 5 cents left.

We cannot subtract units from tens, but we can find how many units a given number of tens is equal to, and then subtract from that number of units.

62. *Method of writing Numbers and performing Problems requiring no Reduction.*

(*a.*) Although the result is not affected by the manner of writing the numbers, it is convenient to place those of the same

denomination near each other, or to write units under units, tens under tens, &c., in simple numbers, and pounds under pounds, shillings under shillings, yards under yards, feet under feet, &c., in compound.

(*b.*) For the sake of uniformity, we usually place the minuend above the subtrahend, and the remainder beneath; beginning at the right hand to subtract.

(*c.*) The following examples will illustrate the application of these principles: —

1. What is the value of 4697 — 3265?

Solution. — Beginning at the right hand, and considering the denominations separately, we have 5 units from 7 units leave 2 units; 6 tens from 9 tens leave 3 tens; 2 hundreds from 6 hundreds leave 4 hundreds; 3 thousands from 4 thousands leave 1 thousand.

Having thus subtracted the numbers in all the denominations, we know that the remainder must be 1 thousand, 4 hundred, 3 tens, and 2 units, or 1432.

The work would be written thus: —

$$\begin{array}{rl} 4697 & = \text{Minuend.} \\ 3265 & = \text{Subtrahend.} \\ \hline 1432 & = \text{Remainder.} \end{array}$$

Second Example. — What is the value of £17 8 s. 9 d. — £3 4 s. 6 d.?

Solution. — Beginning with the lowest denomination, we have 6 d. from 9 d. = 3 d.; 4 s. from 8 s. = 4 s.; £3 from £17 = £14.

The answer is, therefore, £14 4 s. 3 d.

Or, beginning at the left, £17 — £3 = £14; 8 s. — 4 s. = 4 s.; 9 d. — 6 d. = 3 d. *Ans.* £14 4 s. 3 d.

The work would be written thus: —

£.	s.	d.	
17	8	9	Minuend.
3	4	6	Subtrahend.
14	4	3	Remainder.

63. *Methods of Proof.*

(*a.*) From the nature of subtraction, it is evident, that if *the minuend* were divided into two parts, such that one

should equal the subtrahend, the other would equal the remainder.

(*b.*) We have, therefore, the following methods of testing the correctness of the work: —

First Method. — Add the remainder to the subtrahend, and if the sum thus obtained is equal to the minuend, the work is probably correct; but if it is not, there is an error in either the subtraction or the addition, and possibly in both.

Second Method. — Subtract the remainder from the minuend, and if the result thus obtained equals the subtrahend, the work is probably correct.

WRITTEN WORK AND PROOF OF THE FIRST EXAMPLE.

4697	Minuend.
3265	Subtrahend.
1432	Remainder.
4697	Sum of Rem. and Sub. = Minuend.
3265	Diff. of Rem. and Min. = Subtrahend.

WRITTEN WORK AND PROOF OF SECOND EXAMPLE.

£.	s.	d.	
17	8	9	Minuend.
3	4	6	Subtrahend.
14	4	3	Remainder.
17	8	9	Sum of the Sub. and Rem. = Minuend.
3	4	6	Difference of Rem. and Min. = Subtrahend.

64. *Problems requiring no Reduction.*

What is the value of each of the following?

1. 854736 — 721423?
2. 9863764 — 420423?
3. 2948769 — 1432526?
4. $5476.92 — $1261.40?

5. 736.87 yd. — 415.24 yd.?

6. 698.795 bu. — 243.521 bu.?

7. 3 lb. 11 oz. 15 dwt. 18 gr. — 1 lb. 8 oz. 13 dwt. 4 gr.?

8. 6 T. 17 cwt. 2 qr. 26 lb. 13 oz. 11 dr. — 2 T. 6 cwt. 15 lb. 8 oz. 3 dr.?

9. 16 ℔ 8 ℥ 5 ʒ 2 ℈ 18 gr. — 5 ℔ 4 ℥ 2 ʒ 1 ℈ 6 gr.?

10. 1757 gal. 3 qt. 1 pt. 3 gi. — 1323 gal. 1 qt. 2 gi.?

11. 18 rd. 4 yd. 2 ft. 11 in. — 6 rd. 2 yd. 1 ft. 5 in.?

65. *Simple Subtraction. Method when Reductions are necessary.*

When, as is often the case, a figure in the subtrahend represents a greater value than the corresponding figure of the minuend, we take one of a higher denomination in the minuend, reduce it to the required denomination, add its value to the value of the figure already expressed, and subtract the value of the subtrahend figure from the sum thus obtained.

First Example. — What is the difference between 62.7 and 35.86?

WRITTEN WORK.

5 11 . 16,10	Minuend, changed in form.
6 2 . 7	Minuend.
3 5 . 8 6	Subtrahend.
2 6 . 8 4	Difference.

Explanation of Process. — As there are no hundredths expressed in the minuend, we reduce one of the 7 tenths to hundredths, leaving 6 tenths. 1 tenth = 10 hundredths, from which subtracting 6 hundredths, leaves a remainder of 4 hundredths.

As 8 tenths cannot be subtracted from 6 tenths, we reduce one of the 2 units to tenths, leaving 1 unit. 1 unit = 10 tenths, which added to the 6 tenths left in the tenths' place equal 16 tenths; 8 tenths from 16 tenths = 8 tenths.

As 5 units cannot be taken from the 1 unit left in the units' place, we reduce one of the 6 tens to units, leaving 5 tens. 1 ten = 10 units, which added to 1 unit equal 11 units; 5 units from 11 units = 6 units 3 tens from 5 tens leave 2 tens.

The answer, then, is 2 tens, 6 units, 8 tenths, and 4 hundredths, or 26.84.

This may be proved in the same way that the preceding examples were.

Questions on the above.

Which expresses the larger number, 7 tenths, or 6 tenths and 10 hundredths, and why? 2 units and 6 tenths, or 1 unit and 16 tenths? 5 tens and 1 unit, or 4 tens and 11 units? 6 tens, 2 units, and 7 tenths, or 5 tens, 11 units, 16 tenths, and 10 hundredths?

How then will the remainder, obtained by subtracting 35.86 from 62.7, compare with the remainder obtained by subtracting it from 5 tens, 11 units, 16 tenths, and 10 hundredths?

Second Example. — How many are 83004 dollars minus 24765 dollars?

WRITTEN WORK.

7	12	9	9	14	= Minuend, changed in form.
$8	3	0	0	4	= Minuend.
$2	4	7	6	5	= Subtrahend.
$5	8	2	3	9	= Remainder.

Explanation. — As we cannot take 5 dollars from 4 dollars, and as there are no tens or hundreds expressed in the minuend, we take 1 thousand from the 3 thousands, leaving 2 thousands; 1 thousand = 10 hundreds, and taking 1 of these hundreds to reduce to tens, we have 9 hundreds left. 1 hundred = 10 tens, and taking 1 of these tens to reduce to units we have 9 tens left. 1 ten = 10 units, which added to the 4 units in the units' place = 14 units.

Now, by subtracting, we have

9 units from 14 units = 5 units.

6 tens from 9 tens = 3 tens.

7 hundreds from 9 hundreds = 2 hundreds.

4 thousands cannot be taken from 2 thousands, therefore we take 1 ten-thousand from the 8 ten-thousands in the minuend, leaving 7 ten-thousands. 1 ten-thousand = 10 thousands, which added to the 2 thousands in the thousands' place = 12 thousands.

4 thousands from 12 thousands = 8 thousands.

2 ten-thousands from 7 ten-thousands = 5 ten-thousands.

The remainder is, therefore, 58239 dollars.

Questions upon the above. — How can it be shown that 3004 is equal to 2 thousands, 9 hundreds, 9 tens, and 14 units? That 83004 is equal to 7 ten-thousands, 12 thousands, 9 hundreds, 9 tens, and 14 units?

66. *Compound Subtraction.*

First Example. — Find the difference between 14 lb. 6 oz. 5 dwt. 7 gr. and 8 lb. 7 oz. 18 dwt. 23 gr.

WRITTEN WORK.

13	17	34	41	Minuend changed in form.
lb.	oz.	dwt.	gr.	
14	6	15	17	Minuend.
8	7	18	23	Subtrahend.
5	10	16	18	Remainder.

Explanation. — As we cannot subtract 23 gr. from 17 gr., we take 1 dwt. from the 15 dwt., which reduced to grains and added to the 17 gr. equals 41 gr.; 23 gr. from 41 gr. = 18 gr.

But as 18 dwt. cannot be taken from the 14 dwt. left in the minuend, we take 1 oz. from the 6 oz. and reduce it to pennyweights; 1 oz. = 20 dwt., which added to the 14 dwt. equal 34 dwt.; 18 dwt. from 34 dwt = 6 dwt.

As 7 oz. cannot be taken from the 5 oz. left in the minuend, we take 1 lb. from the 14 lb. and reduce it to ounces; 1 lb. = 12 oz., which added to the 5 oz. equal 17 oz.; 7 oz. from 17 oz. = 10 oz.

8 lb. from 13 lb. = 5 lb.

The answer is, therefore, 5 lb. 10 oz. 6 dwt. 8 gr., which may be proved as before.

Questions. — Which expresses the greater quantity, 15 dwt. 17 gr., or 14 dwt. 41 gr., and why? 6 oz. 14 dwt., or 5 oz. 34 dwt.? 14 lb. 5 oz., or 13 lb. 17 oz.? 14 lb. 6 oz. 15 dwt. 17 gr., or 13 lb. 17 oz. 34 dwt. 41 gr.?

How would the remainder obtained by subtracting 8 lb. 7 oz. 18 dwt. 23 gr. from 14 lb. 6 oz. 15 dwt. 17 gr. compare with that obtained by subtracting it from 13 lb. 17 oz. 34 dwt. 41 gr.?

Second Example. — A farmer took 8 bu. 3 pk. 5 qt. of corn from a bin containing 17 bushels. How many bushels, pecks, and quarts remained?

Reasoning Process. — If the bin contained 18 bu., and he took out 8 bu. 3 pk. 5 qt., there would remain the difference between 17 bu. and 8 bu. 3 pk. 5 qt. This shows that 17 bu. is the minuend, and 8 bu. 3 pk 5 qt. the subtrahend.

16	3	8	Minuend, changed in form.
bu.	pk.	qt.	
17	0	0	Minuend.
8	3	5	Subtrahend.
8	0	3	Remainder.

Explanation. — As there are no pecks or quarts expressed in the minuend, we take 1 bu. from the 17 bu. and reduce it to lower denominations. 1 bu. = 3 pk. 8 qt. Therefore 17 bu. = 16 bu. 3 pk. 8 qt. The subtraction can now be performed as before.

(*a.*) In examples involving fractional denominations, it will usually be more convenient to make all the reductions and changes in the minuend before beginning to subtract, as in the following: —

Third Example. — What is the difference between 8 rd 3 yd. 1 ft. 4 in. and 2 rd. 4 yd. 2 ft. 5 in.?

WRITTEN WORK.

7	8	2	10	= Minuend, changed in form.
rd.	yd.	ft.	in.	
8	3	1	4	= Minuend.
2	4	2	5	= Subtrahend.
5	4	0	5	= Remainder.

Explanation. — Since there are more yards, feet, and inches expressed in the subtrahend than in the minuend, we will take 1 rd. from the 8 rd. and reduce it to lower denominations. 1 rd. = $5\frac{1}{2}$ yd. = 5 yd. 1 ft. 6 in., which added to the 3 yd. 1 ft. 4 in. equal 8 yd. 2 ft. 10 in. Therefore 8 rd. 3 yd. 1 ft. 4 in. — 2 rd. 4 yd. 2 ft. 5 in. = 7 rd. 8 yd. 2 ft. 10 in. — 2 rd. 4 yd. 2 ft. 5 in.

NOTE. — Had not the 1 rod been reduced to yards, and the $\frac{1}{2}$ yard to feet and inches, before commencing the subtraction, the answer would have taken the form of 5 rd. $3\frac{1}{2}$ yd. 1 ft. 11 in., from which, by reducing the $\frac{1}{2}$ yard to feet and inches, we should get 5 rd. 3 yd. 2 ft. 17 in. = 5 rd 4 yd. 0 ft. 5 in. = the answer obtained directly by first method.

67. *Problems for Solution.*

What is the value of —

1. 3743 — 2554?
2. 839.74 — 213.78?
3. 37.9623 — 21.978?
4. 300.67 — 25.38?

.7235 — .0649 ?

6. .00632 — .00274 ?

7. 87432 — 52841 ?

8. 92846 — 24547 ?

9. 40006 — 14308 ?

10. 294.6 — 87.942 ?

11. 320.06 — 5.947 ?

12. 824.57 — 347.28 ?

What is the value of —

13. £17 6 s. 8 d. 1 qr. — £13 17 s. 3 d. 2 qr. ?

14. 25 cwt. 1 qr. 13 lb. 10 oz. 7 dr. — 13 cwt. 2 qr. 15 lb 3 oz. 11 dr. ?

15. 8 lb. 4 oz. 17 dwt. 13 gr. — 2 lb. 8 oz. 19 dwt. 20 gr. ?

16. 14 ℔ 9 ℥ 4 ʒ 2 ℈ 12 gr. — 4 ℔ 10 ʒ 4 ʒ 2 ℈ 16 gr. ?

17. 4147 bu. 1 pk. 4 qt. — 2878 bu. 2 pk. 7 qt. 1 pt. ?

18. 49 w. 3 da. 19 h. 13 m. 45 sec. — 18 w. 1 da. 22 h. 40 m. 53 sec. ?

19. £487 6 s. 0 d. 1 qr. — £236 11 s. 8 d. 3 qr. ?

20. 27° 24′ 47″ — 19° 37′ 51″ ?

21. 38 m. 4 fur. 23 rd. 4 yd. 0 ft. 3 in. — 32 m. 5 fur 28 rd. 5 yd. 1 ft. 5 in. ?

22. 54 m. 6 fur. 3 rd. 8 in. — 48 m. 3 fur. 3 rd. 2 yd. 2 ft. 10 in. ?

23. 3 R. 14 sq. rd. 7 sq. yd. 2 sq. ft. 19 sq. in. — 23 sq. rd. 11 sq. yd. 5 sq. ft. 138 sq. in. ?

24. 15 rd. 5 yd. 2 ft. 11 in. — 16 rd. 1 ft. 4 in. ?

68. *The Changed Minuend not usually written.*

(*a.*) The changed form of the minuend has been written in the preceding examples to insure that the nature of the reductions and changes shall be understood by the pupil. It is not, however, customary to write it. The full explanation is the same whether it is written or omitted; but when it is omitted, and every step of the process is understood and mastered, abbreviated explanations like the following may be adopted : —

WRITTEN WORK OF FIRST EXAMPLE UNDER **65.**

62.7	= Minuend.
35.86	= Subtrahend.
26.84	= Remainder.

First Abbreviated Explanation. — 6 hundredths cannot be taken from 0 hundredths, but 1 tenth = 10 hundredths, and 6 hundredths from 10 hundredths leave 4 hundredths.

8 tenths cannot be taken from 6 tenths, but one unit equal 10 tenths, and 6 tenths added = 16 tenths. 8 tenths from 16 tenths = 8 tenths.

5 units cannot be taken from 1 unit, but 1 ten = 10 units, and 1 unit added = 11 units. 5 units from 11 units leave 6 units.

3 tens from 5 tens leave 2 tens.

Hence the answer is 26.84.

Second Abbreviated Explanation. — 6 hundredths from 10 hundredths leave 4 hundredths; 8 tenths from 16 tenths leave 8 tenths; 5 units from 11 units leave 6 units; 3 tens from 5 tens leave 2 tens.

This gives 26.84 for an answer, as before.

(*b.*) All explanations should finally be dropped, and only results named, thus: — 4 *hundredths,* 8 *tenths,* 6 *units,* 2 *tens; giving for the answer* 26.84, *as before.*

WRITTEN WORK OF FIRST EXAMPLE UNDER **66.**

lb.	oz.	dwt.	gr.	
14	6	15	17	= Minuend.
8	7	18	23	= Subtrahend.
5	10	16	18	= Remainder.

First Abbreviated Explanation. — 23 gr. cannot be subtracted from 17 gr.; but 1 dwt. = 24 gr., and 17 gr. added are 41 gr. 23 gr. from 41 gr. = 18 gr.

18 dwt. cannot be taken from 14 dwt.; but 1 oz. = 20 dwt., and 14 dwt. added = 34 dwt. 18 dwt. from 34 dwt. = 16 dwt.

7 oz. cannot be taken from 5 oz.; but 1 lb. = 12 oz., and 5 oz. added are 17 oz. 7 oz. from 17 oz. = 10 oz.

Hence the remainder is 5 lb. 10 oz. 16 dwt. 18 gr.

69. *Subtrahend Figure may be increased instead of diminishing Minuend.*

It is obvious that the result would be the same, if, instead of considering the minuend figure of a denomination from which a reduction has been made to be one less, we should consider the corresponding subtrahend figure to be one greater. In the former case, we subtract 1 (on account of the reduced unit) before subtracting the subtrahend figure while in the latter we add 1 to the subtrahend figure, and subtract both together.

Thus, in subtracting the tenths of the first example, we may subtract 1 tenth from the 7 tenths before subtracting the 8 tenths, or we may add the 1 tenth to the 8 tenths, and subtract both at once. Many always subtract by the last method. One method is as convenient as the other, but the one to which we are most accustomed will *seem* the easiest.

70. *Another, and usually shorter, Method of subtracting Compound Numbers.*

(*a.*) When in compound subtraction reductions are necessary, and the changed minuend is not written, one of the following methods is often, if not usually, easier than that hitherto taken.

First. Subtract from the value of the reduced unit, and add the remainder to the minuend figure; or,

Second. Subtract as many as possible from the written minuend figure, and the rest from the value of the reduced unit.

(*b.*) Applying the first of the above methods to the example just considered gives the following work: —

Subtracting 17 of the 23 gr. from the 17 gr. leaves 6 gr. to be taken from 24 gr., (the value of the reduced unit.) 6 gr. from 24 gr. = 18 gr.

Subtracting 14 of the 18 dwt. from the 14 dwt. leaves 4 dwt. to be subtracted from 20 dwt., (the value of the reduced unit.) 4 dwt. from 20 dwt. = 16 dwt.

Subtracting 5 of the 7 oz. from the 5 oz. leaves 2 oz. to be taken from 12 oz., (the value of the reduced unit.) 2 oz. from 12 oz. = 10 oz.

8 lb. from 13 lb. leave 5 lb.

(*c.*) In practice, the above method may be abbreviated thus: —

Subtracting 17 of the 23 gr. leaves 6 gr., and 6 gr. from 1 dwt., or 24 gr., = 18 gr.

Subtracting 14 of the 18 dwt. leaves 4 dwt., and 4 dwt. from 1 oz., or 20 dwt., = 16 dwt.

Subtracting 5 of the 7 oz. leaves 2 oz., and 2 oz. from 1 lb., or 12 oz., = 10 oz.

8 lb. from 13 lb. = 5 lb.

(*d.*) Abbreviating still more, we have —

17 from 23 = 6, and 6 gr. from 1 dwt., or 24 gr., = 18 gr.

14 from 18 = 4, and 4 dwt. from 1 oz., or 20 dwt., = 16 dwt.
5 from 7 = 2, and 2 oz. from 1 lb., or 12 oz., = 10 oz.
3 lb. from 13 lb. = 6 lb., giving, as before, 5 lb. 10 oz. 16 dwt. 18 gr.

(*e.*) Adopting the second method, we have —

23 gr. from 1 dwt., or 24 gr., = 1 gr., which added to the 17 gr. gives 18 gr.

18 dwt. from 1 oz., or 20 dwt., = 2 dwt., which added to the 14 dwt. gives 16 dwt.

7 oz. from 1 lb., or 12 oz., = 5 oz., which added to the 5 oz. gives 10 oz.

8 lb. from 13 lb. = 5 lb., giving, as before, 5 lb. 10 oz. 16 dwt. 18 gr.

NOTE. — Practice will make the student so familiar with all these methods, that he will be able to see at once which is best adapted to the case he is considering.

71. *Problems for Solution*

What is the value of —

1. 3743 — 2917 ?	7. 93.46 — 2.78457 ?
2. 94276 — 46324 ?	8. 6.0004 — 4.000563 ?
3. 867004 — 328527 ?	9. 100000000 — 7 ?
4. 8674.5 — 2594.326 ?	10. 7000000 — .000007 ?
5. 45842.7 — 6243.984 ?	11. 847.96 — 47.96823 ?
6. 7564.001 — 756.4002 ?	12. 487.6307 — 48.76307 ?

13. A land company bought 8479 acres of wild land, and sold 3896 acres of it. How many did they have left?

Reasoning Process. — If they bought 8479 acres, and sold 3896 acres of it, they must have left the difference between 8479 acres and 3896 acres, to find which we subtract 3896 from 8479.

14. A ship is valued at $27648, and its cargo at $49325. How much more is the cargo worth than the ship ?

Reasoning Process. — If the ship is worth $27648, and the cargo is worth $49325, the cargo must be worth as many dollars more than the ship as there are in the difference between $49325 and $27648, to find which the latter must be subtracted from the former.

15. Census returns show that the United States contained 3929827 inhabitants in the year 1790; 5305941 in 1800; 7239814 in 1810; 9638191 in 1820; 12866020 in 1830;

17069453 in 1840; and 23263488 in 1850. How many more did they contain in 1800 than in 1790?

16. How many more in 1810 than in 1800?
17. How many more in 1820 than in 1810?
18. How many more in 1830 than in 1820?
19. How many more in 1840 than in 1830?
20. How many more in 1850 than in 1840?
21. How many more in 1850 than in 1790?
22. How many more in 1850 than in 1800?
23. How many more in 1820 than in 1790?

What is the value of —

24. 27 bu. 2 pk. 2 qt. — 18 bu. 3 pk. 3 qt.?
25. 83 yd. 2 qr. 1 na. — 47 yd. 3 qr. 2 na.?
26. 8 cwt. 2 qr. 15 lb. 7 oz. 3 dr. — 2 cwt. 3 qr. 24 lb. 3 oz. 9 dr.?
27. 37 gal. 2 qt. 1 pt. 2 gi. — 12 gal. 3 qt. 1 pt. 3 gi.
28. 37 lb. 2 dwt. — 4 lb. 7 oz. 5 dwt. 13 gr.?
29. 24 ℔ 3 ℥ — 5 ℔ 11 ℥ 4 ʒ 2 ℈ 14 gr.?
30. 83 yd. — 2 qr. 3 na.?
31. £64 — £28 14 s. 7 d. 2 qr.?
32. 187 T. 3 qr. 13 lb. — 67 T. 17 cwt. 1 qr. 18 lb. 5 oz. 13 dr.?
33. 27 sq. rd. 5 sq. ft. 17 sq. in. — 26 sq. rd. 30 sq. yd. 7 sq. ft. 53 sq. in.?
34. 6 fur. 8 rd. 3 yd. 1 ft. 1 in. — 1 fur. 8 rd. 4 yd. 2 ft. 11 in.?
35. 18 yd. 1 na. — 14 yd. 2 qr. 3 na.?
36. 6 m. 1 ft. — 5 m. 7 fur. 39 rd. 5 yd. 1 ft. 2 in.?
37. 18 m. — 17 m. 7 fur. 39 rd. 5 yd. 1 ft. 5 in.?
38. 231 A. 19 sq. rd. — 197 A. 3 R. 27 sq. rd. 15 sq. yd. 5 sq. ft. 97 sq. in.?
39. $127^\circ\ 18'\ 14'' - 113^\circ\ 47'\ 25''$?
40. 307 T. 8 cwt. 2 qr. 23 lb. 8 oz. 12 dr. — 213 T. 15 cwt. 23 lb. 11 oz. 6 dr.?
41. 527 yd. 1 qr. 2 na. 1 in. — 431 yd. 2 qr. 3 na. 2 in.?
42. 63 m. — 27 m. 7 fur. 39 rd. 5 yd. 2 ft. 3 in.?

43. Bought 7 T. 14 cwt. 1 qr. 19 lb. of hay, from which I sold 3 T. 7 cwt. 3 qr. 26 lb. How much had I left?

44. A goldsmith bought 7 lb. 7 oz. of gold. How much will he have left after manufacturing and selling 3 lb. 5 gr. of it?

45. A trader sold 9 yd. 3 qr. 2 na. of cloth from a piece containing 27 yd. 1 qr. 1 na. How much was left in the piece?

46. A man set on foot to travel from Boston to Springfield, the distance being 98 miles. The first day he travelled 28 m. 7 fur. 19 rd., the second 24 m. 6 fur. 28 rd., the third 29 m. 4 fur. 36 rd. How far was he from Springfield at the end of the third day?

47. A man undertook to dig a ditch for a certain price per rod. On completing it he demanded payment for a ditch 37 rd. 0 ft. 3 in. long. His employer, doubting his honesty, measured it, and found it to be but 36 rd. 5 yd. 1 ft. 9 in. long. A dispute arising between them, they called in Mr. Jenks to settle it, agreeing to abide by his decision. He measured the ditch, and found its length to be 36 rd. 16 ft. 9 in. What was the difference in their measurements?

72. *Subtraction from Left to Right.*

We can begin at the left to subtract as well as at the right, if we are only careful to reserve one for reduction from each denomination in the minuend when it is required by the lower denominations. This reduction will be necessary when the figures in the subtrahend at the right of the denomination considered are greater than the corresponding ones of the minuend.

Example. — How many are 508.935 — 249.748?

508.935 = Minuend.
249.748 = Subtrahend.

259.187 = Remainder.

Explanation. — Reserving 1 hundred from the 5 hundreds, we have, 2 hundreds from 4 hundreds = 2 hundreds. Reducing the 1 hundred reserved to tens, and reserving 1 ten for further reduction, we have, 4

tens from 9 tens = 5 tens. Reducing the 1 ten to units, and adding 8 units, we have, 9 units from 18 units = 9 units. Reserving 1 tenth, we have 7 tenths from 8 tenths = 1 tenth. Reducing the 1 tenth reserved to hundredths, and adding 2 hundredths, (1 hundredth being reserved,) we have, 4 hundredths from 12 hundredths = 8 hundredths. Reducing 1 hundredth to thousandths, and adding the 5 thousandths, we have, 8 thousandths from 15 thousandths = 7 thousandths. The answer is, therefore, 259.187.

As soon as the process is sufficiently well understood to justify it, the explanations should be omitted, and the results only named.

Thus, in the example explained above, the pupil should say, 2 hundreds, 5 tens, 9 units, 1 tenth, 8 hundredths, 7 thousandths. The answer is, therefore, 259,187.

It is also a good mental exercise to read the results at once by inspecting minuend and subtrahend.

Having the numbers to be subtracted written thus, —

8274.01 = Minuend,
2357.23 = Subtrahend,

perform the subtractions mentally, beginning at the left, and read at once 5916.78. Practice will make this very easy.

Perform the following subtractions by beginning at the left: —

1. 89704 — 29821.
2. 85.1 — 22.563.
3. \$56 — \$8.73.
4. 800635 — 32070.
5. 472968 — 381489.
6. 521.732 — 23.547.
7. 42.736 — 5.749.
8. \$65.28 — \$47.
9. .4103 — .00627.
10. 678432 — 189146.

73. *Subtraction of several Numbers at once.*

A good method of proceeding when several numbers are to be subtracted is, to subtract the sum of each column of the subtrahend from the appropriate part of the minuend, reducing and changing denominations, as before explained.

How many are 862 — 28 — 59 — 38 — 56?

WRITTEN WORK.

$$\begin{array}{rl} 862 & \text{Minuend.} \\ \left.\begin{array}{r} 28 \\ 59 \\ 38 \\ 56 \end{array}\right\} & \text{Subtrahends.} \\ \hline 681 & \text{Remainder.} \end{array}$$

Explanation. — Adding the units of the subtrahends, we have, 6, 14, 23, 31 units, which cannot be taken from 2 units. To obtain as many as 31 units, we must take 3 tens from the 6 tens. 3 tens = 30 units, which added to the 2 units equals 32 units; 31 from 32 leaves 1.

Now, it makes no difference whether we take 3 tens (on account of those we reduced to units) from the 6 tens, and afterwards take the tens in the tens column of the subtrahend, or whether we take all together. Adopting the latter method, and adding the tens column, we have 3, 8, 11, 16, 18 tens, which cannot be taken from 6 tens, but reducing 2 hundreds to tens, and adding the 6 tens, we have 26 tens, from which 18 tens being taken, there will remain 8 tens.

2 hundreds from 8 hundreds = 6 hundreds. The answer is, therefore, 681.

When the above is well understood, omit in practice a part of the explanation, as follows: —

6, 14, 23, 31 from 32 leaves 1 unit; 3, 8, 11, 16, 18 tens from 26 tens = 8 tens; 2 hundreds from 8 hundreds = 6 hundreds.

The following form may also be taken: —

6, 14, 23, 31, *and* 1 are 32 units; write 1 in the units' place;
3, 8, 11, 16, 18, *and* 8 are 26 tens; write 8 in the tens' place;
2 *and* 6 are 8 hundreds; write 6 in the hundreds' place.

Perform the operations indicated in the following examples by the method explained above: —

1. 87642 — 273 — 4827 — 285 — 437 — 869 — 245.

2. 98402 — 2701 — 2596 — 1874 — 987 — 1283 — 5876.

3. 276.385 — 31.278 — 13.691 — 12.586 — 57.84 — 32.798 — 7.302.

4. 287000 — 328.7 — 221.3 — 344.37 — 2.851 — 117.06 — 576.823.

5. 283.587 — 1.27 — .328 — 9.063 — 57.063 — .00876 — 70.07 — .826.

6. Subtract .87 + 4.73 + 826 + 42.71 + 9.854 + 3.27 from 9012031.

7. Subtract 8837 + 1429 + 6372 + 8406 + 9785 + 4203 from 9120301.

8. Subtract 8375.94 + 276.483 + 5427.98 + 386.421 + 279.43 + 81 679 + .4237 + 4598.7 from 846271.3.

9. Mr. Ewell has in his possession $9478.63, but he owes $143.27 to Mr. Webster, $549.71 to Mr. May, $581.375 to Mr. Kingsbury, and $378.875 to Mr. Bryant. How much will he have left after paying his debts?

10. A man travelled 8725.67 miles in the following conveyances, viz.: 1285.89 miles in railroad cars, 876.81 miles in a canal boat, 587.86 miles in a stage coach, 725.18 miles on horseback, 647.25 miles on foot, 3147.82 miles in a steamboat, and the rest in a ship. How many miles did he travel in a ship?

11. Messrs. Howes and Baker bought 27147 bushels of Indian corn. After selling, at private sale, 1438 bushels to one man, 2627 to another, 3781 to another, and 864 to another, they sold the rest at auction. How many bushels did they sell at auction? They received $719 for the first lot, $1313.50 for the second, $1890.50 for the third, $432 for the fourth, and enough to make up $13573.50 for what they sold at auction. How much did they receive for that which they sold at auction?

SECTION VI.

MULTIPLICATION.

74. *Definitions and Illustrations.*

(*a.*) MULTIPLICATION IS A PROCESS BY WHICH WE ASCERTAIN HOW MUCH ANY GIVEN NUMBER WILL AMOUNT TO, IF TAKEN AS MANY TIMES AS THERE ARE UNITS IN SOME OTHER GIVEN NUMBER.

(*b.*) The following are questions in multiplication: —

How many are 7 times 6?

What is the value of 9 multiplied by 6?

What is the value of 7 × 5 ?
How much will 8 books cost at 3 dollars apiece ?

(*c.*) The number *supposed to be taken* is called the *multiplicand*, the number showing how many times the multiplicand is supposed to be taken is called the *multiplier*, and the result is called the *product*.

(*d.*) The multiplier and multiplicand are called *factors* of the product.

(*e.*) The product is said to be a *multiple* of its factors.

Illustrations. — In the first of the above examples, 6 is the multiplicand, 7 is the multiplier, and the answer, 42, is the product. 7 and 6 are factors of 42, and 42 is a multiple of 7 and 6.

In the second example, 9 is the multiplicand, 6 is the multiplier, and the answer, 54, is the product. 9 and 6 are factors of 54, and 54 is a multiple of 9 and 6.

(*f.*) The last example would be solved thus : —

If 1 book costs 3 dollars, 8 books will cost 8 times 3 dollars, which are 24 dollars.

Here 3 is the multiplicand, 8 is the multiplier, and 24 is the product. 8 and 3 are factors of 24, and 24 is a multiple of 3 and 8.

(*g.*) In performing the operation, the multiplier must always be regarded as an abstract number.

Illustration. — A number can be taken 3 times, 5 times, or 8 times, but it would be absurd to speak of taking it 3 bushels times, 5 houses times, or 8 books times.

(*h.*) The product must be of the same denomination as the multiplicand.

Illustration. — 7 times 8 bushels = 56 bushels ; 9 times 4 books = 36 books ; 7 times 5 tenths = 35 tenths, &c.

NOTE. — It must be observed, that there is an *apparent* exception to the last statement, (*h.*) when the multiplier is a fraction, for .6 times .04 = .024 ; .02 times .0003 = .000006, &c. This will be explained in the section on fractions. (See page 173, Note.)

(*i.*) To examine the nature of the operation on the numbers, let us suppose that a person ignorant of all numerical processes, except that of counting, should be called upon to solve the last question.

(*j.*) If he had a quantity of dollars, he might lay 3 in one place, 3 more in another, 3 more in another, and so go on laying 3 in a place till he should have 8 piles of 3 dollars each. Since the dollars in each pile would buy 1 book, the dollars in all would buy 8 books; he might then, by counting the dollars in the 8 piles, find how much the books would cost.

(*k.*) If he should have no dollars, he might still determine the result in a similar way, by using pebbles, sticks, marks, or any thing else of a like character. After learning how to *add*, he might obtain the result by adding 8 threes together.

(*l.*) If, after having obtained the result in some way similar to the above, he should remember it, he would ever after be able, *without counting or adding*, to give the answer to any question requiring the amount of 3 taken 8 times.

(*m.*) If he should learn in a similar manner the several amounts of 10 and each number below 10, taken as many times as there are units in each successive number from 1 to 10, he would learn the common multiplication table as far as ten. If he should now learn how to apply this knowledge to the decimal system of numbers, he would be master of the process of multiplication.

NOTE. — The very common definition, "Multiplication is a short method of addition," is not a good one, any more than would be, "Multiplication is a short method of counting;" for while it is true that the results obtained by multiplication might be obtained by addition, it is equally true that they might be obtained by counting. It is true that multiplication has a dependence both on addition and counting, but it is equally true that it is as distinct from them as they are from each other, and that when we multiply we neither add nor count.

For instance, when we find the sum of 75798 + 24687 + 39764 + 86328 + 4395 + 283 + 86536, *by the method explained in article* **50**, we *add* them; but when we *merely remember, and state* that their sum is 317791, we do not add them.

So when we call to mind that 4 *and* 4 *are* 8, *and* 4 *are* 12, *and* 4 *are* 16, *and* 4 *are* 20, we *add* 5 fours together; but when we merely remember that 5 fours, or 5 times 4, are 20, we perform no addition, although as a result, we have in the mind the sum of 5 fours.

75. *Product not affected by Change in Order of Factors*

(*a.*) In determining the product of two numbers, it makes no difference which is regarded as the multiplicand, provided the other is regarded as the multiplier.

Thus: 6 times 4 = 4 times 6, or 6 fours = 4 sixes = 24.
Again: 5 times 3 = 3 times 5, or 5 threes = 3 fives = 15.

(*b.*) The principle may be proved true for all numbers, by the following arrangement of dots: —

. . . . .
. . . . .
. . . . .

Considering the dots as being arranged in horizontal rows, there are 3 rows with 5 dots in each row; considering them as being arranged in vertical rows, there are 5 rows with 3 dots in each row; and reckoning in either way we include all the dots.

(*c.*) Now, if these rows were extended in either direction, always being kept equal to each other, it is evident that the number of rows reckoned in one direction would always be equal to the number of dots that would be in a row were the rows reckoned in the other direction, and that all the dots would be reckoned in both instances. The number that represents the multiplicand when the rows are reckoned in one direction, will represent the multiplier when they are reckoned in the other, while the product, or number of dots, will be unaltered.

(*d.*) Hence, it must always be true that it makes no difference with the product which of the two factors is taken for a multiplier, provided the other be taken as the multiplicand. It will generally be most convenient to consider the larger factor as the multiplicand, though not always so.

NOTE. — In changing the order of factors, the one taken for the multiplier should always be regarded as an abstract number, (see **74**, *g*,) while the other should take the denomination of the original multiplicand. Thus, 4 times 3 apples = 12 apples; or changing the order of the factors, we should have 3 times 4 apples = 12 apples. In the first case, 4 is an abstract, and 3 a concrete, number; but in the second, 4 is a concrete, and 3 an abstract, number.

So 6 times $8 = 8 times $6; 4 times $.09 = 9 times $.04; 5 times $4.69 = 469 times $.05; &c.

76. *Simple Multiplication. — When only one Factor is a large Number.*

When either factor is a large number, it will be well to consider its denominations separately, and, if we write the results as we obtain them, to begin with the lowest denomination.

What will 7 acres of land cost at $75.69 per acre?

Reasoning Process. — If 1 acre costs $75.69, 7 acres will cost 7 times $75.69, which can be found by multiplying it by 7.

In performing the requisite multiplication, the numbers are usually written in some convenient way, as the following: —

$75.69	= Multiplicand.
7	= Multiplier.
$529.83	= Product.

Explanation. — Beginning at the right hand, or lowest denomination of the multiplicand, we have 7 times 9 hundredths = 63 hundredths, or 6 tenths and 3 hundredths.

Writing the 3 in the hundredths' place, and reserving the 6 tenths to add to the product of the tenths by 7, we have 7 times 6 tenths = 42 tenths, and 6 tenths added, = 48 tenths, = 4 units and 8 tenths.

Writing the 8 tenths, and reserving the 4 units to add to the product of the units by 7, we have 7 times 5 units = 35 units, and 4 units added, = 39 units, = 3 tens and 9 units.

Writing the 9 units, and reserving the 3 tens to add to the product of the tens by 7, we have 7 times 7 tens = 49 tens, and 3 tens added, = 52 tens, which, being our last product, we write. The result, then, is 529.83.

NOTE. — As soon as practicable, the explanation should be abbreviated, so as to name only results. Thus, 63 hundredths; 42, 48 tenths, 35, 39 units; 49, 52 tens. *Ans.* $529.83.

77. *Multiplication of Compound Numbers.*

What will 8 casks of wine cost at £3 9 s. 7 d. per cask?

Reasoning Process. — If 1 cask costs £3 9 s. 7 d., 8 casks will cost 8 times £3 9 s. 7 d., which can be found by multiplying it by 8.

WRITTEN WORK.

£.	s.	d.	
3	9	7	= Multiplicand.
		8	= Multiplier.
27	16	8	= Product.

Explanation. — Beginning with the lowest denomination, we have 8 times 7 d. = 56 d., which, since 12 d. = 1 s., must equal as many shillings as there are times 12 in 56, which are 4 times, with a remainder of 8. Hence, 56 d. = 4 s. 8 d.

Writing the 8 d., and reserving the 4 s. to add to the shillings of the

next product, we have 8 times 9 shillings = 72 shillings, and 4 shillings from the former product added, are 76 shillings, which, since 20 s = £1, must equal as many pounds as there are times 20 in 76, which are 3 times, with a remainder of 16. Hence, 76 s. = £3 16 s.

Writing the 16 s., and reserving the £3 to add with the pounds of the next product, we have 8 times £3 = £24, and £3 added, = £27, which, being the last product, we write.

78. *Methods of Proof.*

First Method. — Go over the work a second time in the same manner as before.

Second Method. — Consider the multiplicand as the multiplier, and see if this gives the same result as before.

The figures being in this way presented in a different order, we shall not be liable to repeat any mistake we may have made in the first work.

Third Method. — Write out by itself the product of the multiplication of each denomination, beginning either at the left or right, and afterwards add these products together. The sum should equal the former product.

Below is the written work of the examples in **76** and **77**, as proved by beginning at the left, and writing each denomination of the product separately.

Example 1. 75.69 × 7

490.	= Product of 70 by 7.
35.	= Product of 5 by 7.
4.2	= Product of .6 by 7.
.63	= Product of .09 by 7.
529.83	= Product of 75.69 by 7 = former Product.

Example 2.

£ 3 9 s. 7 d. × 8

£24			= £24	= Product of £3 by 8.
	72 s.		= £ 3 12 s.	= Product of 9 s. by 8
		56 d.	= 4 s. 8 d.	= Product of 7 d. by 8.
£24	72 s.	56 d.	= £27 16 s. 8 d.	= Product of £3 9 s. 7 d by 8 = former Product.

Fourth Method. — Another method of proof is, after hav-

ing written out the work as at first performed, to begin at the left hand, thus:—

$$\begin{array}{r} 75.69 \\ 7 \\ \hline 529.83 \end{array}$$

7 times 7 tens are 49 tens; but as there are 52 tens in the product, 3 tens must have come from the product of the lower denominations. 3 tens = 30 units, and adding to this the 9 units written in the units place, we find there ought to be 39 units in the product. 7 times 5 units are only 35 units; hence, if the work be right, 4 units must have come from the product of the lower denominations. 4 units = 40 tenths, and adding to this the 8 tenths written in the tenths' place of the product, we find that there ought to be 48 tenths in the product. 7 times 6 tenths are only 42 tenths; hence, if the product be right, 6 tenths must have come from the product of the hundredths. 6 tenths = 60 hundredths, and adding to this the 3 hundredths written in the hundredths' place, we find that there ought to be 63 hundredths in the product; and as 7 times 9 hundredths are 63 hundredths, we may infer that the work is correct.

$$\begin{array}{rrr} £. & s. & d. \\ 3 & 9 & 7 \\ & & 8 \\ \hline 27 & 16 & 8 \end{array}$$

8 times £3 = £24. But as in the written product there are £27, £3 must have come from the lower denominations. £3 = 60 s., and adding to this the 16 s. written in the shillings of the product, we find that there ought to be 76 shillings in the product. 8 times 9 s. = 72 s.; therefore, if the work be right, 4 shillings must have come from the lower denominations. 4 s. = 48 d., and adding to this the 8 d. already written, we find there ought to be 56 d. in the product; and as 8 times 7 d. = 56 d., we infer that the work is correct.

If the computer should find by multiplying numbers in one method a result different from that obtained by multiplying the same numbers in some other method, he may be sure that there is an error in one operation or the other, and he should examine his work patiently till he finds it. No person who is willing to allow an error to pass undetected can be a good arithmetician. (See **54.**)

79. *Problems.—Multiplier a single Figure.*

NOTE.—This article includes reduction descending. (See Note after 36th example.)

1. What is the product of 84687 × 4?
2. What is the product of .0078673 × 7?
3. What is the product of 237.904 × 8?
4. What is the product of 20078 × 9?
5. What is the product of .00978 × 6?
6. What is the product of 796.783 × 7?
7. What is the product of .00978 × 6?
8. What is the product of 37842 × 8?
9. What is the product of .7948 × 8?

10. 1 pound Avoirdupois of distilled water contains 27.7015 cubic inches. How many cubic inches will 8 pounds contain?

Reasoning Process. — If 1 pound contains 27.7015 cubic inches, 8 pounds will contain 8 times 27.7015 cubic inches, which can be found by multiplying it by 8.

11. What will 7 acres of land cost at $94.839 per acre?

12. 1 pound Troy of distilled water contains 22.794377 cubic inches. How many cubic inches will 8 pounds contain?

13. How many cubic inches are there in 7 cubic feet?

14. How much will it cost to build 9 miles of railroad at $19783.27 per mile?

15. How much will 3 farms cost at $3879.86 each?

16. How many feet would a man walk in 6 days at the rate of 56487 feet per day?

17. How many miles would a ship sail in 9 weeks, if she sails at the rate of 1198.47 miles per week?

18. How many inches are there in 4 miles, there being 63360 inches in 1 mile?

19. How many pounds are there in 5 loads of hay, each weighing 2794 pounds?

20. How many acres in 7 lots, each containing 24.74386 acres?

21. How many square rods in a road 754 rods long and 4 rods wide?

Reasoning Process. — Since a space 1 rod long and 1 rod wide contains 1 square rod, a space 754 rods long and 1 rod wide must contain

754 square rods, and a space 754 rods long and 4 rods wide must contain 4 times 754 square rods, which may be found by multiplying 754 by 4. (See **40**, Note.)

22. How many square feet in a walk 796 feet long and 9 feet wide?

23. How many square feet in a wall 437 feet long and 6 feet high?

24. Mr. Haven's garden is 124 feet long and 97 feet wide, and is enclosed by a tight board fence 5 feet high. How many square feet of boards are there in the fence?

Suggestion to the Student. — Draw a plan of the garden.

25. Mr. Haven laid out a gravel walk, 4 feet wide, within the fence, and extending all around the garden. How many square feet did it contain?

Suggestion to the Student. — Draw a plan of the garden and walk.

26. What will 49.67 barrels of apples cost at $3 per barrel?

Reasoning Process. — If 1 barrel costs 3 dollars, 49.67 barrels will cost 49.67 times 3 dollars, which is equivalent to 3 times 49.67 dollars.

The work would be written thus: —

$$\begin{array}{rl} \$49.67 & = \text{Multiplicand.} \\ 3 & = \text{Multiplier.} \\ \hline \$149.01 & = \text{Product.} \end{array}$$

Another Reasoning Process. — It is evident that if each barrel should cost a dollar, all would cost as many dollars as there are barrels bought, which in this instance would be $49.67. But if, at $1 per bbl., they cost $49.67, at $3 per bbl., they would cost 3 times $49.67.

The work may be written thus: —

$$\begin{array}{rl} & 49.67 \text{ bbl. at } \$3 \text{ per bbl.} \\ \hline & \$\ 49.67 = \text{cost at } \$1 \text{ per bbl.} \\ 3 \text{ times } \$49.67 = & \$149.01 = \text{cost at } \$3 \text{ per bbl.} = \textit{Ans.} \end{array}$$

27. What will 3749 lb. of saleratus cost at .08 per lb.?

28. What will 178.69 bbl. of flour cost at $7 per bbl.?

29. What will 27.96 firkins of butter cost at $9 per firkin?

30. What will be the weight of 3794 cannon balls, each ball weighing 8 lb?

31. If a soldier eats 4 lb. of meat in a week, how many pounds will 2896 soldiers eat in the same time?

32. What will 4736 casks of raisins cost at $7 per cask?

33. If a vessel sails uniformly at the rate of 9 miles per hour, how far will she sail in 476 hours?

34. How many bushels in 1487 barrels, if each barrel holds 3 bushels?

35. How much will 3479 window weights weigh, if each weighs 6 pounds?

36. 82 bu. 3 pk. 5 qt. 1 pt. = how many pints?

Reasoning Process. — Since there are 4 pecks for every bushel, there must be 4 times as many pecks as bushels, or, in this case, 4 times 82 pecks, which are 328 pecks, and adding 3 pecks to this gives 331 pecks as the value of 82 bu. 3 pk.

Since there are 8 quarts for every peck, there must be 8 times as many quarts as there are pecks, or, in this case, 8 times 331 quarts, which are, &c.

Another Form. — Since 1 bushel = 4 pecks, 82 bushels must equal 82 times 4 pecks, equivalent to 4 times 82 pecks, which are 328 pecks, and adding 3 pecks to this gives 331 pecks, as the value of 82 bu. 3 pk.

Since 1 peck = 8 quarts, 331 pecks must equal 331 times 8 quarts, equivalent to 8 times 331 quarts, which are, &c.

NOTE. — Questions like the above, requiring that the value of numbers of one denomination shall be expressed in terms of some lower denomination, are called questions in Reduction Descending; but as they do not differ at all from other questions in multiplication, they require no separate treatment. In performing them, there is no need of writing the multipliers, as they may be known from the table of the weight or measure used. In reducing to any denomination of which there are units already expressed, it will usually be more convenient to add those units at the time we make the multiplication.

METHOD OF WRITING THE WORK.

82 bu. 3 pk. 5 qt. 1 pt.

331 pk. = 82 bu. 3 pk.

2653 qt. = 82 bu. 3 pk. 5 qt.

5307 pt. = 82 bu. 3 pk. 5 qt. 1 pt.

The following abbreviated form may be adopted after the above is perfectly familiar:—

82 bu. 3 pk. 5 qt. 1 pt.

331 pk.
2653 qt.
5307 pt. = *Ans.*

37. 97 bu. 2 pk. 7 qt. 1 pt. = how many pints?
38. 187 bu. 1 pk. 6 qt. 0 pt. 3 gi. = how many gills?
39. 238 gal. 3 qt. 1 pt. 2 gi. = how many gills?
40. 596 gal. 2 qt. 1 pt. 3 gi. = how many gills?
41. 9 ℔ 11℥ 6ʒ 2℈ = how many scruples?
42. 8 ℔ 3℥ 7ʒ 1℈ = how many scruples?
43. 937 le. 1 m. 5 fur. = how many furlongs?
44. 286 le. 2 m. 7 fur. = how many furlongs?
45. What will 37 bu. 2 pk. 3 qt. 1 pt. of cherries cost at 4 cents per pint?
46. What will 114 bu. 3 pk. 2 qt. of wheat cost at 1 cent per gill?
47. What will 17 gal. 3 qt. 1 pt. 3 gi. of oil cost at 5 cents per gill?
48. What will 93 gal. 2 qt. 1 pt. 2 gi. of brandy cost at 8 cents per gill?
49. What is the product of £22 18 s. 8 d. 2 qr. by 7?

£.	s.	d.	qr.	
22	18	8	2	= Multiplicand.
			7	= Multiplier.
160	10	11	2	= Product.

Explanation.—7 times 2 qr. = 14 qr., which (since 4 qr. = 1 d.,) equal as many pence as there are times 4 in 14, which are 3 times, with a remainder of 2. Hence, 14 qr. = 3 d. 2 qr.

7 times 8 d. = 56 d., and 3 d. added = 59 d., which (since 12 d. = 1 s.) equal as many shillings as there are times 12 d. in 59 d., which are 4 times, with a remainder of 11. Hence, 59 d. = 4 s. 11 d.

7 times 18 s. = 126 s., and 4 s. added = 130 s., which (since 20 s

= £1) equal as many pounds as there are times 20 in 130, which are 6 times, with a remainder of 10. Hence, 130 s. = £6 10 s.

7 times £22 = £154, and £6 added = £160.

Hence, the product is £160 10 s. 11 d. 2 qr.

NOTE.—When any denomination to be multiplied is very near a unit of the next higher, the work may frequently be much shortened by considering it a unit of that higher denomination, and subtracting for its deficiency in value.

For instance, in the example above: since 18 s. = £1 — 2 s., 7 times 18 s. must equal £7 — 14 s., or £6 6 s., to which adding 4 s., (from the previous product,) we have £6 10 s., as before.

What is the product —

50. Of £29 8 s. 11 d. 1 qr. multiplied by 9?

51. Of 37 T. 19 cwt. 2 qr. 24 lb. 11 oz. 7 dr. multiplied by 3?

52. Of 273 bu. 1 pk. 5 qt. 1 pt. multiplied by 2?

53. Of 9 lb. 8 oz. 13 dwt. 22 gr. multiplied by 7?

54. Of 28 da. 17 h. 27 m. 58 sec. multiplied by 6?

55. Of 47 ℔ 8 ℥ 7 ʒ 2 ℈ 18 gr. multiplied by 4?

56. Of 238 gal. 1 qt. 1 pt. 3 gi. multiplied by 9?

57. Of 674 lb. 4 oz. 19 dwt. 20 gr. multiplied by 8?

58. Of 23 lb. 4 oz. 16 dwt. 22 gr. multiplied by 8?

59. Of 13 cwt. 2 qr. 17 lb. 13 oz. 9 dr. multiplied by 6?

60. Of 6 T. 18 cwt. 1 qr. 24 lb. 2 oz. 1 dr. multiplied by 7?

61. Of 9 bu. 3 pk. 7 qt. multiplied by 9?

62. Of 9 gal. 2 qt. 1 pt. multiplied by 5?

63. Of 8 w. 1 da. 23 h. 59 m. 56 sec. multiplied by 7? *

64. Of £8 19 s. 11 d. 3 qr. multiplied by 8?

65. Of 9 T. 19 cwt. 3 qr. 24 lb. 14 oz. multiplied by 7?

66. Of 7 lb. 11 oz. 19 dwt. 21 gr. multiplied by 4?

67. Of 483 yd. 3 qr. 1 na. multiplied by 9?

68. Of 4978 bu. 3 pk. 5 qt. multiplied by 5?

69. Of 37 lb. 11 oz. 19 dwt. 23 gr. multiplied by 6?

70. Of £5871 18 s. 4 d. 1 qr. multiplied by 2?

---

* In performing this example, much labor may be saved by observing that the multiplicand is only 4 seconds less than 8 w. 2 da. Similar things can frequently be applied, as in several of the subsequent examples.

80. *Multiplication by Factors.*

(*a.*) It often happens that a number used as a multiplier is the product of two or more factors. In such cases it is sometimes convenient to resort to processes similar to those explained below.

NOTE.—In writing the work here, as in several other places throughout the book, we have used letters for convenience of indicating to the eye the operations which have been performed, and the relations which the numbers bear to each other. For instance, in the first four given below, "a = 743," means that the letter "a" stands for 743. "2 × a = b = 1486," means that two times the number "a," i. e., 2 times 743, = the number represented by "b," which is 1486. "6 × b = 12 × a,' means that 6 times the number "b" (i. e., 6 times 1486) equals 12 times the number "a," (i. e., 12 times 743.)

The student will observe that the letter which in one form stands for one number, may in another form stand for a different number. Thus, in the first form "b" is used to represent 1486, while in the second it is used to represent 2972.

How many are 12 times 743?

FIRST METHOD.

$$\begin{array}{r} a = 743 \\ 2 \\ \hline 2 \times a = b = 1486 \\ 6 \\ \hline 6 \times b = 12 \times a = 8916 \end{array}$$

SECOND METHOD.

$$\begin{array}{r} a = 743 \\ 4 \\ \hline 4 \times a = b = 2972 \\ 3 \\ \hline 3 \times b = 12 \times a = 8916 \end{array}$$

THIRD METHOD.

$$\begin{array}{r} a = 743 \\ 3 \\ \hline 3 \times a = b = 2229 \\ 2 \\ \hline 2 \times b = 6 \times a = c = 4458 \\ 2 \\ \hline 2 \times c = 12 \times a = 8916 \end{array}$$

FOURTH METHOD.

$$\begin{array}{r} a = 743 \\ 2 \\ \hline 2 \times a = b = 1486 \\ 2 \\ \hline 2 \times b = 4 \times a = c = 2972 \\ 3 \\ \hline 3 \times c = 12 \times a = 8916 \end{array}$$

Explanations.

First Method. — Since 12 = 6 times 2, 12 times a number must be equal to 6 times 2 times the number.

Second Method. — Since 12 = 3 times 4, 12 times a number must be equal to 3 times 4 times the number.

Third Method. — Since 12 = 2 times 2 times 3, 12 times a number must be equal to 2 times 2 times 3 times the number.

Fourth Method. — Since 12 = 3 times 2 times 2, 12 times a number must equal 3 times 2 times 2 times the number.

(*b.*) Solve the following examples in a similar manner.

What is the value —

1. Of 879 × 18?
2. Of 9874 × 27?
3. Of 8764 × 36?
4. Of 6427 × 42?
5. Of 4.379 × 64?
6. Of 2976.4 × 28?
7. Of 2377 T. 17 cwt. 2 qr. 19 lb. 6 oz. 11 dr. × 63?
8. Of 27 ℔ 8 ℥ 6 ʒ 2 ℈ 17 gr. × 45?
9. Of 19 w. 5 d. 17 h. 38 m. 29 sec. × 36?
10. Of £28 13 s. 10 d. 2 qr. × 35?
11. Of 48 lb. 10 oz. 16 dwt. 19 gr. × 24?
12. Of 837 bu. 3 pk. 6 qt. 1 pt. × 18?

(*c.*) The most useful application of the foregoing principle is made when the multiplier is some multiple of 10.

13. What is the product of 8746 × 400?

Solution. — Since 400 = 4 times 100, 400 times a number must equal 4 times 100 times the number: to obtain which we have only to remove the point two places towards the right, (**24**) and multiply by 4. Hence,

```
8746
  400
-------
3498400
```

14. What is the product of 9.7487 × 7000?

Solution. — Since 7000 = 7 times 1000, 7000 times a number must equal 7 times 1000 times the number; to obtain which we have only to multiply by 7, and remove the point three places towards the right. Hence

```
9.7487
    7000
--------
68240.9
```

(*d.*) In like manner, to multiply by 60, we may multiply by 6, and remove the point one place towards the right; to multiply by 9000000, we may multiply by 9, and remove the point six places towards the right. In any case, all places left vacant between the number and the point must be filled with zeros. (See **15.**)

What is the product —

15. Of 874379 × 20?	19. Of 627.34 × 80?
16. Of 27.9863 × 5000?	20. Of 9137.6 × 30000?
17. Of 714.26 × 90000?	21. Of 84273 × 60?
18. Of 62.794 × 40?	22. Of 7643 × 7000000?

81. *When both Factors are large Numbers.*

(*a.*) We can find the product of two numbers by multiplying one of them by the parts into which we choose to separate the other, and then adding the products thus obtained together.

Illustration. — 8 times 7 = 6 times 7 + 2 times 7 = 3 times 7 + 5 times 7 = 7 times 7 + once 7 = 4 times 7 + 4 times 7 = 56.

(*b.*) This principle, and the one illustrated in article **80**, are ordinarily employed when the multiplier contains several denominations.

Illustrations. — We usually get 83 times a number, by adding together 80 times the number and 3 times the number.

We get 647 times a number by adding together 600 times the number, 40 times the number, and 7 times the number.

We get 8009 times a number by adding together 8000 times the number and 9 times the number.

(*c.*) It can make no difference which part of a number we multiply by first, provided we multiply by all its parts; yet for the sake of uniformity it may be well generally to begin with the lowest denomination.

1. Suppose that we are required to find the product of 5794 × 78?

Explanation. — We may find the product by 8 in the usual manner. To find the product by 70, we have only to multiply by 7, and remove the point one place towards the right, or, which is the same thing, the

figures one place towards the left. Adding these results together will give 78 times the number.

WRITTEN WORK.

$$
\begin{array}{rcl}
a = & 5794 & = \text{Multiplicand.} \\
 & 78 & = \text{Multiplier.} \\
\hline
8 \times a = b = & 46352 & = \text{Product by 8.} \\
70 \times a = c = & 405580 & = \text{Product by 70.} \\
\hline
b + c = 78 \times a = & 451932 & = \text{Product by 78.}
\end{array}
$$

(*d.*) Since the product of the multiplication by the units is sufficient to fix the place of the figures in the subsequent products, the zero at the right of the second product need not have been written. The product would then stand, —

$$
\begin{array}{rcl}
4345 & = & \text{Product by } 5. \\
6083 & = & \text{" " } 70. \\
\hline
65175 & = & \text{" " } 75.
\end{array}
$$

(*e.*) Examples in which the multiplier consists of more than two figures are performed in a similar way.

2. What is the product of 780.69 × 20850 ?

WRITTEN WORK.

$$
\begin{array}{rcl}
a = & 780.69 & = \text{Multiplicand.} \\
 & 20850 & = \text{Multiplier.} \\
\hline
a \times 50 = b = & 39034.50 & = \text{Product by 50.} \\
a \times 800 = c = & 624552. & = \text{Product by 800.} \\
a \times 20000 = d = & 156138 & = \text{Product by 20000.} \\
\hline
b + c + d = & 16277386.50 & = \text{Product by 20850.}
\end{array}
$$

NOTE. — In practice, only the necessary *figures* should be written Thus: —

$$
\begin{array}{r}
780.69 \\
20850. \\
\hline
39034.50 \\
624552. \\
156138 \\
\hline
16277386.50
\end{array}
$$

What is the product of —

3. Of 80276 × 39?
4. Of 298794 × 148?
5. Of 273986 × 27?
6. Of 4943 × 78?
7. Of 23879 × 2741?
8. Of 84976 × 203?
9. Of 25873 × 506?
10. Of 47.296 × 37?
11. Of 423758 × 6200?
12. Of 594.27 × 30200?
13. Of 30678 × 427?
14. Of 5.796 × 238?
15. Of 45.718 × 432?
16. Of .00876 × 74?
17. Of 67.968 × 327?
18. Of 970062 × 37?
19. Of 29743 × 806?
20. Of 8427 × 3076?
21. Of 279437 × 27623?
22. Of 8.64298 × 43000?

(*f.*) It is obvious that the product obtained by multiplying one number by the difference of two numbers, is equivalent to the difference of the products obtained by multiplying the numbers separately by the two numbers.

Illustration. — 5 times 3 = 7 times 3 — 2 times 3 = 8 times 3 — 3 times 3 = 15 times 3 — 10 times 3 = 29 times 3 — 24 times 3, &c.

(*g.*) This principle is the reverse of that stated in (*a*,) and can often be advantageously applied, as illustrated below.

(*h.*) To multiply by 99, observe that since 99 = 100 — 1, 99 times a number must equal 100 times the number minus once the number. For example, —

99 times 837 = 100 times 837 — 837 = 83700 — 837 = 82863.

(*i.*) To multiply by 999, observe that since 999 = 1000 — 1, 999 times a number must equal 1000 times the number minus once the number. For example, —

999 times 14.67 = 1000 times 14.67 — 14.67 = 14670 — 14.67 = 14655.33.

(*j.*) To multiply by 699, observe that since 699 = 700 — 1, 699 times a number must equal 700 times the number — once the number For example,

699 × 5784 = 700 × 5784 — 5784 = 4048800 — 5784 = 4043016

(*k.*) In like manner we should have —

49 × 785 = 50 × 785 — 785.

98 × 4697 = 100 × 4697 — 2 × 4697.

79996 × 394845 = 80000 × 394845 — 4 × 394845.

What is the product —

23. Of 48673 × 29?
24. Of 37848 × 99?

25. Of 69435 × 69?
26. Of 29485 × 999?
27. Of 7486 × 998?
28. Of 4278 × 3999?
29. Of 6786 × 49?
30. Of 4296 × 79?
31. Of 28643 × 999?

82. *Abbreviated Method.*

(*a.*) When the multiplier consists of more than one denomination, much labor in writing figures may be saved by applying the principles illustrated in the following examples: —

1. What is the product of 8356 multiplied by 79?

Preliminary Explanation. — It is evident that, in performing the required multiplication, we shall obtain units by multiplying 6 units by 9 units. We shall obtain tens by multiplying 5 tens by 9, and 6 units by 7 tens, and we may have some from the product of the units. We shall obtain hundreds by multiplying 3 hundreds by 9, and 5 tens by 7 tens, and we may have some from the former products. We shall obtain the other denominations in a similar manner. We may then proceed thus, writing the figures of each denomination as usual: —

WRITTEN WORK.

8356	Multiplicand.
79	Multiplier.
660124	Product.

Explanation. 9 times 6 units = 54 units. We write 4 units.

9 times 5 tens = 45 tens, + 5 tens (from the product of the units) = 50 tens, + 7 tens times 6, or 42 tens, = 92 tens = 9 hundreds and 2 tens. We write 2 tens.

9 times 3 hundreds = 27 hundreds, + 9 hundreds (from the previous product) = 36 hundreds, + 7 tens times 5 tens, or 35 hundreds, = 71 hundreds = 7 thousand and 1 hundred. We write 1 hundred.

9 times 8 thousands = 72 thousands, + 7 thousands (from the previous product) = 79 thousands, + 7 tens times 3 hundreds, or 21 thousands, = 100 thousands = 10 ten-thousands. We write 0 in the thousands' place of the product.

7 tens times 8 thousands = 56 ten-thousands, + 10 ten-thousands (from the previous product) = 66 ten-thousands, which we write.

The multiplication being now completed shows the answer to be 660124

(*b.*) The following exhibits the necessary operations on the numbers, and is practically a much more convenient solution than the full form above given.

$$9 \times 6 = 54 \text{ units.}$$

5 (from last product) $+ 9 \times 5, + 7 \times 6 = 5 + 45 + 42 = 92$ tens.

9 (from last product) $+ 9 \times 3, + 7 \times 5 = 9 + 27 + 35 = 71$ hundreds.

7 (from last product) $+ 9 \times 8, + 7 \times 3 = 7 + 72 + 21 = 100$ thousands.

10 (from last product) $+ 7 \times 8 = 10 + 56 = 66$ ten-thousands.

This gives for an answer 660124, as did the first method.

(*c.*) The last process being understood, the work may be still further abbreviated by omitting to name the factors used.

Thus, 54 units = 5 tens and 4 units.

$45 + 5 = 50, + 42 = 92$. 92 tens = 9 hundreds and 2 tens.

$27 + 9 = 36, + 35 = 71$. 71 hundreds = 7 hundreds and 1 thousand.

$72 + 7 = 79, + 21 = 100$. 100 thousands = 10 ten-thousands and 0 thousands.

$56 + 10 = 66$. 66 ten-thousands.

Answer, as before, 660124.

(*d.*) Finally, the work may be abbreviated so as to name only results: —

54 units = 5 tens and 4 units.

45, 50, 92 tens = 9 hundreds and 2 tens.

27, 36, 71 hundreds = 7 thousands and 1 hundred.

72, 79, 100 thousands = 10 ten-thousands and 0 thousands.

56, 66 ten-thousands.

NOTE. — The above methods are much more expeditious than is the method of writing the product by each figure of the multiplier separately, and are no more liable to inaccuracy.

2. How much will 97 acres of land cost at $347 per acre?

3 If a cubic yard of sand weighs 2537 lb., how much will 88 cubic yards weigh?

4. How many pounds are there in 18 T. 17 cwt. 1 qr.?

5. If a ship sails 96 miles in one day, how far will she sail in 247 days?

6. Bought 24 bundles of hay, each bundle containing 497 lb. How many pounds were there in all?

7. Bought 2947 gallons of oil at $.84 per gallon. How much did it cost? Sold it for $.97 per gallon. How much was received for it? What was the gain on it?

8. Mr. Russell bought 86 balls of twine, each ball containing 8794 ft., and Mr. Greene bought 57 times as much. How many feet of twine did Mr. Russell buy? How many did Mr. Greene buy?

9. How much will 83 casks of old wine cost at $138.47 per cask?

10. How much will 67 tons of lead cost at $139.48 per ton?

11. Mr. Hovey bought 6247 feet of land, and Mr. Ewell bought 94 times as much. How many feet did Mr. Ewell buy?

12. How many pounds are there in 958 boxes of sugar, each box containing 743.67 lb.?

NOTE.—The products and sums employed in solving the above example are given below, but the pupil should be prepared to give a more full explanation.

56 hundredths = 5 tenths and 6 hundredths.

5 + 48 + 35 = 88. 88 tenths = 8 units and 8 tenths.

8 + 24 + 30 + 63 = 125. 125 units = 12 tens and 5 units.

12 + 32 + 15 + 54 = 113. 113 tens = 11 hundreds and 3 tens.

11 + 56 + 20 + 27 = 114. 114 hundreds = 11 thousands and 4 hundreds.

11 + 35 + 36 = 82. 82 thousands = 8 ten-thousands and 2 thousands.

8 + 63 = 71. 71 ten-thousands.

The answer, therefore, is 712435.86 lb.

13. How many are 8795 times 96543?

NOTE.—By extending the principles before explained, we can write the final product at once, as below.

$$\begin{array}{r} 96543 \\ 8795 \\ \hline 849095685 \end{array}$$

In the following forms, two products are written: —

96543		
8795		
9171585	units	$= 95 \times 96543.$
8399241	hundreds	$= 8700 \times 96543.$
849095685		$= 8795 \times 96543.$

96543		
8795		
76751685	units	$= 795 \times 96543.$
772344	thousands	$= 8000 \times 96543.$
849095685		$= 8795 \times 96543.$

14. If a cubic foot of iron weighs 486.25 lb., how much will 347 cubic ft. weigh?

15. How many square feet are there in a rectangular lot, 4327 feet long and 249 feet wide?

16. How much will 48 acres of land cost at $23.968 per acre?

17. What will 798 tons of hay cost at $14.278 per ton?

18. I bought 287 bales of cloth, each bale containing 247.986 yards. How many yards did they all contain?

19. How many square inches are there in a lot 247 ft. long and 187 ft. wide?

20. What will 47983 yards of cloth cost at $2.83 per yd.?

21. What will 7894 bbl. of flour cost at $6.37 per bbl.?

22. How many solid inches in 5 C. 6 Cd. ft. 12 cu. ft. 1437 cu. in.?

23. How many dr. in 18 T. 16 cwt. 1 qr. 14 lb. 6 oz. 11 dr.?

24. A grain dealer sold 287 bushels of wheat at $1.294 per bushel, and 1479 bushels at $1.267 per bushel. What did he receive for it?

25. I bought 48 yards of broadcloth at $3.875 per yd., 153 yards of doeskin at $1.166 per yd., and 379 yards of cassimere at $1.125 per yd. What was the cost of the whole?

26. Mr. Aldrich owns 4 house lots, the first 328 ft. long and 189 ft. wide; the second 437 ft. long and 249 ft. wide;

the third 129 ft. long and 88 ft. wide; and the fourth 97 ft. long and 86 ft. wide. How many square feet are there in all of them?

27. Mr. Whitney sold 84 acres of land at $34.96 per acre, 138 acres at $27.58 per acre, and 427 acres at $49.64 per acre. How much did he sell the whole for?

28. A city merchant went into the country to purchase flour. He was absent from the city 27 days, and his expenses while absent were $7.386 per day. He bought 175 bbl. of flour at $5.875 per bbl., 516 bbl. at $5.948 per bbl., 1386 bbl. at $6.11 per bbl., and 827 bbl. at $6.087 per bbl. It cost him $.634 per bbl. to get the flour transported to the city. He sold 697 bbl. of it at $7.114 per bbl., 824 bbl. at $7.213 per bbl., and the rest at $6.978 per bbl. Did he gain or lose by the adventure, and how much?

29. A merchant bought 49.5 cases of cassimere, each case containing 297 yd., at $1.1875 per yd. It cost him $.125 per case to have the cloth removed to his store, and $.045 per case to have it hoisted into his loft. One case of the cloth was stolen from him; he sold 23 cases at $1.423 per yd., and the remainder at $1.357 per yd., agreeing to deliver it at a railroad depot, 1 mile from his store. It cost him $.158 per case to have it carried to the depot. Did he gain or lose on the cloth, and how much?

SECTION VII.

DIVISION.

83. *Definitions and Illustrations.*

(*a.*) DIVISION IS A PROCESS BY WHICH WE ASCERTAIN THE NUMBER OF PARTS OF A GIVEN SIZE INTO WHICH A GIVEN NUMBER MAY BE SEPARATED, OR BY WHICH WE

ASCERTAIN THE NUMBER OF UNITS THERE WILL BE IN EACH PART OBTAINED BY DIVIDING A GIVEN NUMBER INTO A GIVEN NUMBER OF EQUAL PARTS.

(*b.*) The following are questions in division: —

1. 42 = how many times 6 ?
2. What is the value of 54 divided by 9 ?
3. What is the value of 35 ÷ 5 ?
4. How many apples at 3 cents apiece can be bought for 24 cents ?
5. What is $\frac{1}{4}$ of 36 ?
6. If 8 apples cost 24 cents, how much will 1 apple cost ?

(*c.*) In the first of the above questions, we are required to find how many 6's, or parts of 6 each, there are in 42; in the second, how many 9's, or parts of 9 each, there are in 54; in the third, how many 5's, or parts of 5 each, there are in 35; in the fourth, how many 3's, or parts of 3 each, there are in 24; while in the fifth we are required to find how many units there will be in each part obtained by dividing 36 into 4 equal parts; and in the sixth, how many units there will be in each part obtained by dividing 24 into 8 equal parts.

(*d.*) This shows that there are two classes of questions in division, viz.: one class in which, knowing the number to be divided, and the number of units which each part is to contain, we are required to find the number of parts; and one in which, knowing the number to be divided, and the number of parts into which it is to be divided, we are required to find how many units there will be in each part; i. e., we are required to find a fractional part of a number. Both classes of questions may be solved by the same numerical process, though in their solution they require different reasoning processes.

(*e.*) The number to be divided is called the *dividend.* In the first class of questions the number which shows the size of each part, and in the second class that which shows the number of parts, is called the *divisor.* The result is called the *quotient.*

Illustrations. — In the first of the above questions, 42 is the dividend, 6 is the divisor, and the answer, 7, is the quotient.

In the second, 54 is the dividend, 9 is the divisor, and the answer, 6, is the quotient.

In the fifth, 36 is the dividend, 4 is the divisor, and the answer, 9, is the quotient.

(*f.*) The fourth example would be solved thus: —

If for 3 cents one apple can be bought, for 24 cents as many apples

can be bought as there are times 3 cents in 24 cents, which are 8 times. Therefore, 8 apples can be bought for 24 cents, when 1 apple can be bought for 3 cents. Here, 24 is the dividend, 3 is the divisor, and the answer, 8, is the quotient. The divisor and dividend are of the same denomination, and the quotient is the number of times the divisor is contained in the dividend, or the number of parts equal to the divisor which the dividend equals.

(*g*.) The sixth example would be solved thus : —

If 8 apples cost 24 cents, 1 apple will cost $\frac{1}{8}$ of 24 cents, which is 3 cents. Hence, 1 apple will cost 3 cents, if 8 cost 24 cents.

Here, 24 is the dividend, 8 is the divisor, and the answer, 3, is the quotient. The dividend and quotient are of the same denomination, and the divisor shows the number of times the quotient is taken in the dividend, or the number of parts equal to the quotient which the dividend equals.

(*h*.) To examine more fully the nature of the processes, let us see by what methods the answers to the fourth and sixth questions could be obtained.

(*i*.) It is obvious that, to determine the answer to the fourth question, we must find how many parts of 3 cents each there are in 24 cents; for each such part is the price of 1 apple.

We can do this by counting 24 cents into piles of 3 cents each, and then counting the number of piles ; — by finding how many threes must be added to make 24 ; — or by finding how many times 3 equal 24, by our knowledge of multiplication. This last process is division.

(*j*.) To determine the answer to the sixth question, we must find how many cents there will be in each part obtained by separating 24 cents into 8 equal parts.

We can do this by laying out 24 cents into 8 equal piles, and then counting the number of cents in each pile ; — or by finding, by our knowledge of multiplication, what number must be taken 8 times to equal 24 ; or, which will give the same numerical answer, by finding how many times 8 equal 24. The last process is division.

NOTE. — It will be seen, that in the first class of questions, the divisor and dividend are of the same denomination, and that the quotient expresses the number of times the divisor is contained in the dividend, or the number of parts equal to the divisor which must be taken to produce the dividend ; while in the second class, the divisor expresses the number of parts into which the dividend is to be divided, and the quotient expresses the number of units in each part; thus making the dividend and quotient of the same denomination.

(*k*.) Practically, division is the reverse of multiplication. In the

latter, the factors are given, and we are required to find the product while in the former, one of the factors and their product are given, and we are required to find the other; or, when the dividend will not exactly contain the divisor, one factor and the product of the two plus the remainder are given, and we are required to find the other factor and the remainder.

(*l.*) The divisor is always the given factor, the quotient is the required factor, and the dividend is the product, or the product plus the remainder. The remainder is always less than the divisor.

(*m.*) The following examples illustrate this: —

1. "3 times 8," is a question in multiplication. The factors 3 and 8 are given, and the product, 24, is required.

2. "How many times 8 = 24," or "24 ÷ 8?" are questions in division, in which the factor 8, and the product 24, are given, and the missing factor, 3, is required.

3. "$\frac{1}{3}$ of 24, or 3 times what number = 24?" are questions in division, in which the factor 3, and the product 24, are given, and the missing factor is required.

4. "In 8 times 7, plus 5," the factors 8 and 7 are given, and their product, plus 5, which is 61, is required.

5. "In 61 ÷ 7," or "61 = how many times 7," the factor 7, and the sum of the product, and remainder, which is 61, are given, and the other factor and remainder are required.

6. "In $\frac{1}{7}$ of 61," or "7 times what number = 61," the factor 7, and the sum of the product, plus the remainder, are given, and the other factor and remainder are required.

84. *Methods of Proof.*

From the preceding illustrations it is evident, —

First. That, where there is no remainder, the divisor multiplied by the quotient must produce the dividend; and that the dividend divided by the quotient must produce the divisor.

Second. That if, when there is a remainder, the divisor and quotient be multiplied together, and the remainder be added to their product, the result will equal the dividend.

Third. That if the remainder be subtracted from the dividend, and the remainder thus obtained be divided by the divisor, the result will equal the quotient; or, if it be divided by the quotient, the result will equal the divisor.

NOTE. — The remainder should always be less than the divisor; for,

if it is not, the dividend will contain the divisor more times than is indicated by the quotient figure.

85. *Examples.* — *Quotient a single Figure.*

(*a.*) How many oranges at 7 cents apiece can be bought for 61 cents?

Reasoning Process. — If for 7 cents 1 orange can be bought, as many oranges can be bought for 61 cents as there are times 7 in 61.

Explanation. — To perform the necessary division, we observe that as $56 = 8$ times 7, 61 must equal 8 times 7, with a remainder of 5; or, since $5 \div 7 = \frac{5}{7}$, $61 = 8\frac{5}{7}$ times 7.

Hence, the quotient is 8, and the remainder is 7, or the complete quotient is $8\frac{5}{7}$, which shows that 8 oranges can be bought, leaving 5 cents unused, or that $8\frac{5}{7}$ oranges can be bought with all the money.

First Proof. — See if the price of 8 oranges at 7 cents apiece, added to 5 cents, will make 61 cents.

Second Proof. — Considering that $8\frac{5}{7}$ oranges are bought, see if they will cost 61 cents at 7 cents apiece.

(*b.*) The above are proofs of the correctness of the entire work; the following only test the correctness of the division.

Third Proof. 8 times $7 = 56$, and 5 added $= 61 =$ dividend.

Fourth Proof. 8 times $7 = 56$; and 56 from 61 leaves $5 =$ remainder.

Fifth Proof. 5 from $61 = 56$, and $56 \div 7 = 8 =$ quotient.

(*c.*) The work may be written thus: —

$$\begin{array}{rl} & \text{Dividend.} \\ \text{Divisor } 7\ ) & \underline{61} - 5 = \text{Remainder.} \\ & 8 = \text{Quotient.} \end{array}$$

(*d.*) Or, by expressing the division of the remainder, by placing it in the form of a fraction, we have, —

$$\begin{array}{rl} \text{Divisor } 7\ ) & \underline{61} = \text{Dividend.} \\ & 8\frac{5}{7} = \text{Quotient.} \end{array}$$

In the first form, the remainder is undivided, and is of the same denomination as the dividend. It is placed opposite the dividend, with the minus sign between, to indicate that all the dividend except that

has been divided. Indeed, it might have been subtracted from the dividend without affecting the quotient.

In second form, the entire dividend is divided. Hence, the $\frac{5}{7}$ is not a remainder, but is a part of the quotient.

NOTE. — The distinction between a remainder and a fractional quotient should be carefully observed.

What is the quotient —

1. Of $37 \div 4$?
2. Of $83 \div 10$?
3. Of $39 \div 7$?
4. Of $17 \div 2$?
5. Of $86 \div 9$?
6. Of $57 \div 6$?
7. Of $43 \div 7$?
8. Of $26 \div 3$?
9. Of $47 \div 5$?
10. Of $48 \div 9$?
11. Of $75 \div 8$?
12. Of $41 \div 7$?

13. How many apples at 3 cents apiece can be bought for 28 cents?

14. How many barrels of flour at $9 per barrel can be bought for $62?

15. How many yards of broadcloth at $6 per yard can be bought for $53?

NOTE. — The student should practise on examples like the above, till he can perform them with ease and rapidity.

86. *Dividend a large Number.*

(*a.*) When large numbers are to be divided, we begin with the highest denominations, and proceed as in the following example.

1. How many barrels of flour at $8 per barrel can be bought for $58455?

Reasoning Process. — If for $8, 1 barrel can be bought, as many barrels can be bought for $58455 as there are times 8 in 58455.

WRITTEN WORK.

$$\begin{array}{rl} & \text{Dividend.} \\ \text{Divisor} = 8\,) & 58455 - 7 = \text{Remainder.} \\ \hline & 7306 = \text{Quotient.} \end{array}$$

Explanation of Process of Dividing. 8 is contained in 58 thousands 7 thousand times, with a remainder of 2 thousands, (for 7000 times 8 = 56000, and 2000 added = 58000.) We therefore write 7 as the thousands' figure of the quotient.

Reducing the 2 thousands remaining to hundreds, and adding to them the 4 hundreds, gives 24 hundreds to be next divided. 8 is contained in 24 hundreds 3 hundred times, (for 300 times 8 = 2400.) We therefore write 3 as the hundreds' figure of the quotient.

As 8 is not contained as many as ten times in 5 tens, we write 0 as the tens' figure of the quotient, and reduce the 5 tens to units, considering them with the 5 units.

8 is contained in 55 units 6 times, with a remainder of 7 units, (for 6 times 8 = 48, and 7 added = 55.) We therefore write 6 as the units' figure of the quotient.

This gives 7306 for the quotient, and 7 for the remainder; or, since $7 \div 8 = \frac{7}{8}$, it gives $7306\frac{7}{8}$ for a quotient.

Hence, 7306 barrels of flour can be bought, leaving \$7 unused; or $7306\frac{7}{8}$ barrels can be bought with all the money.

The methods of proof are the same as those before given.

(*b.*) When the above explanation and the principles involved are fully understood, the names of the denominations may be omitted in writing the work. Thus: —

8 is contained 7 times in 58, with 2 remainder.
8 is contained 3 times in 24, with no remainder.
8 is contained 0 times in 5, with 5 remainder.
8 is contained 6 times in 55, with 7 remainder.

Hence, the quotient is 7306, and the remainder is 7; or the complete quotient is $7306\frac{7}{8}$.

(*c.*) In the following form, only the figures of the quotient are named.

7 thousands, 3 hundreds, 0 tens, 6 units, and 7 remaining; giving same result as before.

2. If 8 acres of land cost \$9707, what will 1 acre cost?

Reasoning Process. — If 8 acres of land cost \$9707, 1 acre will cost $\frac{1}{8}$ of \$9707.

This may be found by dividing by 8, as in the last example; or it may be found by the following method.*

* The student should observe that the real work performed, and the figures used in writing it, are the same by one method as by the other.

WRITTEN WORK.

$$\text{Divisor } 8\,)\,\underline{\$9707} \quad \text{Dividend} = \text{cost of 8 acres.}$$
$$\$1213\tfrac{3}{8} \quad \text{Quotient} = \text{cost of 1 acre.}$$

We divide thus: $\frac{1}{8}$ of 9 thousands = 1 thousand, with a remainder of 1 thousand. 1 thousand = 10 hundreds, and 7 hundreds added = 17 hundreds. $\frac{1}{8}$ of 17 hundreds = 2 hundreds, with a remainder of 1 hundred. 1 hundred = 10 tens, and there are no tens to add. $\frac{1}{8}$ of 10 tens = 1 ten, with a remainder of 2 tens. 2 tens = 20 units, and 7 units added = 27 units. $\frac{1}{8}$ of 27 units = 3 units, with a remainder of 3 units.

Therefore, the quotient is $1213, and there is a remainder of $3; or, since $3 \div 8 = \frac{3}{8}$, we may say that the quotient is $$1213\frac{3}{8}$. Hence, 1 acre will cost $$1213\frac{3}{8}$, when 8 acres cost $9707.

NOTE.—It is obvious that in getting $\frac{1}{8}$ of the above number, we have really divided it by 8.

(*d.*) By omitting the names of the denominations, we should have —

$\frac{1}{8}$ of 9 = 1, with 1 remaining.
$\frac{1}{8}$ of 17 = 2, with 1 remaining.
$\frac{1}{8}$ of 10 = 1, with 2 remaining.
$\frac{1}{8}$ of 27 = 3, with 3 remaining.
This gives 1213, with 3 remaining, or $1213\frac{3}{8}$, as before.

(*e.*) By only naming the quotient figures, we have —

1 thousand, 2 hundreds, 1 ten, 3 units, and 3 remainder; which gives the same answer as before.

Proof.—If 1 acre costs $$1213\frac{3}{8}$, 8 acres will cost 8 times $$1213\frac{3}{8}$; or, which is the same thing, 8 times $1213, plus $3. This will be $9707, if the work is correct.

87. *Decimal Fractions in the Quotient.*

(*a.*) The answer to each of the above questions might have been obtained in another form, by reducing the final remainder to lower denominations.

(*b.*) In the first example, the 7 units remaining = 70 tenths, and 8 is contained in 70 tenths 8 tenths times, with a remainder of 6 tenths. But 6 tenths = 60 hundredths, and 8 is contained in 60 hundredths 7 hundredths times, with a remainder of 4 hundredths But 4 hundredths

= 40 thousandths, and 8 is contained in 40 thousandths 5 thousandths times. This would give the following written work: —

Divisor $8) $58455.000 = Dividend.

7306.875 = Quotient = number of barrels.

(*c*.) In the second example, the 3 units remaining equal 30 tenths, and $\frac{1}{8}$ of 30 tenths = 3 tenths, with a remainder of 6 tenths. But 6 tenths = 60 hundredths, and $\frac{1}{8}$ of 60 hundredths = 7 hundredths, with a remainder of 4 hundredths. But 4 hundredths = 40 thousandths, and $\frac{1}{8}$ of 40 thousandths = 5 thousandths. This would give the following written work: —

Divisor = 8) $9707.000 = Dividend.

$1213.375 = Quotient = cost of 1 acre.

(*d*.) The answers in this form can be proved in precisely the same way as were the former answers.

88. *Compound Division.*

(*a*.) All questions in compound division must of necessity belong to the second class.

A benevolent society divided 10 cwt. 3 qr. 20 lb. of flour equally among 6 poor persons. What was each person's share?

Reasoning Process. — If 6 persons had 10 cwt. 3 qr. 20 lb. of flour, 1 person would have $\frac{1}{6}$ of 10 cwt. 3 qr. 20 lb.

WRITTEN WORK.

	cwt.	qr.	lb.	oz.	
6)	10	3	20		= share of 6 persons.
	1	3	7	8	= share of 1 person.

Explanation of Process. $\frac{1}{6}$ of 10 cwt. = 1 cwt., with a remainder of 4 cwt. But 4 cwt. = 16 qr., and 3 qr. added = 19 qr. $\frac{1}{6}$ of 19 qr. = 3 qr., with a remainder of 1 qr. But 1 qr. = 25 lb., and 20 lb. added = 45 lb.; $\frac{1}{6}$ of 45 lb. = 7 lb., with a remainder of 3 lb. But 3 lb. = 48 oz., and $\frac{1}{6}$ of 48 oz. = 8 oz.

Hence, the quotient is 1 cwt. 3 qr. 7 lb. 8 oz.

Second Explanation. 6 is contained in 10 once, with 4 remainder. 4 cwt. = 16 qr., and 3 qr. added = 19 qr. 6 is contained 3 times in 19, with 1 remainder. 1 qr. = 25 lb., and 20 lb. added = 45 lb. 6 is contained 7 times in 45, with 3 remainder. 3 lb. = 48 oz., and 6 is con-

tained 8 times in 48. Hence, the quotient is 1 cwt. 3 qr. 7 lb. 8 oz., as before.

(*b*.) This may be proved as were the examples in simple numbers.

NOTE. — Had the division been carried no farther than to pounds, the answer would have been 1 cwt. 3 qr. 7 lb., with a remainder of 3 lb., or 1 cwt. 3 qr. $7\frac{3}{8}$ lb.

89. *Problems for Solution, including Reduction Ascending.*

What is the quotient —

1. Of 87543 ÷ 7?
2. Of 59487 ÷ 3?
3. Of 38640 ÷ 6?
4. Of 827546 ÷ 9?
5. Of 217483 ÷ 8?
6. Of 348724 ÷ 4?
7. Of 37927 ÷ 6?
8. Of 29684 ÷ 9?
9. Of 20034 ÷ 6?
10. Of 7248396275436002031 ÷ 9?
11. How many yards of German broadcloth at $6 per yard can be bought for $174564?
12. How many sheep at $7 apiece can be bought for $7833?
13. If 8 melons cost $1, how many dollars will 3744 melons cost?
14. When 6 books are bought for $1, how many dollars must be paid for 1743 books?
15. How many kegs, each holding 9 gallons, can be filled from 4752 gallons of molasses?
16. How many hours will it take a vessel to sail 3937 miles, if she sails 8 miles per hour?
17. How many clocks at $6 apiece can be bought for $3528?
18. How many bags of coffee at $9 per bag can be bought for $7487?
19. How many casks of raisins at $7 per cask can be bought for $6594?
20. How many hours would it take a man who walks at the rate of 4 miles per hour to walk 4739 miles?

21. How many hats at $4 apiece can be bought for $1372?

22. 31741 quarts = how many bushels, pecks, and quarts?

WRITTEN WORK.

```
8 ) 31741 — 5 qt.
4 )  3967 — 3 pk.
      991 bu.
```

Reasoning Process and partial Explanation. — Since 8 qt. = 1 pk., 31741 qt. must equal as many pecks as there are times 8 in 31741, which, found by the usual method, gives 3967 pk. 5 qt. We now reduce the pecks to bushels. Since 4 pk. = 1 bu., 3967 pk. must equal as many bushels as there are times 4 in 3967, which, found by the usual method, gives 991 bu. 3 pk. Hence, 31741 qt. = 991 bu. 3 pk. 5 qt.

NOTE. — Problems like the above, in which it is required to find the value of a number of units of one denomination in terms of some higher, are ordinarily called *Problems in Reduction Ascending;* but they do not differ in their nature from other problems in division.

23. 87633 days = how many weeks?
24. 698472 inches = how many feet?
25. 98464 ʒ = how many ounces?
26. 7987536 square feet = how many square yards?
27. 97875 quarters = how many yards and quarters?
28. 87637 quarts = how many bushels, pecks, and quarts?
29. 798953 gills = how many gal., qt., pt., and gi.?
30. 793527 ℈ = how many ℔, ℥, ʒ, and ℈?
31. 587537 qr. = how many £, s., d., and qr.?
32. 23975 fur. = how many le., m., and fur.?
33. 15951 na. = how many yd., qr., and na.?
34. 63359 pt. = how many bu., pk., qt., and pt.?
35. 57984 gi. = how many gal., qt., pt., and gi.?
36. 7951 qr. = how many £., s., d., and qr.?
37. 4375 ℈ = how many ℔, ℥, ʒ, and ℈?
38. A farmer put 75 gal. 3 qt. 1 pt. 2 gi. of cider into bottles, each containing 1 pt. 2 gi. How many bottles did it take to contain it?

Reasoning Process. — Since it takes 1 bottle to hold 1 pt. 2 gi., it must take as many bottles to hold 75 gal. 3 qt. 1 pt. 2 gi. as there are times 1 pt. 2 gi. in 75 gal. 3 qt. 1 pt. 2 gi.

NOTE. — Before performing the division, each quantity must be reduced to gills.

39. A grain dealer put 498 bu. 3 pk. of grain into bags, each holding 1 bu. 3 pk. How many bags did he fill?

40. I bought 536 yd. 2 qr. 2 na. of tape, which I cut into pieces 2 qr. 1 na. long. How many pieces did it make?

41. If 9 men earn $4375.26 in a year, how much will 1 man earn in the same time?

Reasoning Process. — If 9 men earn $4375.26 in a year, 1 man will earn $\frac{1}{9}$ of $4375.26 in the same time, which may be found by dividing by 9.

42. If 7 cases of cloth cost $6545, how many dollars will 1 case cost?

43. If 18579 lb. of hay are obtained from 9 acres, how many pounds are obtained from 1 acre?

44. If 7 horses can draw 16786 lb., how many pounds can 1 horse draw?

45. If it cost $173549 to build 9 miles of railroad, how much will it cost to build 1 mile?

46. A father divided his estate, valued at $879643, equally among his 5 children. What was the share of each?

47. What is $\frac{1}{7}$ of 975646?

48. What is $\frac{1}{8}$ of 39547 bushels?

49. What is $\frac{1}{9}$ of 7354.278 tons?

50. What is $\frac{1}{6}$ of 12456.78 miles?

51. If 6 piano-fortes cost $1837.44, how much will 7 cost?

Reasoning Process. — If 6 piano-fortes cost $1837.44, 1 will cost $\frac{1}{6}$ of $1837.44, which (found by dividing by 6) is $306.24. If 1 piano-forte costs $306.24, 7 will cost 7 times $306.24, which (found by multiplying by 7) is $2143.68.

52. If 9 wagons cost $1378.71, what will 4 cost?

53. If 8 gold watches cost $775.36, what will 9 cost?

54. If 1 acre of land costs $327.52, what will 3 roods cost?

55. What is the cost of 5 cord feet of wood at $7.376 per cord?

56. How far will a man travel in 6 days, if he travels at the rate of 174 79 miles per week?

57. If 1 bushel contains 2150.4 cubic inches, how many cubic inches will 3 pecks contain?

58. If 1 peck contains 537.6 cubic inches, how many cubic inches will a six-quart basket contain?

59. If in 1 quart, liquid measure, there are 57.75 cubic inches, how many cubic inches are there in 1 qt. 1 gi., or 9 gills?

60. If 1 man can perform a piece of work in 2944 hours, in how many hours can 8 men perform the same work?

NOTE. — It is obvious that 8 men will perform it in $\frac{1}{8}$ of the number of hours required by 1 man.

61. If it takes 1 laborer 3474 days to earn $5211, how many days will it take 6 laborers to earn the same sum?

62. In a certain fort there are provisions enough to keep 1 company of 76 soldiers 7256 days. How long can 8 companies of the same size be kept on them?

63. 10 men bought a stack of hay weighing 4 T. 16 cwt. 0 qr. 1 lb. 14 oz. 6 dr. What ought each man's share to weigh?

64. A farmer had 9 equal bins filled with grain. They all contained 2047 cu. ft. 1436 cu. in. How many feet and inches did each bin contain?

65. The brig Maria sailed from Philadelphia to Boston, with 207 T. 13 cwt. 2 qr. 19 lb. of coal, $\frac{1}{7}$ of which was landed at one wharf, and the remainder at another. What was the weight of that landed at the first wharf? at the second?

66. If John walks $\frac{1}{7}$ as fast as George, how far will John walk while George is walking 97 m. 6 fur. 38 rd.?

67. A company of 9 California gold diggers found in 1 month 37 lb. 4 oz. 16 dwt. 17 gr. of gold, which they divided equally. What was the share of each?

68. 8 men bought 17 T. 18 cwt. 3 qr. 16 lb. of sugar, which they divided equally. What was the share of each man?

69. If 7 cows eat 16 T. 5 cwt. 1 qr. 18 lb. of hay in 1 year, how much will 9 cows eat in the same time?

70. If 6 equal pieces of cloth contain 185 yd. 2 qr. 2 na., how much will 5 pieces of the same size contain?

71. If 5 pieces of broadcloth cost £87 13 s. 9 d., how much will 7 pieces cost?

72. What is $\frac{1}{4}$ of 374 lb. 11 oz. 19 dwt. 20 gr.?

73. What is $\frac{1}{6}$ of 857 m. 5 fur. 36 rd. 4 yd. 2 ft. 6 in.?

74. What is $\frac{1}{8}$ of 17 yr. 135 da. 14 h. 37 m. 56 sec.?

75. What is $\frac{1}{9}$ of $15° 35' 54''$?

76. What is $\frac{1}{5}$ of 19 ℔ 11 ℥ 6 ʒ 2 ℈ 15 gr.?

90. *Division by Factors.*

(*a.*) When the divisor is the product of factors, it will sometimes be convenient to divide by its factors, instead of dividing by the whole number at once.

(*b.*) To understand the process of division, and method of getting the true remainder, reduce 15262 quarts to bushels and quarts, by first reducing it to bushels, pecks, and quarts, as before explained, and compare the explanation and work with the explanation and work of the following example.

(*c.*) What is the quotient of 15262 divided by 32?

WRITTEN WORK.

8) 15262 — 6, the first remainder.

4) 1907 times 8, — 3 times 8, or 24, the 2d remainder.

476 times 32 = quotient sought.

$24 + 6 = 30 =$ true remainder.

Explanation. — Since $32 = 4$ times 8, we may divide by 8, and then by 4. Dividing by 8 gives 1907 for a quotient and 6 for a remainder, i. e., 1907 times 8, or parts of 8 each, with 6 units remaining undivided.

Dividing 1907 by 4 gives 476 for a quotient and 3 for a remainder, i. e., 476 parts each equal to 4 of the former ones, or to 4 times 8, or 32, with 3 times 8, or 24, undivided. Adding 24 to the former remainder gives 30 for the true remainder. Hence, $15262 = 476$ times 32, with a remainder of 30, or it equals $476\frac{30}{32}$ times 32.

(*d.*) Had the problem been to find $\frac{1}{32}$ of 15262, the first quotient would have represented the number of units in each part obtained by

dividing 15262 into 8 equal parts; and the second the number of units in each part obtained by dividing each of these 8 parts into 4 others, or, which is the same thing, the number of units there would be in each part obtained by dividing 15262 into 32 equal parts.

To get the true remainder, observe that as each of the 8 parts gives a remainder of 3 units, all of them must give a remainder of 8 times 3, or 24, which, added to the 6 left by first division, gives 30 for the true remainder.

The work would be written thus: —

8) 15262 — 6 first remainder.

4) 1907 units in each of 8 parts — 3 remainder on [each part, or 8 times 3 in all.

476 units in each of 32 parts.

24 + 6 = true remainder.

(*e.*) From the above, it is evident that the true quotient will be obtained by dividing the dividend by one factor, the quotient of this division by another, the quotient of the last division by another, and so on, till all the factors of the set considered are used.

(*f.*) The true remainder may be obtained by multiplying the remainder of each division by the divisors of all the preceding divisions, and adding the products to the remainder of the first division.

(*g.*) What is the value —

1. Of $74385 \div 49$?
2. Of $67849 \div 35$?
3. Of $4535 \div 24$?
4. Of $\frac{1}{28}$ of 59358?
5. Of $\frac{1}{27}$ of 39475?
6. Of $57482 \div 15$?
7. Of $28654 \div 21$?
8. Of $5477 \div 18$?
9. Of $\frac{1}{45}$ of 20249?
10. Of $\frac{1}{16}$ of 172783?

(*h.*) The most important application of the division by factors is made when the divisor is a multiple of 10, or of some power of 10. In such cases we first divide by the power of 10, and then that quotient by the other factor of the divisor, getting the true remainder as before explained.

11. What is the quotient of $578635 \div 800$?

Explanation. — Dividing by 100, by removing the point two places to the left, gives 5786 for a quotient and 35 for a remainder, and dividing

this quotient by 8, gives a final quotient of 723 and a remainder of 2 times 100, or 200, which, added to 35, the first remainder, gives 235 as the true remainder.

The work may be written thus:—

$$\left.\begin{array}{r} 100\;)\;\underline{578635}\;-\;35 \\ 8\;)\;\underline{5786}\;-\;2 \\ 723 \end{array}\right\} = 2 \times 100 + 35 = 235.$$

(*i.*) What is the quotient —

12. Of $6379 \div 40$?	16. Of $174326 \div 50$?
13. Of $27476 \div 300$?	17. Of $927673 \div 7000$?
14. Of $427875 \div 9000$?	18. Of $42700 \div 9000$?
15. Of $258647 \div 80000$?	19. Of $23750000 \div 30000$?

91. *Divisor a large Number.— Trial Divisor.*

(*a.*) When the divisor is a large number, it is not always easy to tell at once what is the true quotient figure. In such cases, the number expressed by one or two of the left hand figures of the divisor may be selected as a sort of *trial divisor;* but to determine whether the quotient figure thus obtained is the true one, it will be necessary to find the product of the divisor by the quotient, and the remainder after subtracting this product from the dividend.

(*b.*) The product of the divisor by the quotient should either be equal to, or less than, the dividend. When it is equal to the dividend, the division can be exactly performed: but when it is less, there is a remainder, which may be found by subtracting it from the dividend.

(*c.*) If, in any case, the product of the divisor by the supposed quotient is greater than the dividend, it shows that the divisor is not contained as many times in the dividend as the supposed quotient indicates.

(*d.*) We will now apply these principles to a few examples.

1. What is the quotient of $387 \div 43$?

Solution.— Since 43 differs but little from 4 tens, we may infer that the entire part of the quotient of $387 \div 43$ will be nearly or precisely

the same as that of 38 ÷ 4, which is 9. To ascertain whether this be correct, we multiply 43 by it, which gives exactly 387, and proves our work.

The figures may be written thus : —

Divid.
Divisor = 43) 387 (9 = Quotient.
387 = 9 times 43.

2. What is the quotient of 6169 ÷ 825?

Solution. — Since 825 differs but little from 8 hundreds, we may infer that the entire part of the quotient of 6169 ÷ 825 will be nearly or precisely the same as that of 61 ÷ 8, which is 7. To ascertain if this is correct, we multiply 825 by 7, which gives 5775 for a product. This being less then the dividend shows that the quotient figure is not too large.* Subtracting this product from the dividend gives 394 for a remainder, which, being less than the divisor, shows that the quotient figure is correct.†

Hence, the quotient is 7, and the remainder is 394, or the complete quotient is $7\frac{394}{825}$.

WRITTEN WORK.

Divid.
Divisor = 825) 6169 (7 = Quotient.
5775 = 7 times 825.
394 = Remainder.

3. What is the quotient of 55673 ÷ 6349?

Solution. — Making 6 the trial divisor, we have 9 for a trial quotient. Multiplying 6349 by 9 gives 57141 for a product, which, being greater than the dividend, shows that the quotient figure is too large. Selecting 8 as the quotient figure, and multiplying as before, gives 50792 for a product, which, being less than the dividend, shows that 8 is not too large. Subtracting 50792 from 55673 gives a remainder of 4881.

Hence the quotient is 8, and the remainder is 4881, or the complete quotient is $8\frac{4881}{6349}$.

WRITTEN WORK.

Divid.
Divisor = 6349) 55673 (8 = Quotient.
50792 = 8 times 6349
4881 = Remainder.

* For it shows that the divisor taken 7 times is less than the dividend.

† For it shows that if the divisor had been taken once more, the result would have been more than 6169.

4. What is the quotient of 3913 ÷ 482?

Solution. — Since the divisor is nearer to 5 hundreds than 4 hundreds, we select 5 as the trial divisor. This gives 7 for a trial quotient, and multiplying gives 3374 for a product, which, being less than the dividend, shows that 7 is not too large. Subtracting 3374 from 3913 gives a remainder of 539, which, being greater than the divisor, shows that 7 is too small. Substituting 8 in its place, and multiplying and subtracting as usual, gives the following written work —

```
482 ) 3913 (8
      3856 = 8 × 482.
      ----
        57 = Remainder.
```

Hence, the quotient is 8, and the remainder is 57, or the complete quotient is $8\frac{57}{482}$.

What is the quotient —

5. Of 5279 ÷ 847?
6. Of 19476 ÷ 2784?
7. Of 35947 ÷ 8912?
8. Of 394687 ÷ 182578?
9. Of 807436 ÷ 294367?
10. Of 42974 ÷ 8523?

11. What is the quotient of 15078 ÷ 276?

WRITTEN WORK.

```
276 ) 15078 ( 54
      1380  = 50 × 276
      ----
       1278
       1104 = 4 × 276
       ----
        174 = Remainder.
```

Solution. — Making 3 the trial divisor, we may infer that as 3 is contained 5 times in 15, 276 must be contained 5 tens times in 1507 tens. Writing 5 as the tens' figure of the quotient, we multiply the divisor by it, and place the product, which is 1380, under the tens of the dividend, to show that its denomination is tens. Subtracting this from 1507 tens leaves a remainder of 127 tens, to which adding the 8 units gives 1278 units to be divided.

Since 3 is contained 4 times in 12, we may infer that 276 is contained 4 times in 1278. Writing 4 as the units' figure of the quotient, we multiply the divisor by it, and write the product, which is 1104, under the dividend. Subtracting leaves a remainder of 174, which is less than the divisor.

As we have now considered all the denominations of the dividend, we may consider the division complete, (unless we wish to have the

quotient appear in the form of a decimal fraction.) Hence, the quotient is 54, and the remainder is 174, or the complete quotient is $54\frac{174}{276}$.

NOTE.—It is obvious that in performing the above work, we subtracted 50 times the divisor, and then 4 times the divisor, which is equivalent to 54 times the divisor, from the dividend, and that this left a remainder of 174.

(*e*.) Had it been desirable to have the answer contain a decimal fraction, we should have continued the division, thus:—

174 units = 1740 tenths, and as 3 is contained 5 times in 17, we may infer that 276 is contained 5 tenths times in 1740 tenths. Multiplying the divisor by 5, and subtracting the product as before, gives a remainder of 360, which, being greater than the divisor, shows that the divisor would have been contained more than 5 tenths times in the dividend. We therefore substitute 6 for 5 in the tenths place of the quotient, and having erased the last product and remainder, multiply the divisor by 6, subtracting the product as before. This gives a remainder of 84 tenths, which may be reduced to hundredths, and divided as already explained.

In the following form, the division is carried out to millionths.

```
276 ) 15078. ( 54.630434
      1380 = 5 tens × 276
      ----
       1278.
       1104. = 4 times 276
       ----
        174.0
        165.6 = 6 tenths × 276
        -----
          8.40
          8.28 = 3 hundredths × 276
          ----
           .1200
           .1104 = 4 ten-thousandths × 276
           -----
              960 = hundred-thousandths
              828 = 3 hundred thousandths × 276.
              ---
              1320 = millionths
              1104 = 1 millionth × 276
              ----
               216 = millionths = Remainder.
```

NOTE.—It is obvious that we have subtracted from the dividend the products of the divisor multiplied by 5 tens, by 4 units, by 6 tenths, by 3 hundredths, by 4 ten-thousandths, by 3 hundred-thousandths, and by 1 millionth; and that this leaves a remainder of 44 millionths. Hence, if the work be correct the sum of these several products added to the final remainder will equal the dividend. The methods of proof before explained will also apply.

92. *Long Division.*

(*a.*) When, as in **85** to **91**, we write only the divisor, dividend, quotient, and final remainder, the process is called Short Division; but when, as in **91**, we write the divisor, dividend, and quotient, and also the products of the divisor by the successive figures of the quotient, together with the remainders obtained by subtracting these several products from the corresponding denominations of the dividend, the process is called Long Division.

(*b.*) Long Division differs from Short Division in these respects, viz.: First. That having obtained a quotient figure, we multiply the divisor by it, writing the product under the part of the dividend considered.

Second. That we subtract this product from the part of the dividend considered, *writing the remainder.*

Third. That we write the next figure of the dividend after this remainder, to form the next partial dividend.

(*c.*) The real difference between them may be stated thus: In long division more of the work is written than in short division.

(*d.*) Long division is generally employed when the divisor is a large number.

(*e.*) If at any time the product of the divisor by a quotient figure should be less than the part of the dividend considered, it would show that the quotient figure was too large. (See 3d example, **91.**)

(*f.*) If the remainder obtained by subtracting from the dividend the product of the divisor by any quotient figure should be greater than the divisor, it would show that the quotient figure was too small. (See 4th example, **91.**)

(*g.*) Perform the following examples by long division. See solution to 11th example in **91.**)

What is the quotient —

1. Of 3784 ÷ 21?	5. Of 386427 ÷ 5287?
2. Of 59378 ÷ 82?	6. Of 2700684 ÷ 19743?
3. Of 386495 ÷ 289?	7. Of 438277 ÷ 34?
4. Of 43625 ÷ 714?	8. Of 6293876 ÷ 2874?

9. How many square feet are equal to 42912 square inches?

10. How many boxes, each containing 73 quarts, can be filled from 658764 quarts of meal?

11. How many watches at \$48 apiece can be bought for \$3456?

12. If a boy can perform 37 examples in one day, how many days will it take him to perform 8764 examples?

13. If a vessel sails 147 miles per day, how many days will it take her to sail 3579 miles?

14. If 252 pints of cider will fill a barrel, how many barrels can be filled from 876438 pints?

15. How many weights, each weighing 56 lb., will be required to balance 7 loads of iron, each weighing 9472 lb.?

16. James can pump 37 quarts of water per minute, and William can pump 43 quarts of water per minute. How many minutes will it take William to pump as much as James can pump in 473 minutes?

17. What will 1 acre of land cost if 237 acres cost \$236526?

18. If 687 bushels of potatoes cost \$515.25, what will 236 bushels cost?

19. If 67 yd. of silk cost \$75.375, what will 48 yd. cost?

20. If 89 barrels of flour contain 17444 pounds, how many pounds will 258 barrels contain?

93. *Abbreviated Process.*

(*a.*) By performing the subtraction at the same time that we do the multiplication, much of the written work can be avoided.

(*b.*) The subtraction may be performed in accordance with the principles explained in **73.** The following example will illustrate it. We have, however, given only the mechanical process, leaving it with the pupil to explain the various steps.

1. What is the quotient of 1947.63 ÷ 396?

Process. 396 is contained 4 times in 1947. 4 times 6 = 24, which subtracted from 27 leaves 3; this we write in the units' place. 4 times 9 = 36, and 2 added make 38, which subtracted from 44 leaves 6. 4 times 3 = 12, and 4 added make 16, which subtracted from 19 leaves 3. The first remainder is, therefore, 363. Reducing this to tenths, and adding the 6 tenths, we have 3636 tenths, which divided by 396 gives 9 tenths for a quotient. We multiply and subtract as before, and thus proceed till the division is completed, or till we have as many decimal places in the quotient as we wish.

WRITTEN WORK.

396) 1947.63 (4.9182 = Quotient.
363.6
7.23
3.270
.1020
.0228 = Remainder.

(*c.*) Perform the division in the following examples in the same way:—

2. What is the quotient of 78643 ÷ 47?
3. What is the quotient of 137648 ÷ 326?
4. What is $\frac{1}{37}$ of 65427?
5. What is $\frac{1}{579}$ of 3865726?
6. What is the value of 764379 cubic inches, expressed in cubic feet and inches?
7. How many square feet and inches are equal to 954376 square inches?
8. If the wages of 96 laborers for 1 year are $26470.08, what will be the wages of 1 laborer for the same time?
9. If 875 pairs of boots cost $2737.16, how much will 1 pair cost?
10. How many pairs of shoes at $2.37 per pair can be bought for $184.86?

NOTE.—In solving such examples as the above, both divisor and dividend should be reduced to cents, thus: if for 237 cents one pair of shoes can be bought, for 18486 cents as many pairs can be bought as there are times 237 in 18486, which may be found by methods already *explained.*

11. How many pounds of tea at \$.47 per pound can be bought for \$464.23?

12. How many bushels of wheat at \$1.85 per bushel can be bought for \$287.35?

13. How many pounds of raisins at \$.125 per pound can be bought for \$39.875?

14. How many days must a man labor at \$1.75 per day to earn \$596.75?

15. I gave 17 acres of land worth \$46.98 per acre in exchange for wild land at \$4.59 per acre. How many acres of wild land did I receive?

16. 1475869 cu. in. = how many C., Cd. ft., cu. ft., and cu. in.?

17. 8383794 oz. = how many T., cwt., qr., lb., oz., and dr.?

18. 67458 sq. rd. = how many A., R., and sq. rd.?

19. 57864 gr. = how many lb., oz., dwt., and gr.?

20. 457986 cu. in. = how many C., Cd. ft., cu. ft., and cu. in.?

SECTION VIII.

CONTRACTIONS AND MISCELLANEOUS PROBLEMS.

Introductory Note. — It is frequently the case that by carefully examining the numbers we are to operate upon, we can discover some abbreviated method of performing the work. We have already suggested several such methods, and it is the design of this section to suggest others.

94. *Multiplier a convenient fractional Part of* 10, 100 1000, *&c.*

When the multiplier is a convenient fractional part of 10, 100, 1000, or a unit of any higher denomination, the following principles may be advantageously applied.

1. How many are 25 times 679?

Solution. — Since 25 equals one fourth of 100, 25 times 679 must equal one fourth of 100 times 679, or $\frac{1}{4}$ of 67900, which equals 16975.

2. How many are 25 times 657?
3. How many are 50 times 657?
4. How many are $12\frac{1}{2}$ times 834? ($12\frac{1}{2} = \frac{1}{8}$ *of* 100.)
5. How many are $16\frac{2}{3}$ times 957? ($16\frac{2}{3} = \frac{1}{6}$ *of* 100.)
6. How many are $33\frac{1}{3}$ times 871? ($33\frac{1}{3} = \frac{1}{3}$ *of* 100.)
7. How many are $14\frac{2}{7}$ times 249? ($14\frac{2}{7} = \frac{1}{7}$ *of* 100.)
8. How many are $3\frac{1}{3}$ times 249? ($3\frac{1}{3} = \frac{1}{3}$ *of* 10.)
9. How many are $2\frac{1}{2}$ times 822? ($2\frac{1}{2} = \frac{1}{4}$ *of* 10.)
10. How many are $6\frac{1}{4}$ times 944? ($6\frac{1}{4} = \frac{1}{16}$ *of* 100.)
11. How many are $333\frac{1}{3}$ times 9478? ($333\frac{1}{3} = \frac{1}{3}$ *of* 1000.)
12. How many are 125 times 8767? ($125 = \frac{1}{8}$ *of* 1000.)
13. How many are 250 times 6894? ($250 = \frac{1}{4}$ *of* 1000.)

95. *One Part of Multiplier a Factor of another Part.*

When one part of the multiplier is a factor of another part, much labor can often be saved by applying the principles illustrated in the following examples: —

1. What is the product of 7678 multiplied by 427?

Solution. — By examining the multiplier, we perceive that it equals 420 + 7 = 42 tens + 7 units, and that 42 tens = 6 tens or 60 times 7. Hence, we have the following written work.

$$\begin{array}{rr}
\text{a} = & 7678 \\
\text{b} = & 427 \\
\hline
7 \text{ times a} = \text{c} = & 53746 \\
6 \text{ tens, or } 60 \text{ times c} = 420 \text{ times a} = \text{d} = & 322476 \\
\hline
\text{c} + \text{d} = \text{b times a} = \text{e} = & 3278506
\end{array} = \textit{Ans.}$$

2. What is the product of 72144 times 874369?

Explanation. — By examining the multiplier, we perceive that it equals 72 thousands + 144, and that as 144 = twice 72, it must equal 2 times 001 of 72 thousands. Hence,

a =	874369
b =	72144
72000 × a = c =	62954568000
2 times .001 of c = 144 times a = d =	125909136
c + d = b times a = e =	63080477136

3. How many are 1875625125 times 97643721785 ?

a =	97643721785
b =	1875625125
125 times a = c =	12205465223125
5000 times c = 625000 times a = d =	61027326115625
3000 times d = 1875000000 times a = e =	183081978346875
c + d + e = b times a =	183143017878455848125

How many are —

4. 14874 times 376437 ?
5. 19899 times 879438 ?
6. 17525 times 43678 ?
7. 83415 times 476437 ?
8. 43821973 times 976327864 ?
9. 36324 times 54289732 ?
10. 1998999 times 463829748 ?
11. 1752512½ times 83743954 ?

96. *To divide by* 99, 999, *&c.*

(*a.*) Since 10 = 9 + 1, and 100 = 99 + 1, and 1000 = 999 + 1, &c., it follows that 40 = 4 times 9 + 4; that 2800 = 28 × 99 + 28; that 37900 = 379 × 99 + 379; that 786000 = 786 times 999 + 786; &c. On this principle the following processes are based.

1. What is the quotient of 437 ÷ 99 ?

Solution. 437 = 400 + 37; but 400 = 4 times 99 + 4. Hence, 437 = 4 times 99 + 4 + 37 = 4 times 99 + 41 = $4\frac{41}{99}$ times 99 = *Answer.*

2. What is the quotient of 15378 ÷ 99 ?

Solution. 15378 = 15300 + 78 = 153 times 99 + 153 + 78 = 153 times 99 + 231. But 231 = 200 + 31 = 2 times 99 + 2 + 31

= 2 times 99 + 33. Now, by adding 153 times 99 to 2 times 99, we have 155 times 99. Hence, 15378 = 155 times 99 + 33 = $155\frac{33}{99}$ times 99.

The following exhibits a convenient form for writing the work: —

$$\begin{array}{r|l} 153 & 78 \\ 2 & 31 \\ \hline 155 & 33 = 155\frac{33}{99} \end{array}$$

The full explanation is the same as that already given The necessary numerical operations are as follows: Separating the hundreds from the tens and units by a vertical line, we add the number at the right of the line to that at the left. This gives 153 + 78 = 231 = first remainder. Writing this beneath the dividend, and adding as before, we have 2 + 31 = 33, which, being less than 99, is the final remainder. The sum of the numbers at the left of the line, or 153 + 2 = 155, the quotient.

(*h.*) In a similar manner we can divide by 999, 9999, &c.

(*i.*) Let the pupil find, if he can, the application of a similar principle to the division by 98, 97, 96, &c.; also to 998, 997, &c., and afterwards perform the following examples: —

What is the quotient —

3. Of 186738 ÷ 99?
4. Of 49763842 ÷ 999?
5. Of 763852748 ÷ 9999?
6. Of 9842987483 ÷ 99999?
7. Of 54783 ÷ 98?
8. Of 2987637 ÷ 96?
9. Of 248763 ÷ 997?
10. Of 69874325 ÷ 9998?

97. *To divide by any convenient fractional Part of* 10, 100, *or* 1000.

(*a.*) Since 100 ÷ 25 = 4, 900 ÷ 25 must equal 9 times 4; 1700 ÷ 25 must equal 17 times 4; 49600 ÷ 25 must equal 496 times 4.

(*b.*) Since 100 ÷ $12\frac{1}{2}$ = 8, 3800 ÷ $12\frac{1}{2}$ must equal 38 times 8; 49700 ÷ $12\frac{1}{2}$ must equal 497 times 8, &c.

(*c.*) Since 1000 ÷ 125 = 8, 479000 ÷ 125 must equal

479 times 8; 3785000 ÷ 125 must equal 3785 times 8, &c.

(*d.*) These principles are applied in the following processes.

1. What is the quotient of 9738 ÷ 25?

Solution. 9738, 9700 + 38. But 9700 = 97 times 100 = 97 times 4 times 25 = 388 times 25; and 38 = $1\frac{13}{25}$ times 25. Hence, 9738 ÷ 25 = 388 + $1\frac{13}{25}$ = $389\frac{13}{25}$.

What is the quotient —

2. Of 86794 ÷ 25?	6. Of 54737 ÷ 125?
3. Of 3475 ÷ $12\frac{1}{2}$?	7. Of 98734 ÷ $33\frac{1}{3}$?
4. Of 6950 ÷ $16\frac{2}{3}$?	8. Of 4765465 ÷ $333\frac{1}{3}$?
5. Of 42725 ÷ $6\frac{1}{4}$?	9. Of 657847 ÷ 250?

98. *Miscellaneous Problems.*

1. A man bought a house lot and garden for $1378.24; he paid $4796.87 for building a house, $1274.38 for building a stable and carriage house, $438.47 for fencing the lot, $578.37 for laying out the garden and grounds, $1287.63 for a span of horses and a carriage, $1328.56 for furnishing his house, and then had $47289.43 left. How much money did he have at first?

2. Mr. French bought a house for $6742.38, and after paying $138.47 for having it painted, and $527.94 for repairs, he sold it for $8472. Did he gain or lose, and how much?

3. Mr. Hall bought 437 cords of wood at $3 per cord, and sold it at $4.75 per cord. What did he gain by the speculation?

4. A man started on a journey of 1164 miles, and travelled 37 miles per day for the first 21 days. How many days would it take him to finish the journey if he should travel at the rate of 43 miles per day?

5. Bought 43 acres of land at $28.73 per acre, and sold it at $37.73 per acre. How much did I gain?

6. I bought 487 yards of cloth at $2 per yard, and 627 yards at $4 per yard; I sold the whole of it at $4 per yard. How much did I gain?

7. A trader bought 528 barrels of flour at \$7 per barrel, 875 barrels at \$8 per barrel, and 497 barrels at \$9 per barrel. He sold the whole of it at \$8 per barrel. Did he gain or lose, and how much?

8. How much will a house lot, 137 feet long and 89 feet wide, cost, at 2 cents per square foot?

9. Which is the larger $13 \times 17 \times 28 \times 43$, or $28 \times 13 \times 43 \times 17$, and how much?

10. A man who had \$4376 invested it in flour at \$8 per barrel. He sold 238 barrels of the flour at \$9 per barrel, and the rest for enough to make up \$5232. For how much did he sell the last lot per barrel?

11. A man bought a horse for \$237; he kept him 19 weeks at an expense of \$2.50 per week, and then exchanged him for another horse, receiving \$48 for their difference in value. After keeping the second horse 4 weeks, at an expense of \$2.25 per week, he sold him for \$225. Now, allowing that the use of each horse was worth \$1.75 per week, did he gain or lose by the transaction, and how much?

12. To 12 times 487 add 8 times 683, multiply the sum by 7, and subtract 4279 from the product.

13. James Smith bought of John Brown 7 hogsheads of molasses, each containing 147 gallons, at \$.27 per gallon, and gave in payment \$100 in money, and the rest in nails at 5 cents per pound. How many pounds of nails did it take?

14. How many cords of wood at \$4 per cord can be bought for 82 barrels of apples at \$2 per barrel?

15. John earns \$4 per week, and William earns \$7. How many weeks will it take John to earn as much as William can earn in 48 weeks?

16. Gave 3 hogsheads of oil, each containing 167 gallons, worth \$1.68 per gallon, and \$200 in money, for 4 acres of land. How much ought the land to be worth per acre, that I may neither gain nor lose?

17. George had 378 oranges, which sold at 2 cents apiece. With the money thus received he bought a box of oranges, which he found contained 427. He ate 9 of these, gave away

7, had 4 stolen from him, sold 294 at 3 cents apiece, and the rest at 2 cents apiece. Did he gain or lose on the box, and how much?

18. How many times will a carriage wheel. 12 feet 6 inches in circumference, revolve in going 5 miles, 4 fur., 18 rd., 4 yd., 1 ft., 1 in.?

19. A boy, riding with his father, ascertained that the hinder wheel of the carriage, which was 10 feet 4 inches in circumference, revolved 1297 times in passing from one village to another. How far apart were the villages, reckoning the distance in miles, furlongs, rods, &c.?

20. Mr. Clarke's house lot is 83 feet wide, and contains 8051 square feet. How long is it?

21. Mr. Angell says that his house lot is 97 feet long, but that if it were 100 feet long, it would contain 168 square feet more than it now does. How many feet does it now contain?

22. Multiply 837 by 5, add 247 to the result, divide this by 4, and calling the result dollars, find how many yards of cloth at $2 per yard you could buy with it.

23. By buying a cargo of coal at $6 per ton, and selling it at $8 per ton, I gained $198. How much did I pay for it?

24. A silversmith bought 13 lb., 4 oz., 2 dwt., 5 gr. of silver, and after mixing with it 1 lb., 5 oz., 15 dwt., 19 gr. of alloy, made it into spoons, each weighing 1 oz., 4 dwt., 17 gr. How many spoons did he make?

25. I bought the wood standing on 9 acres of land, paying for it at the rate of $67.25 per acre. I paid $.625 per cord for having it cut, and $.75 per cord to have it carted to a railroad depot, where I sold it for $4.50 per cord. If there was an average of 30 cords to the acre, how much did I gain by the transaction?

26. A trader mixed 536 lb. of sugar, worth 9 cents per pound, with 52 lb., worth 6 cents per pound, and 432 lb., worth 7 cents per pound. For how much per pound ought he to sell the mixture so as neither to gain nor lose?

11 *

27. A trader bought 597 gallons of vinegar at 14 cents per gallon, and after mixing with it 24 gallons of water, sold it for 15 cents per gallon. How much did he gain by the transaction?

28. I bought a pile of wood, 144 feet long, 4 feet wide, and 6 feet high, at $3.92 per cord. What did it cost me?

29. How many square feet in the walls of a room 18 feet long, 16 feet wide, and 12 feet high?

30. If William walks at the rate of 16 rods per minute, and Joseph at the rate of 19 rods per minute, how long will it take Joseph to overtake William, when William has 12 minutes the start.

99. *Bills of Goods.*

(*a.*) When a man sells goods, he usually gives the purchaser a written statement of the articles bought, and the prices he is to pay for them. Such a statement is called a "Bill of the Goods," or simply a "Bill."

(*b.*) A bill, like every other business paper, should contain the date, i. e., the time and place of the transaction, and also the names of the parties, and an account of the transaction. If the goods are paid for, the bill should be receipted; i. e., the words *Received Payment* being written at the bottom, the seller should affix his name.

(*c.*) The following example will illustrate this:—

Mr. George W. Dodge is a trader, residing in Lancaster. On the 1st of January, 1855, he sold to Mr. Humphrey Barrett, for cash, 3 yards of broadcloth at $3.75 per yard, 6 yards of doeskin at $1.87 per yard, 32 yards of sheeting at 11 cents per yard, 9 yards of black silk at 97 cents per yard, and 6 linen handkerchiefs at 34 cents apiece.

Mr. Dodge made out the bill as follows:—

Lancaster, Jan. 1, 1855

Mr. Humphrey Barrett,

Bought of Geo. W. Dodge.

3	yds.	Broadcloth	at	$3.75	$11.25
6	"	Doeskin	"	1.87	11.22
32	"	Sheeting	"	.11	3.52
9	"	Black Silk	"	.97	8.73
6	"	Linen Hdks	"	.34	2.04
					$36.76

Received Payment,

Geo. W. Dodge

(*d.*) If Mr. F. W. Spofford, Mr. Dodge's clerk, had received the money due on the bill, he would have receipted it by some form similar to the following: —

Received Payment,

Geo. W. Dodge,

by F. W. Spofford.

Or,

F. W. Spofford,

for Geo. W. Dodge.

(*e.*) If the goods had been bought on credit, the words "Received Payment" might have been written, but no name would have been affixed. It is the *signing* of the receipt by the seller, or his authorized agent, which gives validity to it. The receipted bill should be kept by the person who pays it, as evidence of the payment.

(*f.*) The following form of heading a bill is often used instead of the above: —

Lancaster, Jan. 1, 1855.

Mr. Humphrey Barrett,

To Geo. W. Dodge, Dr.

For 3 yds. Broadcloth, at, &c.

NOTE. — A person is my *debtor* when he owes me money, and my *creditor* when I owe him.

(*g.*) When articles are bought or services rendered at different times, the bill may be headed and receipted as before, and the dates of the various transactions written at the left, opposite the entries.

(*h.*) Theodore Gay is a trader living in Dedham. He sold to Samuel French the following articles, viz.: Jan. 3, 1855, 5 bags of meal at $1.58 per bag, and 2 bags of corn at $1.54 per bag; Jan. 27, 8 lbs. of coffee at 16 cents per pound; Feb. 7, 3 lbs. tea at 59 cents per pound; Feb. 21, 1 bbl. of flour for $10.37, and 14 lbs. of brown sugar at 9 cents per pound. On the 1st of March, Mr. Gay being in want of money, made out a bill, and sent it to Mr. French, who paid it on the 3d of March.

(*i.*) The bill was made out in the following form: —

Dedham, March 1, 1855.

Mr. Samuel French,

To Theodore Gay, Dr.

1855.				
Jan. 3,	For 5 Bags Meal,	at	$1.58,	$7.90
" "	" 2 " Corn,	"	1.54,	3.08
27,	" 8 lbs. Coffee,	"	.16,	1.28
Feb. 7,	" 3 lbs. Tea,	"	.59,	1.77
" 21,	" 1 bbl. Flour,	"		10.37
" "	" 14 lb. Sugar,	"	.09,	1.26
				$25.66

March 3, 1855.

Received Payment,

Theodore Gay

(*j.*) If Mr. French had paid Mr. Gay $2 on the 1st of February, and worked for him the 4 days ending Feb. 24 at $1.50 per day, the first part of the bill would have been made out as before, and then the *credit* entries would have been made as follows: —

* Feb. 21, For 14 lb. Sugar, at \$.09,		\$ 1.26
		\$25.66
	Cr.	
Feb 1, By Cash,	. . . . .	\$ 2.00
" 24, " 4 days' labor, at \$1.50,		\$ 6.00
		\$ 8.00
	. Bal. due T. G., .	\$17.66

March 3, Received Payment,

Theodore Gay.

(*k.*) The mere sending of a bill to a person, except at his request, or at the time of sending the articles for which the bill is made out, is equivalent to a request that the money due on it should be paid.

(*l.*) A bill is said to be against the person who owes, and in favor of the one who is to receive the money due on it. Thus the first of the preceding bills is against Humphrey Barrett, and in favor of Geo. W. Dodge.

100. *Examples for Practice.— Due Bills.*

Make out the proper bills for each of the following examples: —

1. L. H. Holmes is a dry goods dealer, residing in Bridgewater. Oct. 7, 1854, he sold to E. C. Hewett, for cash, 3 yds. of broadcloth at \$3.62, 3 yds. of doeskin at \$1.68, 1 cravat for \$1.50, 1 vest for \$6.00, 1 pair of gloves for \$1.00, and 1 pair of boots for \$4.50.

2. A. B. Curry & Son, of Providence, sold to Geo. A. Richards, June 13, 1855, the following articles, viz.: 35

* This entry is repeated so as to make the form of the bill more apparent.

lbs. live geese feathers, at 50 cents per pound; 12 common chairs, at $.42 each; 6 cane seat, at $1.00 each; 6 mahogany spring seat, at $3.00 each; 3 common bedsteads, at $3.00 each; 2 cottage bedsteads, at $5.00 each; 2 best hair mattresses, 25 lb. each, at $.50 per lb.; 2 palm leaf mattresses at $4 each, 2 husk at $5.00 each, 2 straw at $2.50 each, 1 sofa, $35; 1 pair tete-a-tetes, $75; 1 gilt mirror, $75; 1 marble centre table, $42; 1 secretary, $45; 1 painted chamber set, $45; 1 enamelled chamber set, $125.00; 3 common bureaus at $8.00 each; 1 marble toilet bureau, $42.00; 1 extension dining table, $45.00; 12 oak dining chairs at $3.50; 1 time-piece, $8.00; and 1 whatnot, $25.00.

3. John Smith sold to David Brown the following articles, viz.: Oct. 7, 1854, 13 bushels of potatoes at $.56, 19 bu. corn at $.97, and 3 bbl. of apples at $2.42; Nov. 1, 4 tons of hay at $19, and 3 tons at $18.50; Dec. 17, 40 bushels of potatoes at $.67, 34 bu. of corn at $.98, and 21 bbl. of apples at $2.75 per barrel. Jan. 1, 1855, the account was settled by a due bill.

NOTE. — A due bill is not a promise to pay a debt, but merely an acknowledgment that it is due. It is intended to cut off after disputes as to the debt for which it is given, by furnishing the creditor additional means of establishing the justice of his claim. Every due bill, or written promise to pay money, should contain the words "value received," to show that the person who signs it has received an equivalent for it. Indeed, it is a principle in law, that no claim is valid unless it is based on some service rendered, or consideration given, to the person against whom it is made, or on his account.

To illustrate the form of a due bill, we will suppose that John Smith owes James Brown $25, and that he gives him a due bill for the amount, as follows: —

$25. Boston, Jan. 1, 1855.

Due to James Brown twenty-five dollars, for value received.

John Smith.

The receipt, when a due bill is given, might be, "Received payment by due bill," or, "Settled by due bill."

4. May 7, 1855, Hill & Saunders, of Boston, sold to James Drew, 1 cask linseed oil, 24 gal., at $1.50 per gal.; 1 bag Java coffee, 122 lbs., at 16 cents per lb.; 1 hhd. of N. O. molasses, 127 gals., at 34 cents per gal.; 1 chest tea, 84 lb., at 48 cents per lb.; 1 bag pepper, 24 lbs., at 11 cents per lb.; 1 box N. O. sugar, 278 lbs., at 6 cents per lb.

5. April 21, 1855, French & Simmons, of New York, sold to Clark & Hubbard, 1 piece super. broadcloth, 26 yds., at $3.75 per yd.; 1 piece fancy cassimere, 31 yds., at $1.34 per yd.; 1 piece black cassimere, 23 yds., at $1.62 per yd.; 6 pieces English prints, 29, 31, 30, 33, 29, and 31 yds., or 183 yds., at $.12 per yd.; 3 pieces Merrimack prints, 28, 32, 31 yds., or 91 yds., at $.09 per yd.; 2 pieces Scotch gingham, 42 and 41 yds., or 83 yds., at $.17 per yd.; 1 piece sarsenet cambric, 32 yds., at $.28 per yd.; 3 dozen cotton hose at $1.87 per doz.; and 2 lbs. Marshall's linen thread at $1.75 per lb.

6. Geo. Stevens, of Worcester, sold to Daniel Barnard, Sept. 15, 1854, 28 bu. of potatoes at $.87; Oct. 1, 43 bu. of potatoes at $.65; Oct. 7, 7 tons of hay at $19.50, 4 tons at $18.37, and 2 tons at $17.25; and Nov. 3, 23 bbl. of apples at $1.75, and 19 bbl. at $1.94. Mr. Barnard paid Mr. Stevens, Oct. 5, 1854, $20, Oct. 17, $13.50; and Nov. 20, he sold him 14 lb. sugar at 7 cents, 12 lb. at 9 cents, 8 lb. at 11 cents, 7 lb. coffee at 15 cents, 4 lb. tea at 54 cents, 3 lb. at 47 cents, 4 lb. chocolate at 19 cents, and 1 bag salt for $1.25. Jan. 1, 1855, Mr. Stevens made out his bill.

SECTION IX.

PROPERTIES OF NUMBERS, TESTS OF DIVISIBILITY, FACTORS, MULTIPLES, DIVISORS.

101. *Definitions.*

(*a.*) A FACTOR of any given number is such a number as taken an entire number of times will produce the given number; or, the FACTORS of a number are the numbers which multiplied together will produce it.

Thus, 2 is a factor of 4, 6, 8, &c.; 3 is a factor of 6, 9, 12, 15, &c.

(*b.*) A DIVISOR of a number is any number which will exactly divide it.

NOTE. — Every divisor of a number must be a factor of it, and every factor of a number a divisor of it. The terms *factor* and *divisor*, as here used, are only applied to entire numbers.

(*c.*) A PRIME NUMBER is one which has no other factors besides itself and unity.

Thus, 1, 2, 3, 5, 7, 11, 13, 17, &c., are prime numbers.

(*d.*) A COMPOSITE NUMBER is one which has other factors besides itself and unity.

Thus, 4, 6, 8, 9, 10, 12, 14, 15, 16, &c., are composite numbers.

(*e.*) Any entire number of times a given number is a MULTIPLE of it; or, a MULTIPLE of a number is any number which can be exactly divided by it.

Thus, 12 is a multiple of 1, 2, 3, 4, 6, and 12, because it is an exact number of times each of them, or because it can be divided by each without a remainder.

(*f.*) Two numbers are PRIME TO EACH OTHER when they have no common factor.

For example, 4 and 9 are prime to each, as are 8 and 15, 24 and 35, &c.

Again, 6 and 9 are not prime to each other, because they have the common factor 3; 8 and 12 are not prime to each other, because they have the common factor 4, &c.

NOTE. — It is obvious from the foregoing, that every number is a factor of all its multiples, and a multiple of all its factors.

102. *Demonstration of Principles.*

Proposition First. — If one of two numbers is a factor of another, it must be a factor of any number of times that other number.

For to find any number of times a given number, we have only to multiply the number by some new factor, without striking out any of the former ones.

Illustrations. — Since 2 is a factor of 12, it must be a factor of any number of times 12, as 24, 36, 48, &c.

Since 7 is a factor of 14, it must be a factor of any number of times 14, as 28, 42, 56, &c.

Proposition Second. — If each of two numbers is a multiple of a third number, their sum and their difference must also be multiples of that third number.

For, adding an exact number of times a given number to, or subtracting it from, an exact number of times the same number, must give an exact number of times that number.

Illustrations. 8 times 9, or 72, + 3 times 9, or 27, = 11 times 9, or 99. So 8 times 9, or 72, — 3 times 9, or 27, = 5 times 9, or 45.

Again. Both 12 and 20 are multiples of 4, and so is their sum, 32, and their difference, 8.

Both 42 and 28 are multiples of 7, and so is their sum, 70, and their difference, 14.

Proposition Third. — If one of two numbers is a multiple of a third number, and the other is not, neither their sum nor their difference will be a multiple of that third number.

For both the sum and the difference of an entire, and a fractional, number of times a given number, must equal a fractional number of times that given number.

Illustrations. 8 times 6, or 48, + $2\frac{1}{2}$ times 6, or 15, = $10\frac{1}{2}$ times 6, or 63.

8 times 6, or 48, — $2\frac{1}{2}$ times 6, or 15, = $5\frac{1}{2}$ times 6, or 33.

$7\frac{1}{3}$ times 9, or 66, — 4 times 9, or 36, = $3\frac{1}{3}$ times 9, or 30.

$7\frac{1}{3}$ times 9, or 66, + 4 times 9, or 36, = $11\frac{1}{3}$ times 9, or 102.

Again. 20 is a multiple of 5, and 13 is not; hence, neither their sum, 33, nor their difference, 7, is a multiple of it.

38 is not a multiple of 6, and 24 is; hence, neither their sum, 62, nor their difference, 14, is a multiple of it.

Proposition Fourth. — If neither of two numbers is a multiple of a third, their sum or their difference may or may not be a multiple of it.

The truth of this proposition can best be made manifest by a few illustrations.

1. Neither 7 nor 23 is a multiple of 2; yet both their sum, 30, and their difference, 16, are multiples of 2.

2. Neither 5 nor 28 is a multiple of 3; yet their sum, 33, is, and their difference, 23, is not, a multiple of 3.

3. Neither 8 nor 17 is a multiple of 3; yet their sum, 25, is not, and their difference, 9, is, a multiple of 3.

4. Neither 14 nor 27 is a multiple of 4; and neither their sum, 41, nor their difference, 13, is a multiple of 4.

103. *Tests of the Divisibility of Numbers.*

Application of the foregoing propositions.

I. *Divisibility by* 2, 5, $3\frac{1}{3}$, *or by any other number which will exactly divide* 10.

Every number greater than ten is composed of a certain number of tens, plus the units expressed by its right hand figure. But the part which is made up of tens must (**102**, Prop. I.) be divisible by any divisor of ten; and hence, (**102**, Prop. II. and III.) the divisibility of the entire number will depend on the part expressed by the right hand figure.

Therefore, a number is divisible by 2, 5, $2\frac{1}{2}$, $1\frac{2}{3}$, *or by any other number which will exactly divide* 10, *when its right hand figure is thus divisible.*

Illustration. 4125 is divisible by 5, by $2\frac{1}{2}$, and by $1\frac{2}{3}$, because each of these numbers will exactly divide 10, and also 5, the right hand figure of the given number.

II. *Divisibility by* 4, 20, 25, 50, $12\frac{1}{2}$, $16\frac{2}{3}$, *or any other number which will exactly divide* 100.

Every number greater than 100 is composed of a certain number of hundreds, plus the number expressed by its two right hand figures. But the part which is made up of hundreds must (**102**, Prop. I.) be divisible by any divisor of one hundred, and hence, (**102**, Prop. II. and III.) the divisibility of the entire number must depend on the part expressed by the two right hand figures.

Therefore, a number is divisible by 4, 20, 25, 50, $12\frac{1}{2}$, *or by any other number which will exactly divide* 100, *where its two right hand figures are thus divisible.*

III. *Divisibility by* 8, 40, 125, 250, 500, $333\frac{1}{3}$, $166\frac{2}{3}$, *or by any other number which will exactly divide* 1000.

Every number greater than 1000 is composed of a certain number of thousands, plus the number expressed by its three right hand figures. But the part which is composed of thousands must (**102**, Prop. I.) be divisible by any divisor of 1000, and hence, (**102**, Prop. II. and III.) the divisibility of the entire number must depend on the three right hand figures.

Therefore a number is divisible by 8, 125, 250, 500, $166\frac{2}{3}$, $333\frac{1}{3}$, *or by any number which will exactly divide* 1000, *when its three right hand figures are thus divisible.*

NOTE. — Similar tests for determining the divisibility of numbers by any divisor of 10,000, 100,000, &c., could be established, but as they could very rarely be applied to advantage, we omit them.

IV. *Divisibility by* 9.

(*a.*) If 1 be subtracted from a unit of any decimal denomination above unity, the remainder will be expressed entirely by 9's, and will therefore be a multiple of 9.

Illustrations. —

$$10 - 1 = 9$$
$$100 - 1 = 99$$
$$1000 - 1 = 999$$
$$10000 - 1 = 9999$$
&c., &c.

(*b.*) But unity, or 1, $= 0 \times 9 + 1$; and if, for convenience of statement, we regard 0×9, or 0, as a multiple of 9, it

will follow that a unit of any decimal denomination is 1 more than a multiple of 9.

Illustrations. —

$$1 = 0 + 1$$
$$10 = 9 + 1$$
$$100 = 99 + 1$$
$$1000 = 999 + 1$$
&c., &c.

(*c.*) As a unit of any decimal denomination is 1 more than a multiple of 9, 2 units must be 2 more than such a multiple, 3 units must be 3 more, 4 units 4 more, 5 units 5 more, &c.

Illustrations. — Since 1000 = 1 more than a multiple of 9, 7000 must equal 7 more.

Since 1000000 = 1 more than a multiple of 9, 7000000 must equal 7 more, &c.

(*d.*) But the digit figures of a number express the number of units of its various denominations, and therefore any number must be as many more than a multiple of 9 as there are units in the sum of its digit figures.

Illustrations. 8235 = 8000 + 200 + 30 + 5.
8000 = 8 more than a multiple of 9.
200 = 2 more than a multiple of 9.
30 = 3 more than a multiple of 9.
5 = 5 more than a multiple of 9.

Therefore, 8235 = 8 + 2 + 3 + 5, or 18 more than a multiple of 9, or it equals a multiple of 9, plus 18, and must therefore (since 9 is a divisor of 18) be a multiple of 9.

Again. 57864 = 50000 + 7000 + 800 + 60 + 4.
50000 = 5 more than a multiple of 9.
7000 = 7 " " " " "
800 = 8 " " " " "
60 = 6 " " " " "
4 = 4 " " " " "

Therefore, 57864 = 5 + 7 + 8 + 6 + 4, or 30 more than a multiple of 9, and is therefore (since 9 is not a divisor of 30) not a multiple of 9.

(*e.*) From these principles it follows, —

1. *That every number is equal to a multiple of* 9, *plus the sum of its digit figures.*

2. *That if the sum of the digit figures of any number be subtracted from it, the remainder will be a multiple of* 9.

3. *That a number is divisible by* 9 *when the sum of its digit figures is thus divisible.*

4. *That the remainder obtained by dividing the sum of the digit figures of any number by* 9, *is the same as that obtained by dividing the number itself by* 9.

5. *That the difference of any two numbers, the sums of whose digits are alike, will be a multiple of* 9.

6. *That the divisibility of a number by* 9 *will not be affected by any change in the order of its digits.*

7. *That if the digit figures of a number be added together, and then the digit figures of the result, and so on till the sum is expressed by a single figure, that figure will either be* 9, *or the remainder obtained by dividing the original number by* 9. *If the figure is* 9, *the number is a multiple of* 9.

Remark. — Finding the excess of any number over a multiple of 9 is called casting out the 9's.

(*f.*) *Consequences of the foregoing.* — From the foregoing properties of the number 9, considered in connection with the principles established in **102**, come some convenient methods of proving numerical operations; a few of which we will mention, leaving the pupil to find out the reasons for each.

1. *To prove Addition.* — Cast out the 9's from the several numbers added, add the results, and cast out the 9's from their sum. Then cast out the 9's from the number obtained as the answer to the question, and if the work be correct, the last two results will be equal.

2. *To prove Subtraction.* — Cast out the 9's from the subtrahend and remainder, add the results, and cast out the 9's from their sum. Then cast out the 9's from the minuend, and if the work is correct, the last two results will be equal.

3. *To prove Multiplication.* — Cast out the 9's from the several factors employed, multiply the results together, and cast out the 9's from their product. Then cast out the 9's from the product of the original multiplication, and if the work is correct, the last two results will be equal.

4. *To prove Division.* — Cast out the 9's from the divisor, quotient, and remainder; to the product of the first two results add the last result, and cast out the 9's. Then cast out the 9's from the dividend, and if the work is correct, the last two results will be equal.

(g.) Dependent on the same principles is the following, which the pupil may use *with the uninitiated* as an arithmetical puzzle.

To tell what figure has been erased. — Tell a person to write any number whatever, without informing you what it is; to subtract the sum of its digit figures from it; to erase from this result any digit figure, other than zero, and write zero in its place; and finally, to add together the digit figures of the number thus obtained, and tell you their sum.

The difference between this sum and the next higher multiple of 9 will show the figure removed. Thus, if the sum is 29, 7 was erased; if it is 45, 9 was erased; &c.

Let the pupil explain the reasons of this.

V. *Divisibility by* 3.

Since 9 is a multiple of 3, every number which is a multiple of 9 must also be a multiple of 3, (Prop. I.) Therefore, a unit of any decimal denomination must be 1 more than a multiple of 3; and hence *a number is a multiple of* 3 *when the sum of its digit figures is such a multiple.*

VI. *Divisibility by* 11.

(a.) A unit of any decimal denomination is either 1 or 10 more than a multiple of 11. Thus, —

$$\begin{aligned} 1 &= 0 \times 11 + 1 \\ 10 &= 0 \times 11 + 10 \\ 100 &= 9 \times 11 + 1 \\ 1000 &= 90 \times 11 + 10 \\ 10000 &= 909 \times 11 + 1 \\ 100000 &= 9090 \times 11 + 10 \end{aligned}$$

Or, arranging the numbers with reference to the remainders, we have —

$$\begin{aligned} 1 &= 0 \times 11 + 1 \\ 100 &= 9 \times 11 + 1 \\ 10000 &= 909 \times 11 + 1 \\ 1000000 &= 90909 \times 11 + 1 \end{aligned}$$

$$\begin{aligned} 10 &= 0 \times 11 + 10 = 1 \times 11 - 1 \\ 1000 &= 90 \times 11 + 10 = 91 \times 11 - 1 \\ 100000 &= 9090 \times 11 + 10 = 9091 \times 11 - 1 \\ 10000000 &= 909090 \times 11 + 10 = 909091 \times 11 - 1 \end{aligned}$$

(b.) From which we see that a unit of any denomination expressed by a figure occupying the 1st, 3d, 5th, or any other odd place from the point, is 1 more than a multiple of 11; and that a unit of any denomination expressed by a figure

occupying the 2d, 4th, or any other even place from the point, is 1 less than a multiple of 11.

(*c.*) Hence, on account of the figures occupying *odd* places from the point, a number is as many more than a multiple of 11 as there are units in the sum of these figures, while on account of the figures occupying *even* places, it is as many less than a multiple of 11 as there are units in the sum of those figures.

(*d.*) Hence, every number is equal to some multiple of 11, plus the sum of its digit figures occupying odd places from the point, minus the sum of those occupying even places. If these sums are alike, the additions will equal the subtractions, and the number will be a multiple of 11. If these sums are unlike, their difference will be the excess of the additions over the subtractions, or of the subtractions over the additions.

(*e.*) If, then, the difference of the sums of the alternate digits is a multiple of 11, the whole number will be either the sum or the difference of two multiples of 11, and hence a multiple of 11; but if this difference is not such a multiple, the whole number will be either the sum or the difference of two numbers, one of which is, and the other is not, a multiple of 11, and hence will not be a multiple of 11.

(*f.*) *Hence, a number is a multiple of* 11, *when the sums of its alternate digits are equal, or when their difference is a multiple of* 11.

First Example. — Is 15873 a multiple of 11?

Solution. — The sum of the digits in the odd places is $3 + 8 + 1$, or 12; the sum of those in the even places is $7 + 5$, or 12. Hence, the two sums are alike, and 15873 is a multiple of 11.

Second Example. — Is 274854 a multiple of 11?

Solution. — The sum of the digits in the odd places is $4 + 8 + 7 = 19$; the sum of the digits in the even places is $5 + 4 + 2 = 11$; the difference between 19 and 11 is 8, which is not a multiple of 11. Hence, 274854 is not a multiple of 11.

VII. *If a number is divisible by each of two numbers*

which are prime to each other, it will be divisible by their product.

For dividing by one cannot (since the numbers are prime to each other) cast out the other, or any factor of it.

Illustrations. 1. A number which is divisible by 4 and 9 must be divisible by 4 × 9, or 36.

2. A number which is divisible by 8 and 15 must be divisible by 8 × 15, or 120

3. A number is divisible by 12, when it is by 3 and 4.

4. A number is divisible by 35, when it is by 7 and 5.

5. A number is divisible by 42, when it is by 6 and 7.

VIII. *If a number is divisible by each of two numbers which have a common factor, it will not of necessity be divisible by their product.*

For dividing by one must cast out the common factor of the two numbers, and if that factor be not taken more than once as a factor of the original number, the quotient will not be divisible by the other of the two numbers.

Illustrations. 84 is divisible by 4 and 6, but not by their product, 24. 72 is divisible by 4 and 6, and also by their product, 24.

IX. *If one number is not divisible by another, it will not be divisible by any multiple of that other number.*

For if a number does not contain once another number, it cannot contain any number of times that other number.

Illustrations. — A number which is not divisible by 2, is not divisible by 4, 6, 8, 10, &c. A number which is not divisible by 3, is not divisible by 6, 9, 12, 15, 18, 21, &c.

X. *A number which is divisible by any composite number is divisible by all the factors of that composite number.*

For dividing by any composite number is merely dividing by the product of its factors.

104. *Recapitulation, for convenience of reference.*

I. *Any number is divisible by* 2, 5, $3\frac{1}{3}$, *or any other number which will exactly divide* 10, *when its right hand figure is thus divisible.*

II. *Any number is divisible by* 4, 20, 25, 50, $12\frac{1}{2}$, $16\frac{2}{3}$, *or any other number which will exactly divide* 100, *when the number expressed by its two right hand figures is thus divisible.*

III. *Any number is divisible by* 8, 4, 125, 250, $333\frac{1}{3}$, *or any other number which will exactly divide* 1000, *when the number expressed by its three right hand figures is thus divisible.*

IV. *Any number is divisible by* 3 *or* 9, *when the sum of its digits is thus divisible.*

V. *Any number is divisible by* 11, *when the sums of its alternate digits are equal, or their difference is a multiple of* 11.

VI. *A number which is divisible by each of two numbers which are prime to each other, is divisible by their product.*

VII. *A number which is divisible by each of two numbers not prime to each other, is not of necessity divisible by their product.*

VIII. *A number which is not divisible by another is not divisible by any multiple of that other number.*

IX. *A number which is divisible by any composite number is also divisible by all the factors of that composite number.*

105. *Definitions of Factors, Powers, &c.*

(*a.*) A number is said to be *divided into factors* when any factors which will produce it are found.

Thus, in $36 = 4 \times 9$, 36 is divided into the factors 4 and 9; but in $36 = 2 \times 6 \times 3$ it is divided into the factors 2, 6, and 3.

(*b.*) A number is said to be *divided into its prime factors* when it is divided into factors which are all prime numbers.

Illustrations. $36 = 2 \times 2 \times 3 \times 3$ $8 = 2 \times 2 \times 2$
$30 = 2 \times 3 \times 5$ $84 = 2 \times 2 \times 3 \times 7$

(*c.*) When any number is taken more than once as a factor to produce another number, we may express the number of times it is taken as a factor, by placing a small figure above it and a little to the right.

Illustrations. 3^2 means the same as 3×3; i. e., that 3 is to be taken twice as a factor.

3^4 means the same as $3 \times 3 \times 3 \times 3$; i. e., that 3 is to be taken 4 times as a factor.

$3^3 \times 2^4$ means the same as $3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2$; i. e., that the product of 3 taken 3 times as a factor is to be multiplied by the product of 2 taken 4 times as a factor.

NOTE. — The student should notice the difference between taking a number *as a factor* a certain number of times, and merely taking it a number of times.

Thus, 5 *taken 3 times as a factor* = 5 times 5 times 5 = 125, but 5 *taken 3 times* = 3 times 5 = 15.

(*d.*) The product of a number taken any number of times as a factor is called a POWER *of the number.*

Illustrations. 9 is the second power of 3, because it is the product of 3^2, i. e., of 3 taken twice as a factor.

2^5 is the fifth power of 2, because it is the product of 2^5, i. e., of 2 taken 5 times as a factor.

(*e.*) The figure indicating how many times a number is taken as a factor is called the EXPONENT *of the power to which the number is raised.*

Thus, in 2^5, the exponent is 5; in 5^2, it is 2; in 6^4, it is 4; &c.

(*f.*) The following examples indicate the method of reading numbers expressing powers.

3^7 is read *three seventh power*, or *three to the seventh power.*

$4^5 \times 2^3$ is read *four fifth power multiplied by two third power.*

NOTE. — The second power of a number is sometimes called its SQUARE, and the third power its CUBE.

106. *Method of Factoring Numbers.*

1. What are the prime factors of 2772?

Solution. — We see by **104**, II., that 2772 is divisible by 4, and therefore by 2×2, the prime factors of 4.

Dividing by 4 gives 693 for a quotient, which, by **104**, IV., we see is divisible by 9, and therefore by 3×3, the prime factors of 9.

Dividing 693 by 9 gives 77 for a quotient, the prime factors of which are 7 and 11. Hence, the prime factors of 2772 are 2^2, 3^2, 7, and 11; or, which is the same thing, $2772 = 2^2 \times 3^2 \times 7 \times 11$.

2. What are the prime factors of 29766?

Solution. — We see by **104**, I. and IV., that 29766 is divisible by both 2 and 3, and hence by their product, 6. Dividing by 6 gives 4961 for a quotient, which, by **104**, V., is a multiple of 11. Dividing by 11 gives 451 for a quotient, which (**104**, V.) is also a multiple of 11 Dividing by 11 gives 41 for a quotient, which is obviously a prime number. Therefore, $29766 = 2 \times 3 \times 11^2 \times 41$.

3. What are the prime factors of 3871123?

Solution. — We readily see that it is not divisible by 2, 3, 5, or 11,

and therefore we must see whether it is divisible by the other prime numbers, beginning, for convenience sake, with the smallest. By actual trial, we find that 7, 13, 17, 19, and 23 each leave a remainder after division, but that 29 is contained in it exactly 133487 times. Therefore, 29 is one of the prime factors of the given number, and the others will be found in 133487. But no number less than 29 can be a factor of 133487, for none is a factor of the original number. We therefore first try 29, which we find is contained exactly 4603 times. Therefore, 29 is again a factor of the original number, and the remaining factors must be found in 4603. But no number less than 29 can be a factor of 4603, for none was a factor of the original number. Beginning with 29, we try in succession 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, and 71, and find that none of them will exactly divide 4603; and we observe, moreover, that the entire part of the quotient of the division by 71 is less than 71.

Now, as the divisor increases, the quotient must decrease; therefore, if there is any number larger than 71 which will divide it, the quotient must be less than 71. But when a division can exactly be performed, the divisor and quotient must each be a factor of the dividend; and hence if 4603 has a factor greater than 71, it must also have one less than 71. But as we have already found that it has no factor less than 71, we infer that it can have none greater, and that it is a prime number.

Hence, $3871123 = 29^2 \times 4603$.

(*a.*) The labor of testing the divisibility of a number by the various prime numbers may be made much less laborious and tedious, by considering what must be the last figure of the quotient, if the division by any number is possible.

(*b.*) Thus, in the last example, in testing the divisibility of 4603, we may observe that if 29 is a factor of it, the last figure of the other factor must be 7, for 7, or some number ending in 7, is the only number which, multiplied by 29, will give for a product a number ending in 3. Dividing by 29, we have, 29 is contained in 46 once, and 17 remainder; in 170, 5 times, and 25 remainder; and we can readily see that it is contained in 253 more than 7 times.

(*c.*) Again. If 31 is a factor of 4603, the last figure of the other factor must be 3, for 3, or some number ending in 3, is the only number which, multiplied by 31, will give for a product a number ending in 3. Dividing by 31, we have, 31 is contained in 46 once, and 15 remainder; in 150, 4 times, and 26 remainder; and we can see at a glance that it is contained in 263 more than 3 times.

(*d.*) Again. If 37 is a factor of 4603, the last figure of the other factor must be 9. But 37 is contained in 46 once, and 9 remainder; in 90, twice, and 16 remainder; and it is obvious that it cannot be contained as many as 9 times in 163.

(*e.*) From the above, we see that to find the prime factors of a number, we may first divide it by any number which will divide it without a remainder, then divide this quotient by any number which will divide it without a remainder, and so proceed with each successive quotient, till we reach one which is a prime number, or which can be readily divided into its prime factors. The prime factors of the several divisors and of the last quotient will be the prime factors required.

(*f.*) If in any case we cannot discover a divisor of any given number by the tests of **104**, we try in succession 7, and all the prime numbers from 13 upwards, till we find one which is a divisor of the given number, or till the entire part of our quotient is less than the number employed as a divisor, in which case the number is prime.

107. *Exercises for the Student.*

(*a.*) Find the prime factors of the numbers from 1 to 100, writing them as in the following model.

1, prime.	7, prime.
2, prime.	$8 = 2 \times 2 \times 2 = 2^3$.
3, prime.	$9 = 3 \times 3 = 3^2$.
$4 = 2 \times 2 = 2^2$.	$10 = 2 \times 5$.
5, prime.	11, prime.
$6 = 2 \times 3$.	$12 = 2 \times 2 \times 3 = 2^2 \times 3$.

(*b.*) We would recommend that the pupil make out a table of the prime factors of the numbers from 1 to 1000. He will find it a very profitable exercise, and one which will greatly aid him in all his subsequent work.

(*c.*) What are the prime factors —

1. Of 1001?	8. Of 1183?
2. Of 1025?	9. Of 1625?
3. Of 1024?	10. Of 2057?
4. Of 1033?	11. Of 16128?
5. Of 1096?	12. Of 3809?
6. Of 1157?	13. Of 6381?
7. Of 1067?	14. Of 7128?

15. Of 7854?
16. Of 5989?
17. Of 5625?
18. Of 9257?
19. Of 10917?
20. Of 843479?
21. Of 444528?
22. Of 1072181?
23. Of 5764801?
24. Of 6103515625?
25. Of 12168587046?

108. *Common Divisor, Definitions, and Properties.*

(*a.*) A DIVISOR of a number is a number which will exactly divide it.

(*b.*) A COMMON DIVISOR OF TWO OR MORE NUMBERS is a number which is a divisor of each of them.

(*c.*) The GREATEST COMMON DIVISOR OF TWO OR MORE NUMBERS is the largest number which is a divisor of each of them.

(*d.*) From these definitions and the principles previously established, it follows that, —

1. *A divisor of a number can contain only such prime factors as are found in that number.*

2. *A common divisor of two or more numbers can contain only such prime factors as are common to all the numbers.*

3. *The greatest common divisor of two or more numbers is the product of all the prime factors common to all the given numbers.*

109. *Greatest Common Divisor. — Method by Factors.*

1. What is the greatest common divisor of 819 and 1071?

Solution. — The greatest common divisor of 819 and 1071 is the product of all the prime factors common to those numbers.

$$819 = 3^2 \times 7 \times 13$$
$$1071 = 3^2 \times 7 \times 17$$

From which we see that the only common factors are 3^2 and 7. Therefore, $3^2 \times 7$, or 63, is the greatest common divisor required.

What is the greatest common divisor —

2. Of 792 and 936?
3. Of 1125 and 1575?

4. Of 3102 and 3666?
5. Of 287 and 369?
6. Of 4652 and 5544?
7. Of 924 and 1188?

8. What is the greatest common divisor of 72, 108, and 252?

Solution. — The greatest common divisor of 72, 108, and 252, is the product of all the prime factors common to those numbers.

$$72 = 2^3 \times 3^2$$
$$108 = 2^2 \times 3^3$$
$$252 = 2^2 \times 3^2 \times 7$$

From which we see that the only common factors are 2^2 and 3^2. Therefore, $3^2 \times 2^2$, or 36, is the greatest common divisor required.

What is the greatest common divisor —

9. Of 168, 505, and 539?
10. Of 1386, 3234, and 4158?
11. Of 864, 2058, and 2346?
12. Of 686, 1029, and 2401?
13. Of 112, 147, 168, and 189?
14. Of 576, 672, 864, and 1132?

110. *A more brief Method.*

(*a.*) The following solution will usually be found much more brief than the preceding, inasmuch as it avoids the necessity of separating all the numbers into their factors. By it, we find the factors of any one of the numbers, and then see which of them are factors of all the other numbers.

1. What is the greatest common divisor of 756, 840, 1386, and 1596?

Solution. — We first find the factors of 840, because they can be the most readily found. $840 = 2^3 \times 3 \times 5 \times 7$, and we have now to find which of these factors, if any, are factors of all the other given numbers. It is obvious (**104,** I.) that 2 is a factor of all of them, and (**104,** II.) that it is contained but once as a factor in 1386. Hence, 2 will enter once only as a factor of the greatest common divisor. 5 is (**104,** I.) a factor of no other number. 3 being a factor (**104,** IV.) of all the numbers, is a factor of the greatest common divisor. By trial, 7 is found to be a factor of all the numbers, and hence of the greatest common divisor. Hence, the greatest common divisor is $2 \times 3 \times 7$, or 42.

(b.) What is the greatest common divisor —

2. Of 3465, 4375, and 5250?
3. Of 1792, 936, 1224, and 1656?
4. Of 1342, 1738, 2376, and 2596?
5. Of 3312, 6048, 9576, and 6336?
6. Of 4572, 2380, 5272, and 8364?
7. Of 3125, 4379, 8243, and 5975?

111. *Factoring not always necessary.*

(a.) We can frequently find the greatest common divisor of the given numbers without finding their prime factors.

(b.) Thus, in finding the greatest common divisor of 24 and 42, we can see at a glance that 6 is a divisor of both; that $24 = 6 \times 4$, and $42 = 6 \times 7$; therefore, since 4 and 7 have no common factor, 6 must be the greatest common divisor required.

(c.) Again. In finding the greatest common divisor of 693 and 819 we see at once that 9 will divide each.

$$693 = 9 \times 77$$
$$819 = 9 \times 91$$

Now, comparing 77 and 91, we see that 7 will divide each.

$$77 = 7 \times 11$$
$$91 = 7 \times 13$$

And as 7 and 11 are prime to each other, there is no other common factor to the given numbers. Hence, 7×9, or 63, is the greatest common divisor required.

(d.) Again. In finding the greatest common divisor of 36 and 72, we may see that 36 is a divisor of 72, and hence the greatest common divisor required.

(e.) What is the greatest common divisor —

1. Of 12 and 18?
2. Of 28 and 42?
3. Of 18 and 54?
4. Of 48 and 84?
5. Of 324 and 594?
6. Of 2025 and 2916?
7. Of 5544 and 11583?
8. Of 18, 48, 72, and 66?
9. Of 12, 36, 60, and 132?
10. Of 49, 63, 84, and 91?

112. *Method by Addition and Subtraction.*

(*a.*) When the sum or the difference of any two of the given numbers is a small number, or one more easily divided into factors than any of the original numbers, the work may be abbreviated by applying the principles of **102**, Proposition II.

1. What is the G. C. D.* of 8379 and 8484?

Suggestion. — The G. C. D. required must be a divisor of 8484 — 8379, which is $105 = 3 \times 5 \times 7$; and we have only to ascertain which of these factors, if any, are factors of one of the given numbers.

2. What is the G. C. D. of 89437, 95429, and 90537?

Suggestion. — The G. C. D. required must be a divisor of the difference of any two of the given numbers, as of 89437 and 90537, which is $1100 = 2^2 \times 5^2 \times 11$; and we have only to ascertain which of these factors, if any, are factors of all the given numbers.

3. What is the G. C. D. of 56474 and 28526?

Suggestion. — The G. C. D. required must be a divisor of 56474 + 28526, which is $85000 = 2^3 \times 5^4 \times 17$; and we have only to ascertain which of these factors, if any, are factors of one of the given numbers.

4. What is the G. C. D. of 3598, 5383, 6545, and 8617?

Suggestion. — The G. C. D. required must be a divisor of the sum of any two of the given numbers, as of 5383, and 8617, which is $14000 = 2^4 \times 5^3 \times 7$; and we have only to ascertain which of these factors, if any, are factors of all the given numbers.

What is the greatest common divisor —

5. Of 949 and 962?
6. Of 857637 and 857692?
7. Of 5489 and 7689?
8. Of 9709, 10906, and 10241?
9. Of 71227, 72553, 73840, and 75127?
10. Of 930069, 992673, and 1103673?
11. Of 12551 and 25949?
12. Of 169881 and 170119?
13. Of 16181 and 16324?
14. Of 180006, 293694, 468963, and 596862?

* G. C. D. means *greatest common divisor.*

113. *Demonstration of Method by Division.*

(*a.*) *Proposition.* — *The G. C. D. of any two numbers is the same as the G. C. D. of the smaller, and the remainder left after dividing the larger by the smaller.*

(*b.*) To prove this, let the letter a represent any number whatever, and the letter b represent any other number larger than a. Now, whatever numbers a and b represent, it is obvious that a is contained in b some number of times, which we will call c times, and that there may, or may not, be a remainder. If there is no remainder, a will be the G. C. D. of a and b. If, however, there is a remainder, we will call it d, and we have to prove that the G. C. D. of a and b is the same number as the G. C. D. of a and d.

(*c.*) Representing the division by writing the letters as we should the numbers they represent, we have, —

$$\begin{array}{rcl} & \text{Dividend.} & \\ \text{Divisor } a\ ) & b & (\ c = \text{Quotient.} \\ & a \times c & = \text{Product of divisor by quotient.} \\ & \overline{\quad d \quad} & = \text{Remainder.} \end{array}$$

(*d.*) From the nature of division, it follows that —

$$d = b - a \times c$$

$$\text{and} \qquad b = d + a \times c$$

(*e.*) Now, as any divisor of a must (**102**, Prop. I.) be a divisor of $c \times a$, the G. C. D. sought must be a divisor of b and $c \times a$, and hence (**102**, Prop. II.) of $b - c \times a$, or d. Again. Any divisor of d and a must be a divisor of d and $c \times a$, and hence (**102**, Prop. II.) of $d + c \times a$, or b. It therefore follows that the G. C. D. sought is the same number as the G. C. D. of d and a, which, as a and b may represent any numbers whatever, establishes the proposition.

114. *Application of the foregoing Principle.*

(*a.*) When the numbers of which the G. C. D. are required are such as cannot easily be divided into factors, or solved by the methods heretofore given, the principle just demonstrated may be advantageously applied, thus: —

(*b.*) Divide the greater of any two numbers by the less; then will the remainder obtained by this division and the smaller of the two numbers have the same G. C. D. as the numbers themselves.

(*c*) But as the remainder after any division must always be less than the divisor, we may divide the original divisor, i. e., the smaller number, by this remainder. Now, if there be a remainder from this division, it follows, from the proposition, that the G. C. D. sought must be the G. C. D. of this remainder and the last divisor; and we may divide the last divisor by the last remainder.

(*d.*) But as the remainders must constantly be decreasing, it follows that if we continue this process, we shall at last find a remainder which will exactly divide the preceding, and will therefore be the G. C. D., or we shall have a remainder of 1, in which case the numbers are prime to each other.

1. What is the G. C. D. of 4277 and 9737?

WRITTEN WORK.

```
4277 ) 9737 ( 2
       1183 ) 4277 ( 3
               728 ) 1183 ( 1
                      455 ) 728 ( 1
                             273 ) 455 ( 1
                                    182 ) 273 ( 1
                                            91 ) 182 ( 2
                                                  00
```

Explanation. — Dividing the greater number by the less gives 1183 for a remainder. Therefore, the G. C. D. required is the G. C. D. of the smaller number, 4277 and 1183. Dividing 4277 by 1183 gives 728 for a remainder. Therefore, the G. C. D. required is the G. C. D. of 728 and 1183. Dividing 1183 by 728 gives 455 for a remainder. Therefore, the G. C. D. required is the G. C. D. of 728 and 1183.

Proceeding in this way we find that the G. C. D. required is the same as the G. C. D. of 91 and 182, which is 91.

(*e.*) The work can frequently be much shortened by observing that if any remainder contains a prime factor which is not a factor of the preceding divisor, the factor may be cast out without affecting the G. C. D.

Thus: 728, the second remainder in the above example, is obviously a multiple of 8, or 2^3, and 2 is not a factor of 1183, the preceding divi-

sor. We may therefore cast out the factor 8 from 728. The other factor is 91; therefore, the G. C. D. required is the same as the G. C. D. of 91 and 1183, which is 91.

The work, then, would be written thus:—

$$
\begin{array}{l}
4277\ )\ 9737\ (2 \\
\qquad\quad 1183\ )\ 4277\ (\ 3 \\
\qquad\qquad\quad 728 = 8 \times 91\ )\ 1183\ (\ 13 \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 273 \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad 00
\end{array}
$$

Hence, 91 = G. C. D.

(*f.*) What is the greatest common divisor of—

2. 8383 and 9589?
3. 2021 and 6493?
4. 6493 and 8909?
5. 425743 and 584159?
6. 113423 and 836987?
7. 6941849 and 9128111?

115. *Common Multiples, Definitions, and Properties.*

(*a.*) A multiple of a number is any entire number which can be exactly divided by it.

(*b.*) A common multiple of two or more numbers is a number which is a multiple of all of them.

(*c.*) The least common multiple of two or more numbers is the smallest number which is a multiple of all of them.

(*d.*) From these definitions and the principles previously established, it follows that,—

1. *A multiple of a number must contain all the prime factors of that number.*

2. *A common multiple of two or more numbers must contain all the prime factors of each of them.*

3. *The least common multiple of two or more numbers must be the smallest number which contains all the prime factors of each of them.*

116. *Least Common Multiple.—Method by Factors.*

1. What is the L. C. M.* of 504, 756, 924, and 1176?

* L. C. M. means *least common multiple.*

Solution. — The L. C. M. of these numbers is the smallest number which contains all the prime factors of each of them.

$$504 = 2^3 \times 3^2 \times 7$$
$$756 = 2^2 \times 3^3 \times 7$$
$$924 = 2^2 \times 3 \times 7 \times 11$$
$$1176 = 2^3 \times 3 \times 7^2$$

The factors of 1176 are $2^3 \times 3 \times 7^2$, all of which we take. The factors of 924 are $2^2 \times 3 \times 7 \times 11$, all of which, except 11, we have taken. We therefore introduce the factor 11, which gives $2^3 \times 3 \times 7^2 \times 11$. The factors of 756 are $2^2 \times 3^3 \times 7$, all of which, except 3^2, we have taken.. We therefore introduce the factor 3^2, which gives $2^3 \times 3^3 \times 7^2 \times 11$. The factors of 504 are $2^3 \times 3^2 \times 7$, all of which we have. Therefore, the product of $2^3 \times 3^3 \times 7^2 \times 11$, which is 116424, is the L. C. M. required.

NOTE. — When the factors of the L. C. M. have been obtained, much labor in multiplying may be saved, by writing some one of the numbers in place of its factors. Thus, in the example above, by writing 1176 instead of its factors, we should have $1176 \times 3^2 \times 11 = 116424 =$ L. C. M., as before.

What is the least common multiple —

2. Of 18, 24, and 42?
3. Of 36, 48, and 60?
4. Of 132, 144, and 160?
5. Of 98, 126, and 186?
6. Of 364, 637, and 1547?
7. Of 605, 325, 715, and 1001?
8. Of 504, 756, 1008, and 1512?
9. Of 594, 1386, and 1782?
10. Of 735, 945, 1365, and 2310?
11. Of 154, 231, 264, and 392?

117. *Abbreviated Method.*

(*a.*) When the factors of the several numbers can readily be determined, or after they are found, much labor may often be avoided by considering what factors are wanted with any one of the numbers to produce the L. C. M.

(*b.*) Thus, in the example solved in **116**, since the factors cannot easily be recognized, we write them thus: —

$$504 = 2^3 \times 3^2 \times 7$$
$$756 = 2^2 \times 3^3 \times 7$$
$$924 = 2^2 \times 3 \times 7 \times 11$$
$$1176 = 2^3 \times 3 \times 7^2$$

(*c.*) Then, since 1176 contains all the factors of each of the other numbers, except 11×3^2, the L. C. M. must equal $1176 \times 11 \times 3^2$; or if divided by 1176, the quotient would be 11×3^2, or 99.

(*d.*) Again. Since 924 contains all the factors of each of the other numbers, except $2 \times 7 \times 3^2$, the L. C. M. must equal $924 \times 2 \times 7 \times 3^2$; or, divided by 924, the quotient must be $2 \times 7 \times 3^2$, or 126.

(*e.*) Similar considerations will enable us to obtain the L. C. M. from 756 and from 504. A little practice will enable the pupil to determine at a glance from which number the L. C. M. can best be obtained.

1. What is the L. C. M. of 24 and 36?

Solution. — Observing that $36 = 3 \times 12$, and $24 = 2 \times 12$, it will at once be seen that the L. C. M. may be found by multiplying 36 by 2, or 24 by 3, and is 72.

2. What is the L. C. M. of 21, 35, and 56?

Solution. — It is obvious that 56 contains all the factors of 21 and 35, except 3 and 5.* Hence the L. C. M. $= 56 \times 5 \times 3 = 840$.

Second Solution. — Since 21 contains all the factors of 35 and 56, except 5 and 8, the L. C. M. must be $21 \times 5 \times 8 = 21 \times 40 = 840$.

NOTE. — In the above example, it is better to begin with 21 than 56, because it is easier to multiply 21 by 8×5, or 40, than to multiply 56 by 3×5, or 15.

3. What is the L. C. M. of 8 and 9?

Solution. — Since 8 and 9 have no common factors, their L. C. M must be their product, which is 72.

(*f.*) What is the least common multiple —

4. Of 12 and 25?
5. Of 32 and 40?
6. Of 16 and 27?
7. Of 12 and 30?
8. Of 24 and 35?
9. Of 42 and 70?
10. Of 4, 6, and 12?
11. Of 15, 25, and 35?
12. Of 6, 8, and 9?
13. Of 4, 9, and 25?
14. Of 14, 21, and 24?
15. What is the L. C. M. of 15, 18, 30, 12, and 36?

* For $56 = 8 \times 7$, $35 = 5 \times 7$, and $21 = 3 \times 7$; and 8, 5, and 3 are prime to each other.

Solution. — Since 36 is a multiple of 18 and 12, and 30 is a multiple of 15, the L. C. M. required must be the L. C. M. of 30 and 36, which equals 5 × 36, or 6 × 30 = 180.

(*g.*) What is the least common multiple —

16. Of 3, 6, 12, and 18?
17. Of 4, 9, 12, and 18?
18. Of 5, 3, 6, and 15?
19. Of 12, 24, 36, and 72?
20. Of 14, 15, 18, 30, 36, and 42?
21. Of 9, 10, 11, 12, and 18?
22. Of 98, 154, 198, and 284?
23. Of 72, 108, 180, and 252?
24. Of 306, 408, 612, and 1020?
25. Of 1548, 2322, 2580, and 3870?
26. Of 1872, 4212, 6318, and 8424?

118. *When Factors cannot easily be found.*

(*a.*) Since two numbers can have no other common factors than those of their greatest common divisor, it follows that the least common multiple of any two numbers may be found by multiplying one of them by the quotient obtained by dividing the other by their greatest common divisor. When prime to each other, their L. C. M. will be their product.

(*b.*) This principle may be advantageously applied when we wish to find the least common multiple of numbers the factors of which cannot easily be found.

1. What is the L. C. M. of 7379 and 9263?

Solution.—As these numbers cannot readily be divided into their factors, we first find their G. C. D., which is 157. Dividing 7379 by 157 gives 47 for a quotient, which must contain all the factors of the L.C.M. that are not found in 9263. Hence the L.C.M. must equal 9263 × 47 = 435361.

(*c.*) What is the least common multiple —

2. Of 5207 and 7493?
3. Of 2993 and 8651?
4. Of 3337 and 7471?
5. Of 3901 and 9047?
6. Of 6659 and 8083?
7. Of 7379 and 9263?

8. What is the L. C. M. of 3763, 5183, and 7261?

Suggestion. — First find the L. C. M. of any two of the numbers, as 3763 and 5183, and then the L. C. M. of this result and the remaining number.

(*d.*) What is the least common multiple —

9. Of 2881, 4171, and 9313?
10. Of 2419, 4661, and 5609?
11. Of 3811, 6031, 7519, and 7661?
12. Of 8381, 9809, 7361, and 6817?

SECTION X.

FRACTIONS.

119. *Introductory.*

(*a.*) If an apple, an orange, a line, or any other thing, or any number, be divided into *two equal parts*, the parts are called HALVES *of the apple, orange, line, or of whatever may have been thus divided.* If two or any number of things of the same kind are each of them divided into two equal parts, the parts will, as before, be called *halves of a thing of that kind*, and there will be as many times two halves as there are things divided.

(*b.*) If any thing should be divided into *three equal parts*, the parts would be called THIRDS *of the thing.* If the thing divided should be an apple, the parts would be *thirds of an apple.* If each of several apples should be divided into three equal parts, all the parts, or any portion of them, would still be called *thirds of an apple.* Again; if I should cut out of an apple such a part as would be obtained by dividing it into three equal parts, and then cut from another apple such another part, these parts would still be called *thirds of an apple*, although one of them came from one apple and one from another. And so, however many or few such parts we may take from the same apple, or different apples, they would all be called *thirds of an apple.*

(*c.*) Hence, *to have a third or thirds of any thing, quantity, or number, it is only necessary to have one or more such parts as would be obtained by dividing the thing, quantity, or number into three equal parts.*

(*d.*) Such considerations, extended, give the definitions of the following article.

120. *Definitions of Halves, Thirds, &c.*

(*a.*) Such parts as are obtained by dividing any thing or number into *two equal parts*, are called HALVES *of that thing or number.* One such part is called *one half* of it; two such parts are called *two halves* of it; three such, *three halves* of it; &c.

(*b.*) Such parts as are obtained by dividing any thing or number into *three equal parts,* are called THIRDS *of that thing or number.* One such part is called *one third* of it; two such, *two thirds;* three such, *three thirds;* four such, *four thirds;* &c.

(*c.*) In like manner, such parts as are obtained by dividing any thing or number into *four equal parts,* are called FOURTHS *of that thing or number;* such as are obtained by dividing it into *five equal parts,* are called FIFTHS; into *six,* are called SIXTHS; into *seven,* SEVENTHS; into *eight,* EIGHTHS; into *nine,* NINTHS; into *ten,* TENTHS; &c.

(*d.*) 1. What are sevenths?

Answer. — Sevenths of any thing or number are such parts as are obtained by dividing it into seven equal parts.

2. What are fifths?
3. What are thirds?
4. What are thirteenths?
5. What are fourths?
6. What are ninths?
7. What are tenths?
8. What are halves?
9. What are twenty-firsts?

(*e.*) From the method of obtaining halves, thirds, &c., it follows that —

Halves are equal parts of such kind that two of them would equal a unit; thirds are equal parts of such kind that three of them would equal a unit; &c.

(*f.*) Let the pupil now answer all the preceding questions in this article according to the following model.

What are sevenths?

Answer. — Sevenths of any thing or number are equal parts of such kind that it will take seven of them to equal that thing or number.

NOTE. — The explanations of (*d.*) and (*f.*) are alike necessary to a thorough understanding of fractions, and the student should not rest satisfied till he has become so familiar with both, as to be able to use either.

121. *Fractional Parts.*

(*a.*) Such parts as the above are called FRACTIONAL PARTS, and the arithmetical expressions for them are called FRACTIONS; hence, —

1. *Fractional parts of any thing, quantity, or number are such parts as are obtained by dividing it into equal parts.*

2. *Fractional parts of any thing, quantity, or number are equal parts of such kind that a given number of them will equal a unit.*

NOTE. — That from which the fractional parts are obtained is always, when considering the parts, regarded as a unit, and is called the *unit of the fraction.* It may be, —

1. A single object, as *an apple, an orange.*
2. A unit of measure, as *a foot, a yard, a bushel.*
3. The abstract unit, *one.*
4. A number of single objects or units considered as a collection or whole, as 6 *apples*, 18 *feet*, 24 *bushels.*
5. An abstract number, as 5, 8, 12.
6. A fraction, as $\frac{5}{7}$, $\frac{2}{3}$.

When no particular unit is expressed, the abstract unit is the one referred to.

(*b.*) It is obvious that the value of any fractional part depends both on the nature of the unit and the number of such parts which it takes to equal it.

Illustrations. — 1. If a large apple and a small one be each divided into 2, 3, 4, or any other number of equal parts, the parts of the first will be larger than the corresponding parts of the second.

2. If several apples of the same size are divided, one into two equal parts, another into three, another into four, another into five, &c., the parts of the first apple will be larger than those of any other, the parts of the second will be smaller than those of the first, but larger than those of any other, &c.

(*c.*) If two equal units are divided, one into *two* equal parts, and the other into *four*, *each part of the first* will be equal to *two parts of the second;* if one be divided into *two* equal parts, and the other into *three times as many*, *each part of the first* will be equal to *three parts of the second;* and, generally, *each part* obtained by dividing a unit into any num

ber of equal parts, will be *twice as large as each would be if the unit had been divided into twice as many equal parts; three times as large as if divided into three times as many; four times as large as if divided into four times as many; &c.*

(*d.*) In other words, while the unit of the fraction remains the same, each half will be twice as large as each fourth, 3 times as large as each sixth, 4 times as large as each eighth, &c.; each third will be twice as large as each sixth, 3 times as large as each ninth, 4 times as large as each twelfth, &c.; each fourth will be twice as large as each eighth, 3 times as large as each twelfth, &c.

(*e.*) From these considerations, it follows that in order that fractional parts may be of the same denomination, it is necessary, —

1. That they should be parts of the same unit, or of equal units;
2. That they should be obtained by dividing each unit into the same number of equal parts.

122. *Definitions. — Method of writing Fractions, &c.*

(*a.*) *A* FRACTION *expresses the value of such parts as are obtained by dividing a unit into equal parts;*

Or, by definition second, —

A FRACTION *expresses the value of such equal parts, that a certain number of them will equal a unit.*

(*b.*) In writing fractions by figures, two numbers are necessary — one to indicate the number of parts into which the unit is divided, or (which is the same thing) the number of such parts which it will take to equal the unit; the other to indicate how many of the parts are considered.

(*c.*) The first of these is called the DENOMINATOR, because it indicates the denomination of the parts. The second is called the NUMERATOR, because it numbers the parts.

(*d.*) The numerator is usually written above the denominator, and separated from it by a line.

Illustrations. — Five sixths is written $\frac{5}{6}$, 5 being the numerator, and 6 the denominator.

Seventh eighths is written $\frac{7}{8}$, 7 being the numerator, and 8 the denominator.

34 *twenty-firsts* is written $\frac{34}{21}$, 34 being the numerator, and 21 the denominator.

(*e.*) Write the following fractions by figures, and tell the numerator and denominator of each.

1. 3 fourths.	6. 1 third.
2. 9 twenty-seconds.	7. 18 thirteenths.
3. 7 halves.	8. 5 elevenths.
4. 9 seventeenths.	9. 15 thirty-firsts.
5. 22 ninths.	

(*f.*) In DECIMAL FRACTIONS the numerator only is written, the denominator being determined by the position of the decimal point. (See **22**, *a.*)

123. *Exercises in explaining Fractions.*

1. Explain the fraction $\frac{5}{9}$.

Answer. — Five ninths, or five ninths of one, expresses the value of five such parts as would be obtained by dividing a unit into nine equal parts.

Another Form of Answer. — Five ninths, or five ninths of one, expresses the value of five equal parts, such that nine of them would equal a unit.

2. Explain the fraction .07.

First Form of Answer. — Seven hundredths, or seven hundredths of one, expresses the value of seven such parts as would be obtained by dividing a unit into 100 equal parts.

Second Form of Answer. — Seven hundredths, or seven hundredths of one, expresses the value of seven equal parts of such kind that 100 of them would equal a unit.

Explain the following fractions according to the first form of answer, and afterwards according to the second.

3. $\frac{3}{7}$.	6. .09.	9. $\frac{5}{3}$.
4. $\frac{25}{41}$.*	7. .6.	10. .008.
5. $\frac{23}{17}$.	8. .27.	11. $\frac{5}{72}$.

* 25 *forty-firsts*, not 25 forty-oneths, nor 25 forty-ones.

12. $\frac{13}{22}$.*	16. .427.	20. $\frac{17}{21}$.
13. $\frac{11}{5}$.	17. .0628.	21. $\frac{25}{18}$.
14. $\frac{13}{13}$.	18. .000276.	22. .0107.
15. $\frac{263}{179}$.	19. $\frac{44}{55}$.	23. .002006.

124. *Various Kinds of Fractions.*

(*a.*) A simple fraction is one which has but one numerator and one denominator, each of which is a whole number, as $\frac{5}{6}$, $\frac{7}{7}$, $\frac{4}{3}$.

(*b.*) Simple fractions may be proper or improper.

(*c.*) A proper fraction is a fraction whose numerator is less than its denominator, as $\frac{8}{11}$, $\frac{1}{2}$, $\frac{13}{17}$.

(*d.*) An improper fraction is one whose numerator is equal to, or greater than, its denominator, as $\frac{8}{8}$, $\frac{15}{7}$, $\frac{123}{4}$.

(*e.*) A mixed number is a whole number and a fraction, as $2\frac{1}{5}$, which is read *two and one fifth;* $5\frac{3}{7}$, which is read *five and three sevenths.*

1. Which of the fractions in **123** are proper?

2. Which are improper?

(*f.*) A proper fraction is less than 1, because it expresses less parts than it takes to equal a unit.

(*g.*) An improper fraction is equal to, or greater than, 1, because it expresses as many or more parts than it takes to equal a unit.

(*h.*) An improper fraction is so called because it expresses a value, a part or all of which may be expressed in whole numbers.

125. *Illustrations of Operations on Fractions.*

Fractions may be added, subtracted, multiplied, and divided, as whole numbers are.

Thus, $\frac{5}{7} + \frac{4}{7} = \frac{9}{7}$, just as 5 days + 4 days = 9 days.
$\frac{7}{8} + \frac{3}{8} = \frac{10}{8}$, just as 7 qts. + 3 qts. = 10 qts.

* 13 *twenty-seconds*, not 13 twenty-twos.

$\frac{23}{25} + \frac{21}{25} = \frac{44}{25}$, just as 23 lbs. + 21 lbs. = 44 lbs.

Again. $\frac{3}{4} + \frac{1}{8}$ no more equals $\frac{4}{4}$, or $\frac{4}{8}$, than 3 qts. + 1 pt. = 4 qts., or 4 pt.

$\frac{11}{12} + \frac{5}{96}$ no more equals $\frac{16}{12}$, or $\frac{16}{96}$, than 11 ℥ + 5 ʒ = 16 ℥, or 16 ʒ.

Again. $\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$, just as \$8 — \$3 = \$5, or 8 lbs. — 3 lbs. = 5 lbs.

$\frac{15}{17} - \frac{13}{17} = \frac{2}{17}$, just as 15 rods — 13 rods = 2 rods.

$\frac{358}{427} - \frac{183}{427} = \frac{175}{427}$, just as 358 apples — 183 apples = 175 apples.

Again. $\frac{3}{4} - \frac{1}{8}$ no more equals $\frac{2}{4}$, or $\frac{2}{8}$, than 3 pints — 1 gill = 2 pints, or 2 gills.

$\frac{5}{7} - \frac{4}{9}$ no more equals $\frac{1}{7}$, or $\frac{1}{9}$, than 5 apples — 4 pears = 1 apple, or 1 pear.

Again. 5 times $\frac{3}{4} = \frac{15}{4}$, just as 5 times 3 apples = 15 apples.

8 times $\frac{11}{16} = \frac{88}{16}$, just as 8 times 11 oz. = 88 oz.

9 times $\frac{1387}{1728} = \frac{12483}{1728}$, just as 9 times 1387 cubic inches = 12483 cubic inches.

Again. $\frac{2}{7}$ are contained 3 times in $\frac{6}{7}$, just as 2 days are contained 3 times in 6 days.

$\frac{3}{24}$ are contained 7 times in $\frac{21}{24}$, just as 3 grains are contained 7 times in 21 grains.

$\frac{7}{144}$ are contained $18\frac{3}{7}$ times in $\frac{129}{144}$, just as 7 apples are contained $18\frac{3}{7}$ times in 129 apples.

Again. $\frac{1}{4}$ of $\frac{8}{9} = \frac{2}{9}$, just as $\frac{1}{4}$ of 8 apples = 2 apples.

$\frac{1}{5}$ of $\frac{24}{31} = \frac{4\frac{4}{5}}{31}$, just as $\frac{1}{5}$ of 24 bushels = $4\frac{4}{5}$ bushels.

$\frac{1}{9}$ of $\frac{1484}{2147} = \frac{164\frac{8}{9}}{2147}$, just as $\frac{1}{9}$ of 1484 miles = $164\frac{8}{9}$ miles.

126. *Reduction of Whole or Mixed Numbers to Improper Fractions.*

The value of any whole or mixed number may be expressed by an improper fraction.

1. Reduce 8 to fifths.

Solution. — Since $1 = \frac{5}{5}$, 8 must equal 8 times $\frac{5}{5}$, or $\frac{40}{5}$. Therefore, $8 = \frac{40}{5}$.

2. Reduce $9\frac{2}{7}$ to sevenths.

Solution. — Since $1 = \frac{7}{7}$, 9 must equal 9 times $\frac{7}{7}$, or $\frac{63}{7}$, and $\frac{2}{7}$ added are $\frac{65}{7}$. Therefore, $9\frac{2}{7} = \frac{65}{7}$.

3. Reduce 4 to thirds.

4. Reduce $5\frac{2}{3}$ to thirds.

5. Reduce $6\frac{5}{8}$ to eighths.
6. Reduce 8 to eighths.
7. Reduce $9\frac{3}{5}$ to fifths.
8. Reduce $9\frac{3}{4}$ to fourths.
9. Reduce $439\frac{3}{4}$ to fourths.

First Solution. — Since $1 = 4$ fourths, 439 must equal 439 times 4 fourths, which is equivalent to 4 times 439 fourths, or 1756 fourths, and 3 fourths added are 1759 fourths. Therefore, $439\frac{3}{4} = \frac{1759}{4}$.

Second Solution. — Since there are 4 fourths for each unit, there must be 4 times as many fourths in any number as there are units. Hence, $439 = 4$ times 439 fourths, or 1756 fourths, and 3 fourths added are 1759 fourths. Hence, $439\frac{3}{4} = \frac{1759}{4}$.

NOTE. — Compare these reasoning processes with those given on the 85th page.

10. Reduce $578\frac{5}{9}$ to ninths.
11. Reduce $14762\frac{4}{7}$ to sevenths.
12. Reduce $74968\frac{3}{17}$ to seventeenths.
13. Reduce $6483\frac{22}{47}$ to forty-sevenths.
14. Reduce $4386\frac{13}{28}$ to twenty-eighths.
15. Reduce $427\frac{128}{273}$ to two hundred and seventy-thirds.
16. $9 =$ how many times $\frac{1}{4}$? *

First Solution. — Since $1 = \frac{4}{4}$, 9 must equal 9 times $\frac{4}{4}$, or $\frac{36}{4}$, and 1 fourth is contained in 36 fourths 36 times. Hence, $9 = 36$ times $\frac{1}{4}$.

Second Solution. — Since $1 = 4$ times $\frac{1}{4}$, or $\frac{4}{4}$, 9 must equal 9 times 4 times $\frac{1}{4}$, or 36 times $\frac{1}{4}$. Hence, $9 = 36$ times $\frac{1}{4}$.

17. $7 =$ how many times $\frac{1}{8}$?
18. $11 =$ how many times $\frac{1}{5}$?
19. $49 =$ how many times $\frac{1}{6}$?
20. $487 =$ how many times $\frac{1}{25}$?
21. $9 =$ how many times $\frac{1}{10}$?
22. $7 =$ how many times .1?
23. $13 =$ how many times .01?
24. $648 =$ how many times .0001?

127. *Reduction of Improper Fractions to Whole or Mixed Numbers.*

The value of any improper fraction may be expressed by a whole or mixed number.

* It will be seen by the solutions that this question is just equivalent to "$9 =$ how many fourths?"

1. Reduce $\frac{43}{8}$ to ones.

Since $\frac{8}{8} = 1$, $\frac{43}{8}$ must equal as many ones as there are times 8 in 43, which are $5\frac{3}{8}$ times. Hence, $\frac{43}{8} = 5\frac{3}{8}$.

NOTE. — The preceding question is precisely like this: "43 qts. = how many pecks?" as the following solution will show. "Since 8 qts. = 1 pk., 43 qts. will equal as many pecks as there are times 8 in 43, which are $5\frac{3}{8}$ times. Hence, 43 qts. = $5\frac{3}{8}$ pks."

2. Reduce $\frac{83}{9}$ to ones.
3. Reduce $\frac{48}{7}$ to ones.
4. Reduce $\frac{169}{8}$ to ones.
5. Reduce $\frac{427}{5}$ to ones.
6. Reduce $\frac{7693}{23}$ to ones.
7. Reduce $\frac{6785}{1728}$ to ones.
8. Reduce $\frac{76}{11}$ to ones.
9. Reduce $\frac{95}{12}$ to ones.
10. Reduce $\frac{139}{4}$ to ones.
11. Reduce $\frac{896}{12}$ to ones.
12. Reduce $\frac{27946}{13}$ to ones.
13. Reduce $\frac{49864}{144}$ to ones.

128. *Miscellaneous Questions involving Fractions.*

1. How much will 227 yards of cloth cost at $\frac{3}{4}$ of a dollar per yard?

Solution. — Since 1 yard costs $\frac{3}{4}$ of a dollar, 227 yards will cost 227 times $\frac{3}{4}$ of a dollar, equivalent to 3 times $\frac{227}{4}$ of a dollar, which, found by multiplying 227 by 3, is $\frac{681}{4}$ of a dollar, equal (by **127**) to $170\frac{1}{4}$ dollars.

NOTE. — Compare the preceding example and solution with the following: —

How many bushels of grain in 227 bags, each holding 3 pecks?

Solution. — Since 1 bag holds 3 pecks, 227 bags will hold 227 times 3 pecks, equivalent to 3 times 227 pecks, which, found by multiplying 227 by 3, is 681 pecks, equal to 170 bushels, 1 peck.

2. What will 493 lbs. of tea cost at $\frac{5}{8}$ of a dollar per lb.?

3. What will 1286 bu. of apples cost at $\frac{21}{25}$ of a dollar per bu.?

4. What will 347 gal. of burning fluid cost at $\frac{43}{50}$ of a dollar per gal.?

5. How many acres in 169 house lots, each containing $\frac{37}{96}$ of an acre?

6. How many acres in 3 lots, each containing $\frac{42637}{43560}$ acres?

7. How many quarts in 9 boxes, each holding $\frac{48}{49}$ of a quart?

8. How many pounds of silver in 28 bars, each weighing $\frac{3278}{5760}$ of a pound?

9. How many tons of hay in 37 loads, each containing $\frac{1878}{2240}$ of a ton?

10. How much will 7 acres of land cost at \$$275\frac{3}{4}$ per acre?

NOTE. — Compare the last example with this: "How much corn in 7 bins, each holding 275 bushels, 3 pecks?"

11. How much will 6 acres of land cost at \$$493\frac{7}{8}$ per acre?

12. If a ship sails uniformly at the rate of $179\frac{11}{12}$ miles per day, how far will she sail in 5 days?

13. If a steamship sails uniformly at the rate of $4179\frac{347}{396}$ rods per hour, how many rods will she sail in 85 hours?

14. How many baskets, each holding $\frac{7}{8}$ of a peck, can be filled from $82\frac{3}{8}$ pecks of corn?

Solution. — Since $\frac{7}{8}$ of a peck will fill one basket, $82\frac{3}{8}$ pecks will fill as many baskets as there are times $\frac{7}{8}$ in $82\frac{3}{8}$. But $82\frac{3}{8} = \frac{659}{8}$, which contains $\frac{7}{8}$ as many times as 659 contains 7, which is $94\frac{1}{7}$ times. Hence, $94\frac{1}{7}$ baskets can be filled.

NOTE. — Compare the above question and solution with the following: "How many baskets, each holding 7 quarts, can be filled from 82 pk. 3 qt. of corn?"

Solution. — Since 7 quarts will fill one basket, 82 pk. 3 qt. will fill as many baskets as there are times 7 qt. in 82 pk. 3 qt. 82 pk. 3 qt. = 659 quarts, which contains 7 quarts as many times as 659 contains 7, which is $94\frac{1}{7}$ times. Hence, $94\frac{1}{7}$ baskets can be filled.

15. How many vest patterns, each containing $\frac{3}{4}$ of a yard, can be cut from a piece of vesting containing $25\frac{1}{4}$ yards?

16. How many books, at $\frac{5}{8}$ of a dollar a piece, can be bought for $232\frac{1}{8}$ dollars?

17. How many baskets, each holding $\frac{15}{32}$ of a bushel, can be filled from $453\frac{7}{32}$ bushels of apples?

18. How many barrels, holding $2\frac{1}{4}$ bushels each, can be filled from $59\frac{3}{4}$ bushels of apples?

NOTE. — Compare the last question with, — "How many jugs, holding 2 gal. 1 qt. each, can be filled from 59 gal. 3 qt. of fluid?"

19. How many spoons, each weighing $7\frac{5}{8}$ dwt., can be made from $328\frac{7}{8}$ dwt. of silver?

20. How many yards of cloth, at $2\frac{19}{23}$ dollars per yard, can be bought for $89\frac{20}{23}$ dollars?

21. How many days, at $\$2\frac{11}{25}$ per day, must a man labor to earn $\$237\frac{13}{25}$?

22. A man, who owned $247\frac{5}{12}$ dwt. of silver, had it made into spoons, each weighing $4\frac{11}{12}$ dwt. How many spoons did it make?

23. What is the sum of $\frac{11}{12} + \frac{7}{12} + \frac{9}{12} + \frac{4}{12} + \frac{7}{12} + \frac{5}{12}$? * *Ans.* $\frac{43}{12} = 3\frac{7}{12}$.

NOTE. — Compare the above example with 11 in. + 7 in. + 9 in. + 4 in. + 7 in. + 5 in. = 43 in. = 3 ft. 7 in.

What is the sum —

24. Of $\frac{5}{6} + \frac{4}{6} + \frac{1}{6} + \frac{3}{6} + \frac{5}{6}$?

25. Of $\frac{4}{13} + \frac{11}{13} + \frac{7}{13} + \frac{9}{13} + \frac{8}{13} + \frac{10}{13} + \frac{12}{13}$?

26. Of $\frac{22}{23} + \frac{19}{23} + \frac{20}{23} + \frac{17}{23} + \frac{21}{23} + \frac{7}{23} + \frac{18}{23}$?

27. Of $13\frac{3}{7} + 13\frac{5}{7} + 2\frac{4}{7} + 22\frac{6}{7} + 43\frac{2}{7}$?

NOTE. — Compare the 27th example with the following: — 13 w 3 da. + 13 w. 5 da. + 2 w. 4 da. + 22 w. 6 da. + 43 w. 2 da. = 95 w. 6 da. To make the resemblance to compound addition still more apparent, we give two forms of writing the work of the 27th example, and place opposite them the work of the example in this note.

	Ones.	Sevenths.	w.	da.
$13\frac{3}{7} =$	13	3	13	3
$13\frac{5}{7} =$	13	5	13	5
$2\frac{4}{7} =$	2	4	2	4
$22\frac{6}{7} =$	22	6	22	6
$43\frac{2}{7} =$	43	2	43	2
$95\frac{6}{7} =$	95 .	6 = *Ans.*	95	6 = *Ans.*

What is the value —

28. Of $17\frac{5}{11} + 18\frac{6}{11} + 23\frac{9}{11} + 42\frac{7}{11} + 27\frac{10}{11}$?

29. Of $24\frac{7}{8} + 96\frac{3}{8} + 14\frac{5}{8} + 26\frac{1}{8} + 55\frac{7}{8}$?

---

* These, like the compound numbers of **56,** can best be reduced as they are added; thus, $\frac{11}{12} + \frac{7}{12}$ (by adding one of the $\frac{7}{12}$ to $\frac{11}{12}$) $= 1\frac{6}{12}$; and $1\frac{6}{12} + \frac{9}{12}$ (by adding 3 of the $\frac{6}{12}$ to the $\frac{9}{12}$) $= 2\frac{3}{12}$; &c.

30. Of $237\frac{13}{20} + 496\frac{17}{20} + 849\frac{3}{20} + 591\frac{18}{20} + 438\frac{16}{20}$?
31. Of $\frac{8}{9} - \frac{3}{9}$?
32. Of $\frac{128}{197} - \frac{49}{197}$?
33. Of $\frac{83257}{268943} - \frac{49468}{268943}$?
34. Of $24\frac{7}{25} - 19\frac{21}{25}$?

NOTE. — Compare the 34th with the following: — What is the value of 24 qr. 7 lb. — 19 qr. 21 lb.

By writing the work of the 34th example in the following form, and placing opposite it the work of the example in this note, the resemblance of the two is very clearly exhibited.

	Ones.	Twenty-fifths.	Qr.	lb.
$24\frac{7}{25} =$	24	7	24	7
$19\frac{21}{25} =$	19	21	19	21
$4\frac{11}{25} =$	4	11 = *Ans.*	4	11 = *Ans.*

What is the value —

35. Of $247\frac{13}{16} - 159\frac{15}{16}$?
36. Of $4327\frac{28}{45} - 2589\frac{41}{45}$?
37. Of $6732\frac{5276}{9357} - 4694\frac{2159}{9357}$?
38. Of $479\frac{23}{1947} - 214\frac{1573}{1947}$?
39. Of $284\frac{15748}{49876} - 283\frac{15749}{49876}$?
40. Of $9243\frac{24758}{98359} - 3842\frac{98358}{98359}$?

129. *One Number a Fractional Part of another.*

What part of 4 is 1?

Answer. 1 is one fourth of 4, because 4 times 1 are 4.*

What part —

1. Of 6 is 1?
2. Of 2 is 1?
3. Of 9 is 1?
4. Of 12 is 1?
5. Of 16 is 1?
6. Of 243 is 1?

7. What part of 8 is 3?

Solution. — Since $1 = \frac{1}{8}$ of 8, 3 must equal $\frac{3}{8}$ of 8.

What part —

8. Of 9 is 4?
9. Of 11 is 7?
10. Of 3 is 2?
11. Of 8 is 5?
12. Of 6 is 5?
13. Of 13 is 4?

* i. e., Because 1 taken 4 times will equal 4

14. What part of 7 is 9?

Solution. — Since 1 = $\frac{1}{7}$ of 7, 9 must equal $\frac{9}{7}$ of 7, or $1\frac{2}{7}$ times 7.

What part —

15. Of 9 is 13?
16. Of 10 is 87?
17. Of 19 is 39?
18. Of 47 is 63?
19. Of 39 is 19?
20. Of 413 is 527?

21. What part of 8 yards is 1 yard?

First Form of Answer. 1 yard = $\frac{1}{8}$ of 8 yards, because 8 times 1 yard = 8 yards.

Second Form of Answer. 1 yard is the same part of 8 yards that 1 is of 8, which is $\frac{1}{8}$. Hence, 1 yard = $\frac{1}{8}$ of 8 yards.

What part —

22. Of 9 ft. is 1 ft.?
23. Of 4 m. is 3 m.?
24. Of 4 lb. is 3 lb.?
25. Of 4 cwt. is 3 cwt.?
26. Of 10 da. is 7 da.?
27. Of 8 yr. is 5 yr.?

28. What part of $\frac{4}{7}$ is $\frac{3}{7}$?

Answer. $\frac{3}{7}$ is the same part of $\frac{4}{7}$ that 3 is of 4, which is $\frac{3}{4}$. Hence, $\frac{3}{7}$ is $\frac{3}{4}$ of $\frac{4}{7}$.

What part —

29. Of $\frac{4}{9}$ is $\frac{3}{9}$?
30. Of $\frac{4}{177}$ is $\frac{3}{177}$?
31. Of $\frac{14}{19}$ is $\frac{13}{19}$?
32. Of $\frac{14}{163}$ is $\frac{13}{163}$?
33. Of $\frac{7}{13}$ is $\frac{4}{13}$?
34. Of $\frac{5}{11}$ is $\frac{8}{11}$?
35. Of $\frac{2}{5}$ is $\frac{4}{5}$?
36. Of $\frac{23}{58}$ is $\frac{13}{58}$?

37. What part of 1 bushel is 3 pecks?

Solution. 1 bu. = 4 pk., and 3 pk. = $\frac{3}{4}$ of 4 pk. Hence, 3 pk. = $\frac{3}{4}$ of 1 bu.

What part —

38. Of 1 m. is 5 fur.?
39. Of 1 lb. is 7 ℥?
40. Of 1 A. is 3 R.?
41. Of 1 yd. is 2 ft.?
42. Of 1 ft. is 1 in.?
43. Of 1 T. is 13 cwt.?

44. What part of 3 da. is 1 w.?

Solution. 1 w. = 7 da., and 7 da. = $\frac{7}{3}$ of 3 da., or $2\frac{1}{3}$ times 3 da.

What part —

45. Of 5 d. is 1 s.?
46. Of 7 s. is £1?
47. Of 13 dr. is 1 oz.?
48. Of 9 dwt. is 1 oz.?

49. Of 11 ℥ is 1 lb.?

50. Of 8 da. is 1 w.?

51. What part of $\frac{5}{6}$ is 1?

Solution. $1 = \frac{6}{6}$, and $\frac{6}{6}$ is the same part of $\frac{5}{6}$ that 6 is of 5, which is $\frac{6}{5}$. Hence, $1 = \frac{6}{5}$ of $\frac{5}{5}$, or $1\frac{1}{5}$ times $\frac{5}{6}$.

What part —

52. Of $\frac{8}{11}$ is 1?
53. Of $\frac{3}{8}$ is 1?
54. Of $\frac{6}{7}$ is 1?
55. Of $\frac{5}{13}$ is 1?
56. Of $\frac{9}{32}$ is 1?
57. Of $\frac{53}{9}$ is 1?
58. Of $\frac{28}{5}$ is 1?
59. Of $\frac{14}{97}$ is 1?
60. Of $\frac{13}{144}$ is 1?
61. Of $\frac{63}{1829}$ is 1?
62. What part of $1\frac{3}{8}$ is $2\frac{5}{8}$?

Solution. $1\frac{3}{8} = \frac{11}{8}$; $2\frac{5}{8} = \frac{21}{8}$; and $\frac{21}{8} = \frac{21}{11}$ of $\frac{11}{8} = 1\frac{10}{11}$ times $\frac{11}{8}$.

63. What part of 1 m. 3 fur. is 2 m. 5 fur.?

Solution. 1 m. 3 fur. = 11 fur.; and 2 m. 5 fur. = 21 fur.; 21 fur. = $\frac{21}{11}$ of 11 fur., or $1\frac{10}{11}$ times 11 fur.

What part —

64. Of 1 d. 3 qr. is 11 d. 1 qr.?
65. Of $1\frac{3}{4}$ is $11\frac{1}{4}$?
66. Of 8 w. 3 da. is 2 w. 4 da.?
67. Of $8\frac{3}{7}$ is $2\frac{4}{7}$?
68. Of 5 da. 7 h. is 3 da. 11 h.?
69. Of $5\frac{7}{24}$ is $3\frac{11}{24}$?
70. Of 11 h. 27 m. is 21 h. 38 m.?
71. Of 15 pk. is 3 pk. 7 qt.?
72. What part of 1 lb. is 3 oz. 5 dwt. $17\frac{2}{5}$ gr.?

Suggestion. — Reduce 1 lb. and also 3 oz. 5 dwt. $17\frac{2}{5}$ gr. to the lowest denomination mentioned, i. e., to fifths of a grain.

What part —

73. Of 1 gal. is 3 qt. 1 pt. 3 gi.?
74. Of £1 is 17 s. 11 d. 2 qr.?
75. Of 1 w. is 5 da. 11 h. 35 m. $29\frac{2}{3}$ sec.?
76. Of 1 T. is 13 cwt. 2 qr. 19 lb. 13 oz. $11\frac{1}{2}$ dr.?
77. Of 4 yd. 2 ft. 7 in. is 1 yd. 1 ft. 11 in.?
78. Of 17 A. 3 R. 16 sq. rd. is 2 A. 1 R. 28 sq. rd.?

130. *Other Methods of expressing the Value of a Fraction.*

(*a.*) 1. Explain the fraction $\frac{3}{4}$.

Answer. $\frac{3}{4} = 3$ times $\frac{1}{4}$, as $\frac{3}{4}$ of a number $= 3$ times $\frac{1}{4}$ of the number.

In the same way explain the meaning —

2. Of $\frac{2}{3}$.	5. Of $\frac{17}{30}$.	8. Of .4
3. Of $\frac{4}{5}$.	6. Of $\frac{15}{100}$.	9. Of .007
4. Of $\frac{3}{8}$.	7. Of .15	10. Of .0346

(*b.*) Explain each of the above by the following method.

$\frac{3}{4}$, or $\frac{3}{4}$ of 1, $= \frac{1}{4}$ of 3, as $\frac{3}{4}$ of a number $= \frac{1}{4}$ of 3 times the number. For if each of 3 equal things should be divided into 4 equal parts, and 1 of the parts of each taken, the result would equal $\frac{3}{4}$ of 1.

NOTE. — This may be illustrated to the eye, by taking 3 equal lines, and dividing them into 4 equal parts, arranged thus one of the parts will then contain $\frac{1}{4}$ of 3 lines, which, as will be seen, is equal to $\frac{3}{4}$ of a line.

11. To what simple fraction is $\frac{1}{4}$ of 3 equal?

Answer. $\frac{1}{4}$ of $3 = \frac{3}{4}$ of $1 = \frac{3}{4}$.

(*c.*) To what simple fraction is each of the following equal?

12. $\frac{1}{3}$ of 2.	16. $\frac{1}{8}$ of 11.	20. $\frac{1}{17}$ of 832.
13. $\frac{1}{7}$ of 4.	17. $\frac{1}{13}$ of 4.	21. $\frac{1}{7}$ of 98.
14. $\frac{1}{11}$ of 6.	18. $\frac{1}{12}$ of 15.	22. $\frac{1}{29}$ of 493.
15. $\frac{1}{5}$ of 7.	19. $\frac{1}{8}$ of 49.	23. $\frac{1}{47}$ of 861.

131. *To find a Fractional Part of a Number.*

1. What is $\frac{7}{8}$ of 5459?

First Solution. $\frac{7}{8}$ of $5459 = 7$ times $\frac{1}{8}$ of 5459; $\frac{1}{8}$ of 5459, found by dividing by 8, is $682\frac{3}{8}$, and 7 times $682\frac{3}{8} = 4776\frac{5}{8}$.

Second Solution. $\frac{7}{8}$ of $5459 = \frac{1}{8}$ of 7 times 5459; 7 times $5459 = 38213$, and $\frac{1}{8}$ of $38213 = 4776\frac{5}{8}$.

WRITTEN WORK.

By First Solution.

$$\begin{array}{r} 8\ )\ 5459 \\ \hline 682\frac{3}{8} \\ 7 \\ \hline 4776\frac{5}{8} = \textit{Ans.} \end{array}$$

By Second Solution.

$$\begin{array}{r} 5459 \\ 7 \\ \hline 8\ )\ 38213 \\ \hline 4776\frac{5}{8} = \textit{Ans.} \end{array}$$

The following forms of writing the work exhibit very clearly the reason for every step which is taken. The letters a, b, &c., are used as explained in the note on the 88th page.

First Form.	*Second Form.*
$a = 5459$	$a = 5459$
$\frac{1}{8}$ of $a = b = 682\frac{3}{8}$	$7 \times a = b = 38213$
$7 \times b = \frac{7}{8}$ of $a = 4776\frac{5}{8}$	$\frac{1}{8}$ of $b = \frac{7}{8}$ of $a = 4776\frac{5}{8}$

NOTE. — It will be seen that by both methods of solution, we multiply by the numerator, 7, and divide by the denominator, 8. The only difference between them is, that *by the first we divide before we multiply*, while *by the second we multiply before we divide.* The first method involves somewhat smaller numbers than the second.

What is the value of each of the following?

2. $\frac{6}{7}$ of 3843.
3. $\frac{5}{8}$ of 7987.
4. $\frac{3}{5}$ of 7148.
5. $\frac{11}{12}$ of 58437.
6. $\frac{67}{75}$ of 46935.
7. $\frac{137}{144}$ of 87516.
8. What is the value of .0014 of 16.427?

First Solution. .0014 of 16.427 = 14 times .0001 of 16.427; .0001 of 16.427, found by removing the point four places towards the left, is 0016427, and 14 times this is .0229978 = *Ans.*

The work may be written in full after either of the following models

First Model.

$$\begin{aligned} a &= 16.427 \\ .0001 \text{ of } a = b &= .0016427 \\ 14 \text{ times } b = .0014 \text{ of } a &= .0229978 = \textit{Ans.} \end{aligned}$$

Second Model.

$$\begin{array}{r|l} 10000 & 16.427 \\ \hline & .0016427 \\ & \underline{14} \\ & .0229978 = \textit{Ans.} \end{array}$$

Second Solution. .0014 of 16.427 = .0001 of 14 times 16.427; 14 times 16.427 = 229.978, and .0001 of 229.978, found by removing the point four places towards the left, is .0229978 = *Ans.*

The work may be written in full after either of the following models

First Model.

$$\begin{aligned} a &= 16.427 \\ 14 \text{ times } a = b &= 229.978 \\ .0001 \text{ of } b = .0014 \text{ of } a &= .0229978 \end{aligned}$$

Second Model.

$$\begin{array}{r} 16.427 \\ 14 \\ \hline 10000\ )\ 229.978 \\ \hline .0229978 \end{array}$$

NOTE. — It will be seen that by *both* solutions, we *multiply by the numerator*, 14, *and divide by the denominator*, 10000; i. e., we multiply by 14, and remove the point four places towards the left. The only difference between them is, that *in the first we remove the point before we multiply, while in the second we remove it afterwards.* Some of the above written work might have been avoided by multiplying by 14, and changing the position of the point at the time of writing the product, thus: —

$$\begin{array}{r} 16.427 \\ .0014 \\ \hline .0229978 \end{array}$$

In like manner we may get .07 of a number by multiplying by 7, and removing the point two places towards the left; we may get 5.2 * of a number by multiplying by 52, and removing the point one place towards the left; &c.

What is the value of each of the following?

9. .03 of 2589	14. .006 of .006
10. .005 of 3.7479	15. 2.5 of 2.5
11. .28 of 437.96	16. .372 of 5.46
12. 4.2 of 67.93 †	17. .0004 of 200
13. .25 of .25	18. .006 of .006

132. *To multiply by a Fraction.*

1. What is the product of 865 multiplied by $\frac{3}{7}$?

Suggestion. — The product of 865 multiplied by $\frac{3}{7}$ equals $\frac{3}{7}$ times 865, which is the same as $\frac{3}{7}$ of 865, and may be found by methods before explained.

NOTE. — This follows from the definition of multiplication — *A process by which we ascertain how much any given number will amount to, if taken as many times as there are units in some other given number.* In multiplying by more than a unit, then, more than once the multiplicand is taken, while in multiplying by less than a unit, less than once the mul-

* It will be more convenient, for purposes of explanation, to regard this as an improper fraction than as a mixed number.

† The number of which a fractional part is taken may be regarded either *as a mixed* number or an improper fraction.

tiplicand (i. e., some fractional part of it) is taken; the product being always the same part of the multiplicand that the multiplier is of unity.

When, therefore, the multiplier is greater than unity, the product will be greater than the multiplicand; and when the multiplier is less than unity, the product will be less than the multiplicand. In multiplying by $\frac{1}{2}$, $\frac{1}{2}$ of once the multiplicand is taken; in multiplying by $\frac{3}{5}$, $\frac{3}{5}$ of once the multiplicand is taken; &c.

This conclusion is in exact accordance with the principle, that the larger the multiplier the larger the product, and the smaller the multiplier the smaller the product, so long as the multiplicand remains the same; or, to speak more definitely, that the product of a number multiplied by $\frac{1}{2}$ of another, is equal to $\frac{1}{2}$ of its product multiplied by the whole of the other; that the product of a number multiplied by $\frac{3}{4}$ of another, is equal to $\frac{3}{4}$ of its product multiplied by the whole of the other; &c.

Thus, 12 multiplied by 6 = 72.

12 multiplied by $\frac{1}{2}$ of 6, or 3, = 36, which is $\frac{1}{2}$ of 72.
12 multiplied by $\frac{1}{3}$ of 6, or 2, = 24, which is $\frac{1}{3}$ of 72.
12 multiplied by $\frac{2}{3}$ of 6, or 4, = 48, which is $\frac{2}{3}$ of 72; &c.

So, since 12 multiplied by 1 = 12,

12 multiplied by $\frac{1}{2}$ of 1, or $\frac{1}{2}$, must equal $\frac{1}{2}$ of 12, or 6.
12 multiplied by $\frac{1}{3}$ of 1, or $\frac{1}{3}$, must equal $\frac{1}{3}$ of 12, or 4.
12 multiplied by $\frac{2}{3}$ of 1, or $\frac{2}{3}$, must equal $\frac{2}{3}$ of 12, or 8; &c.

NOTE. — By the foregoing explanations, it appears that the change of denomination mentioned in the note under **74**, (*h.*) is caused by the fact that to multiply by a fraction requires a division as well as a multiplication.

What is the product —

2. Of $\frac{4}{7}$ times 8649?
3. Of $\frac{2}{3}$ times 6743?
4. Of $\frac{5}{11}$ times 16872?
5. Of $\frac{4}{15}$ times 6964?
6. Of .7 times 6397?
7. Of .06 times 59.37?
8. What is the product of $3\frac{5}{9}$ times 2948?

Suggestion. $3\frac{5}{9}$ times 2948 = 3 times 2948 + $\frac{5}{9}$ of 2948.

WRITTEN WORK.

First Form.

$$\begin{array}{rr}
a = & 2948 \\
 & 3\frac{5}{9} \\
\hline
3 \times a = b = & 8844 \\
\frac{5}{9} \text{ of } a = c = & 1637\frac{7}{9} \\
\hline
b + c = 3\frac{5}{9} + a = & 10481\frac{7}{9} = \textit{Ans.}
\end{array}
\qquad
\begin{array}{l}
9\ )\ 2948 = a \\
\hline
327\frac{5}{9} = \frac{1}{9} \text{ of } a \\
5 \\
\hline
1637\frac{7}{9} = \frac{5}{9} \text{ of } a
\end{array}$$

Second Form.

$$\begin{array}{rr}
a = & 2948 \\
 & 3\frac{5}{9} \\
\hline
3 \times a = b = & 8844 \\
\frac{1}{9} \text{ of } a = c = & 327\frac{5}{9} \\
4 \times c = d = & 1310\frac{2}{9} = \frac{4}{9} \text{ of } a \\
\hline
{}^{*}b + c + d = 3\frac{5}{9} + a = & 10481\frac{7}{9} = \textit{Ans.}
\end{array}$$

NOTE.—The second of the above forms is preferable to the first, inasmuch as it presents to the eye, in a compact and convenient form, all the numbers which it is necessary to use, and avoids all side work. The references by means of letters may be omitted as soon as the processes and explanations are familiar.

What is the product —

9. Of $8\frac{1}{2}$ times 6794?
10. Of $26\frac{2}{3}$ times 2579?
11. Of $43\frac{7}{8}$ times 45687?
12. Of $3\frac{5}{9}$ times 85476?
13. Of $25\frac{4}{11}$ times 2764?
14. Of $698\frac{3}{4}$ times 29679?

133. *Practical Problems.*

1. How much will $\frac{7}{8}$ of an acre of land cost at $478.36 per acre?

Solution.—If 1 acre costs $478.36, $\frac{7}{8}$ of an acre will cost $\frac{7}{8}$ of $478.36, which, found in the usual way, is $\$418.56\frac{1}{2}$.

First Form.

$$\begin{array}{r}
8\ )\ \$478.36 \\
\hline
\$59.79\frac{1}{2} \\
7 \\
\hline
\$418.56\frac{1}{2} = \textit{Ans.}
\end{array}$$

Second Form.

$$\begin{array}{rll}
a = & \$478.36 & = \text{cost of 1 acre.} \\
\frac{1}{8} \text{ of } a = b = & \$\ 59.79\frac{1}{2} & = \text{cost of } \frac{1}{8} \text{ of an A.} \\
7 \times b = & \$418.56\frac{1}{2} & = \text{cost of } \frac{7}{8} \text{ of an A.}
\end{array}$$

NOTE.—The second form is better than the first, when a full statement and explanation of the successive steps are required; but in other cases the first is preferable.

2. How much will .9 of an acre of land cost at $394.26 per acre?

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* Since $\frac{5}{9} = \frac{1}{9} + \frac{4}{9}$.

Solution.—If 1 acre costs \$394.26, .9 of an acre will cost .9 of \$394.26, which, found as before explained, is \$354.834. The following exhibits the written work.

Full Form.	*Abbreviated Form.*
a = \$394.26 = cost of 1 acre.	\$394.26
.1 of a = b = \$ 39.426 = cost of .1 of an A.	.9
9 × b = \$354.834 = cost of .9 of an A.	
	\$354.834 = *Ans.*

3. How much is $\frac{5}{7}$ of a vessel worth, if the whole vessel is worth \$29951.88?

4. I bought $\frac{3}{4}$ of an acre of land at \$397.36 per acre. How much ought I to pay for it?

5. If a bushel of tomatoes weighs 48.75 lb., what will $\frac{9}{15}$ of a bushel weigh?

6. What will .4 of a ton of iron cost at \$47.93 per ton?

7. What will .8 of a bag of coffee cost at \$27.94 per bag?

8. Mr. Foster bought 4377 bushels of corn, and sold .37 of it to Mr. Gardiner, .43 of it to Mr. Calder, and the rest to Mr. Godfrey. How many bushels did he sell to each?

9. What will $9\frac{7}{8}$ acres of land cost at \$126 per acre?

10. What will $32\frac{28}{75}$ acres of land cost at \$139.50 per acre?

11. The ship Surprise is valued at \$25427. Joseph Ward owns $\frac{2}{5}$ of her, and Samuel Lowe owns the remainder. What is the value of Mr. Ward's share? Of Mr. Lowe's?

12. The brig Maria sailed from Philadelphia to Boston, with 207 T. 13 cwt. 2 qr. 19 lb. of coal, $\frac{5}{7}$ of which was landed at one wharf, and the remainder at another. What was the weight of that landed at each wharf?

13. If John walks $\frac{5}{7}$ as fast as George, how far will John walk while George is walking 97 m. 6 fur. 37 rd. 2 yd.?

14. A company of 9 California gold diggers found in 1 month 37 lb. 4 oz. 16 dwt. 17 gr. of gold, which they divided equally. What was the share of each?

15. An English ship, valued at £8743, carries a cargo which is worth $\frac{7}{8}$ as much as the ship. How many £, s., and d. is the cargo worth? In a storm the sailors were obliged

to throw $\frac{1}{4}$ of the cargo overboard. What was the value of what they threw overboard?

16. A merchant bought 17 T. 14 cwt. 2 qr. 23 lb. 10 oz. of sugar. He sold $\frac{2}{7}$ of it to one man, $\frac{3}{5}$ of the remainder to another, and what there was left to another. What was the weight of that which he sold to the first man? To the second? To the third?

Suggestion. — After selling $\frac{2}{7}$ of the sugar, he must have had $\frac{5}{7}$ of it left, and $\frac{3}{5}$ of this, or what he sold to the second man, must be $\frac{3}{5}$ of $\frac{5}{7}$, or $\frac{3}{7}$ of the original quantity; and the remaining $\frac{2}{5}$ of $\frac{5}{7}$, or $\frac{2}{7}$ of the original quantity, must have been sold to the third man.

Or, the share of the third man may be determined thus: Since the first man took $\frac{2}{7}$ of the sugar, and the second $\frac{3}{7}$, both took $\frac{5}{7}$ of it, which would leave $\frac{2}{7}$ for the third man.

17. A father, dying, left to his son an estate valued at \$87646.44. The first year after the son came into possession of the property, he spent $\frac{1}{7}$ of it in dissipation, the second year he spent $\frac{2}{3}$ of the remainder, and at the end of the third year he was penniless. How much did he spend in the first year? How much in the second year? How much in the third?

Suggestion. — Compare this example with the last.

18. If Mr. Smith uses 9.678 tons of coal per year, and Mr. Parkhurst uses .7 as much, how many tons does Mr. Parkhurst use?

NOTE. — The term *per cent* is often used in arithmetic, and in business transactions, instead of *one-hundredths*. Thus, 6 per cent has the same meaning as 6 one-hundredths. 8 per cent $= .08 = \frac{8}{100}$.

5 per cent of 340 acres = .05 of 340 acres.

6 per cent of \$86.38 = .06 of \$86.38.

Every number is 100 per cent of itself.

19. A teamster carted 7486 bushels of potatoes to a railroad depot, receiving in payment 9 per cent of them. How many bushels did he receive?

20. A man who owned 378.37 acres of land gave 8 per cent of it to his son. How many acres did he give to his son?

21. A city merchant agrees to sell for a country farmer whatever produce the latter may send to him, on condition that he shall receive for his trouble 5 per cent of the money he receives for the produce. Under this arrangement he sells produce to the value of $1768.37. What ought he to receive for his trouble?

NOTE. — A merchant who makes it his business to buy and sell goods for others is called a COMMISSION MERCHANT. The money he receives for his services is called his COMMISSION. For instance: In the last example, the merchant receives a commission of 5 per cent on the value of the produce he sells. He will deduct this commission from the amount of the sales before paying the farmer.

22. A commission merchant sold cloth for a manufacturer to the amount of $7643.79, receiving a commission of 3 per cent. What did his commission amount to? How much money ought he to pay the manufacturer?

23. A commission merchant buys goods for me to the amount of $387.46, for which he charges a commission of 2 per cent. What will his commission amount to? How much money must I send him to pay for the goods and commission?

24. Mr. Moore borrowed some money of Mr. Boyden, agreeing to pay him, for each year's use of it, a sum equal to 6 per cent of the money he had borrowed. He borrowed $125.63, and kept it just one year. How much ought he to pay for the use of it? How much, then, ought he to pay Mr. Boyden in all?

NOTE. — Money paid, as in the above example, for the use of money, is called INTEREST. The money used is called the PRINCIPAL. Interest is usually reckoned at a certain per cent of the principal for each year that it is used. The percentage paid for each year is called the RATE PER CENT. The interest and principal added together form the AMOUNT.

25. What is the interest of $137.84 for 1 year at 6 per cent?

26. What is the interest of $487.31 for 6 months at 6 per cent per year?

Suggestion. — Since the rate for 1 year is 6 per cent, the rate for 6

months must be $\frac{1}{2}$ of 6 per cent, which is 3 per cent. The interest for 6 months will, therefore, be 3 per cent of the principal.

27. Mr. Adams borrowed of Mr. Wales $718.63, for which he agreed to pay interest at 6 per cent. At the end of 6 months he paid the principal and interest. How much did he pay?

28. What is the interest of $47.83 for 4 months at 6 per cent?

134. *Multiplication and Division of the Numerator.*

The foregoing illustrations have shown that multiplying the numerator of a fraction multiplies the fraction, and that dividing the numerator divides the fraction. The same thing may be demonstrated more rigidly, by considering the nature of fractions, thus: —

Since the numerator of a fraction shows how many parts are considered, multiplying or dividing the numerator multiplies or divides the number of parts considered, without affecting their size, and hence multiplies or divides the fraction.

1. Explain the effect of multiplying the numerator of the fraction $\frac{5}{17}$ by 3.

Answer. — Multiplying the numerator of the fraction $\frac{5}{17}$ by 3 gives $\frac{15}{17}$ for a result, which expresses 3 times as many parts, each of the same size as before, and is, therefore, 3 times as large. Hence, the fraction $\frac{5}{17}$ has been multiplied by 3.

See Note after solution of example 8th.

Explain the effect of multiplying the numerator —

2. Of $\frac{5}{12}$ by 2.	4. Of $\frac{8}{9}$ by 4.	6. Of $\frac{3}{7}$ by 9.
3. Of $\frac{7}{13}$ by 8.	5. Of $\frac{37}{97}$ by 11.	7. Of $\frac{13}{49}$ by 8.

8. Explain the effect of dividing the numerator of the fraction $\frac{8}{9}$ by 4.

Solution. — Dividing the numerator of the fraction $\frac{8}{9}$ by 4 gives $\frac{2}{9}$ for a result, which expresses $\frac{1}{4}$ as many parts, each of the same size as before, and is, therefore, $\frac{1}{4}$ as large. Hence, the fraction $\frac{8}{9}$ has been divided by 4.

Note. — The work explained above is really equivalent to 3 times $\frac{5}{17} = \frac{15}{17}$, (just as 3 times 5 apples are 15 apples;) to $\frac{1}{4}$ of $\frac{8}{9} = \frac{2}{9}$, (just as $\frac{1}{4}$ of 8 apples = 2 apples.) The above forms are, however, necessary, and should therefore be mastered.

Explain the effect of dividing the numerator —

9. Of $\frac{24}{25}$ by 8.
10. Of $\frac{10}{12}$ by 2.
11. Of $\frac{18}{23}$ by 9.
12. Of $\frac{15}{17}$ by 3.
13. Of $\frac{12}{13}$ by 6.
14. Of $\frac{72}{83}$ by 12.

135. *Multiplication of the Denominator.*

(*a.*) Since the denominator of a fraction shows into how many parts the unit is divided, or how many parts equal the unit, multiplying the denominator must (**121**, *b.* and *c.*) divide each part, and therefore must divide the fraction.

1. Explain the effect of multiplying the denominator of the fraction $\frac{3}{4}$ by 2.

Answer. — Multiplying the denominator of the fraction $\frac{3}{4}$ by 2 gives $\frac{3}{8}$ for a result, which expresses the same number of parts, each $\frac{1}{2}$ as large as before. Therefore, $\frac{3}{8} = \frac{1}{2}$ of $\frac{3}{4}$, or multiplying the denominator of $\frac{3}{4}$ by 2, has divided the fraction by 2.

Explain the effect of multiplying the denominator —

2. Of $\frac{4}{5}$ by 3.
3. Of $\frac{2}{3}$ by 4.
4. Of $\frac{5}{7}$ by 4.
5. Of $\frac{8}{11}$ by 7.
6. Of $\frac{2}{3}$ by 2.
7. Of $\frac{8}{9}$ by 3.
8. Of $\frac{4}{5}$ by 7.
9. Of $\frac{11}{12}$ by 10.
10. Of $\frac{4}{7}$ by 6.

(*b.*) Hence, *multiplying the denominator of a fraction divides the fraction, by dividing the size of each part, without affecting the number of parts considered.*

136. *Division of the Denominator.*

Since the denominator of a fraction shows into how many parts the unit is divided, or how many such parts are equal to the unit, dividing the denominator must (**121**, *b.* and *c.*) multiply each part, and therefore multiply the fraction.

1. Explain the effect of dividing the denominator of the fraction $\frac{11}{12}$ by 4.

Answer. — Dividing the denominator of the fraction $\frac{11}{12}$ by 4 gives

$\frac{11}{3}$ for a result, which expresses the same number of parts, each 4 times as large as before. Therefore, $\frac{11}{3} = 4$ times $\frac{11}{12}$, or, dividing the denominator of $\frac{11}{12}$ by 4 has multiplied the fraction by 4.

Explain the effect of dividing the denominator —

2. Of $\frac{5}{9}$ by 3.	5. Of $\frac{3}{20}$ by 5.	8. Of $\frac{19}{24}$ by 6.
3. Of $\frac{7}{8}$ by 4.	6. Of $\frac{19}{36}$ by 12.	9. Of $\frac{7}{30}$ by 5.
4. Of $\frac{3}{10}$ by 2.	7. Of $\frac{25}{72}$ by 36.	10. Of $\frac{7}{12}$ by 3.

Hence, *dividing the denominator of a fraction multiplies the fraction, by multiplying the size of each part without affecting the number of parts expressed.*

137. *Recapitulation and Inferences.*

(*a.*) *Multiplying the numerator multiplies the fraction, by multiplying the number of parts considered without affecting their size.*

(*b.*) *Dividing the numerator divides the fraction, by dividing the number of parts considered without affecting their size.*

(*c.*) *Multiplying the denominator divides the fraction, by dividing each part without affecting the number of parts considered.*

(*d.*) *Dividing the denominator multiplies the fraction, by multiplying each part without affecting the number of parts considered.*

(*e.*) Hence, 1. *A fraction may be multiplied either by multiplying the numerator or by dividing the denominator.*

2. *A fraction may be divided either by dividing the numerator or by multiplying the denominator.*

3. *Multiplying both numerator and denominator of a fraction by any number both multiplies and divides the fraction by that number, and, therefore, does not alter its value.*

4. *Dividing both numerator and denominator of a fraction by the same number both divides and multiplies the fraction by that number, and, therefore, does not alter its value.*

138. *Multiplication and Division of both Numerator and Denominator by the same Number.*

1. Explain the effect of multiplying both numerator and denominator of the fraction $\frac{4}{7}$ by 6.

Answer. — Multiplying both numerator and denominator of the frac-

sion $\frac{4}{7}$ by 6 gives $\frac{24}{42}$ for a result, which expresses 6 times as many parts, each $\frac{1}{6}$ as large as before. Therefore, the value of the fraction is unaltered, and $\frac{4}{7} = \frac{24}{42}$.

Explain the effect of multiplying both numerator and denominator of —

2. The fraction $\frac{2}{3}$ by 2.	6. The fraction $\frac{5}{11}$ by 3.
3. The fraction $\frac{7}{8}$ by 9.	7. The fraction $\frac{7}{16}$ by 8.
4. The fraction $\frac{9}{10}$ by 10.	8. The fraction $\frac{13}{27}$ by 9.
5. The fraction $\frac{25}{39}$ by 7.	9. The fraction $\frac{18}{47}$ by 4.

10. Explain the effect of dividing both numerator and denominator of the fraction $\frac{9}{12}$ by 3.

Answer. — Dividing both numerator and denominator of the fraction $\frac{9}{12}$ by 3 gives $\frac{3}{4}$ for a result, which expresses $\frac{1}{3}$ as many parts, each part 3 times as large as before. Therefore, the value of the fraction is unaltered, and $\frac{9}{12} = \frac{3}{4}$.

Explain the effect of dividing both numerator and denominator of —

11. The fraction $\frac{16}{24}$ by 8.	15. The fraction $\frac{91}{133}$ by 7.
12. The fraction $\frac{27}{45}$ by 9.	16. The fraction $\frac{82}{123}$ by 41.
13. The fraction $\frac{8}{24}$ by 8.	17. The fraction $\frac{25}{30}$ by 5.
14. The fraction $\frac{18}{36}$ by 18.	18. The fraction $\frac{78}{273}$ by 39.

139. *Lowest Terms.*

(*a.*) The numerator and denominator are called TERMS of the fraction.

(*b.*) A fraction is said to be reduced to its LOWEST TERMS when its numerator and denominator are the smallest entire numbers which will express its value.

(*c.*) From the preceding explanations, it follows that, —

1. A fraction may be reduced to lower terms by dividing both numerator and denominator by any number which will divide both without a remainder.

2. A fraction may be reduced to its lowest terms by dividing both numerator and denominator by any number which

will divide both without a remainder; then dividing this result in the same way, and so on, continuing the process till a fraction is obtained, the terms of which are prime to each other; or by dividing both numerator and denominator by their greatest common divisor.

3. A fraction will always be reduced to its lowest terms when there is no number greater than 1 which will divide both its numerator and denominator without a remainder.

1. Reduce $\frac{8}{12}$ to its lowest terms.

Solution. — Dividing both numerator and denominator by 4, their greatest common divisor, gives $\frac{2}{3}$, which expresses $\frac{1}{4}$ as many parts, each part 4 times as large as before. Hence, $\frac{8}{12} = \frac{2}{3}$.

2. Reduce $\frac{38412}{50292}$ to its lowest terms.

Solution. — Observing (**104,** II.) that both numerator and denominator are divisible by 4, we first divide by 4, which gives $\frac{9603}{12573}$, or $\frac{1}{4}$ as many parts, each 4 times as large as before.

Observing (**104,** IV.) that both numerator and denominator of the last fraction are divisible by 9, we divide by 9, which gives $\frac{1067}{1397}$, or $\frac{1}{9}$ as many parts, each 9 times as large as before.

Observing (**104,** V.) that both numerator and denominator of the last fraction are divisible by 11, we divide by 11, which gives $\frac{97}{127}$, or $\frac{1}{11}$ as many parts, each 11 times as large as before. As the numerator and denominator of the last fraction are prime to each other, the reduction can be carried no farther, and $\frac{38412}{50292}$ reduced to its lowest terms equals $\frac{97}{127}$.

Second Solution. — The greatest common divisor of 38412 and 50292 found by one of the methods of Section IX.) is 396; and dividing both numerator and denominator by it, gives $\frac{97}{127}$, or $\frac{1}{396}$ as many parts, each part 396 times as large as before. Hence, $\frac{38412}{50292} = \frac{97}{127}$.

NOTE. — The *mechanical* process by the first solution is merely to divide both terms, first by 4, then by 9, then by 11; while by the last it is to find the G. C. D. of both terms, and divide them by it. The first method will usually be the most convenient, when the divisors can be readily perceived.

(*d.*). The pupil should be careful not to decide that any fraction is incapable of reduction till he has carefully tested it by some of the processes of Section IX.

Reduce each of the following fractions to its lowest terms.

3. $\frac{24}{30}$.	7. $\frac{64}{81}$.*	11. $\frac{623}{637}$.
4. $\frac{16}{18}$.	8. $\frac{36}{41}$.	12. $\frac{2021}{2491}$.
5. $\frac{144}{132}$.	9. $\frac{13}{39}$.	13. $\frac{468}{612}$.
6. $\frac{17}{51}$.	10. $\frac{176}{253}$.	14. $\frac{7123}{12989}$.

140. *Cancellation.*

(*a.*) When, as is sometimes the case, the factors of the numerator and denominator are given, labor may be saved by reducing the fraction to its lowest terms before multiplying the factors together.

(*b.*) In writing the work, it is well to draw a line through the factors which have been divided, and to place the quotients above those in the numerator, and beneath those in the denominator.

(*c.*) The numbers by which we divide are said to be *cancelled*, and the process is called *cancellation;* but it is identical in principle with other cases of reducing fractions to their lowest terms.

1. Reduce $\frac{8 \times 15}{9 \times 16}$ to its lowest terms.

Solution. — Cancelling 8 from the factors 8 of the numerator and 16 of the denominator, (i. e., dividing each by 8,) gives 1 in place of 8, and 2 in place of 16, and makes the fraction express $\frac{1}{8}$ as many parts, each 8 times as large as before.†

Cancelling 3 from the factors 15 of the numerator and 9 of the denominator, gives 5 in place of 15, and 3 in place of 9, and makes the fraction express $\frac{1}{3}$ as many parts, each 3 times as large as before.‡ As no further division can be made, the remaining factors are to be multiplied together, which gives $\frac{5}{6}$ for a result.

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* *Solution.* 64 and 81 are prime to each other, and hence $\frac{64}{81}$ cannot be reduced to lower terms.

† For multiplying by $\frac{1}{8}$ of a number gives $\frac{1}{8}$ as large a product as multiplying by the number would give.

‡ For multiplying by $\frac{1}{3}$ of a number gives $\frac{1}{3}$ as large a product as multiplying by the number would give.

The work would be written thus: —

$$\frac{\overset{1}{\cancel{8}} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{16}}} = \frac{5}{6}$$

2. Reduce $\frac{12 \times 7 \times 25 \times 36 \times 11}{35 \times 12 \times 4 \times 11 \times 21}$ to its lowest terms

Solution. — Cancelling the factor 12 from numerator and denominator, gives 1 in place of each. Cancelling 7 from the numerator and from the 35 in the denominator, gives 1 in place of the former, and 5 in place of the latter. Cancelling 5 from the denominator and from the 25 in the numerator, gives 1 in place of the former, and 5 in place of the latter. Cancelling 4 from the denominator and from the 36 in the numerator, gives 1 in place of the former, and 9 in place of the latter. Cancelling 11 from the numerator and denominator, gives 1 in place of each. Cancelling 3 from the 9 in the numerator and from the 21 in the denominator, gives 3 in place of the former, and 7 in place of the latter. As no further reduction can be made, we multiply the remaining factors together, which gives $\frac{15}{7} = 2\frac{1}{7}$.

The written work would be thus: —

$$\frac{\overset{1}{\cancel{12}} \times \overset{1}{\cancel{7}} \times \overset{5}{\cancel{25}} \times \overset{\overset{3}{\cancel{9}}}{\cancel{36}} \times \overset{1}{\cancel{11}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{35}} \times \underset{1}{\cancel{12}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{11}} \times \underset{7}{\cancel{21}}} = \frac{15}{7} = 2\frac{1}{7}$$

Or, by omitting to write the factors which are equal to 1, as we may do without ambiguity, we have the following more convenient form.

$$\frac{\cancel{12} \times \cancel{7} \times \overset{5}{\cancel{25}} \times \overset{\overset{3}{\cancel{9}}}{\cancel{36}} \times \cancel{11}}{\underset{\cancel{5}}{\cancel{35}} \times \cancel{12} \times \cancel{4} \times \cancel{11} \times \underset{7}{\cancel{21}}} = \frac{15}{7} = 2\frac{1}{7}$$

3. Reduce $\frac{5 \times 7 \times 11 \times 12 \times 15 \times 18 \times 2}{4 \times 5 \times 3 \times 11 \times 7 \times 36 \times 5 \times 6}$ to its lowest terms.

Answer —

$$\frac{\cancel{5} \times \cancel{7} \times \cancel{11} \times \overset{\cancel{3}}{\cancel{12}} \times \overset{\cancel{3}}{\cancel{15}} \times \cancel{18} \times \cancel{2}}{\cancel{4} \times \cancel{5} \times \cancel{3} \times \cancel{11} \times \cancel{7} \times \underset{\cancel{2}}{\cancel{36}} \times \cancel{5} \times \underset{2}{\cancel{6}}} = \frac{1}{2}$$

4. Reduce $\frac{48 \times 30 \times 49 \times 64 \times 27}{21 \times 32 \times 36 \times 42}$ to its lowest terms.

Answer. —

$$\frac{\overset{4}{\cancel{48}} \times \overset{10}{\cancel{30}} \times \overset{\cancel{7}}{\cancel{49}} \times \overset{\cancel{2}}{\cancel{64}} \times \overset{\overset{3}{\cancel{9}}}{\cancel{27}}}{\underset{\cancel{7}}{\cancel{21}} \times \cancel{32} \times \underset{\cancel{3}}{\cancel{36}} \times \underset{\underset{\cancel{3}}{\cancel{6}}}{\cancel{42}}} = \frac{120}{1} = 120$$

Reduce each of the following to its lowest terms.

5. $\frac{7 \times 16 \times 18 \times 5 \times 9}{20 \times 14 \times 9 \times 9 \times 11}$

6. $\frac{8 \times 36 \times 28 \times 48}{24 \times 72 \times 13 \times 15}$

7. $\frac{6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12}{7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13}$

8. $\frac{96 \times 65 \times 35}{25 \times 91 \times 48}$

9. $\frac{14 \times 5 \times 9 \times 30 \times 16 \times 17}{8 \times 17 \times 14 \times 27 \times 11 \times 8}$

10. $\frac{72 \times 49 \times 81 \times 33}{77 \times 84 \times 6 \times 27 \times 99}$

11. $\frac{144 \times 103 \times 625 \times 121}{375 \times 64 \times 99 \times 847}$

12. $\frac{315 \times 143 \times 64 \times 221}{119 \times 169 \times 209 \times 125}$

13. $\frac{1331 \times 343 \times 6859}{1463 \times 2527 \times 847}$

14. $\dfrac{4959 \times 3487 \times 2491}{6061 \times 53 \times 2853 \times 47}$

141. *Compound Fractions.*

(*a.*) A COMPOUND FRACTION is a fraction of a fraction as $\frac{3}{4}$ of $\frac{5}{7}$, $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{9}{10}$.

(*b.*) A compound fraction is equivalent to a fraction multiplied by a fraction.

Thus: $\frac{3}{4}$ of $\frac{5}{7} = \frac{3}{4}$ times $\frac{5}{7}$; $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{9}{10} = \frac{2}{3}$ times $\frac{5}{6}$ times $\frac{9}{10}$. (See **132.**)

(*c.*) In a compound fraction, the value expressed by one fraction is made the unit of another. $\frac{3}{4}$ of $\frac{5}{7}$ means $\frac{3}{4}$ of the quantity $\frac{5}{7}$, or 3 such parts as would be obtained by dividing $\frac{5}{7}$ into 4 equal parts. $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{9}{10}$ means $\frac{2}{3}$ of the quantity $\frac{5}{6}$ of $\frac{9}{10}$, and $\frac{5}{6}$ of $\frac{9}{10}$ means $\frac{5}{6}$ of the quantity $\frac{9}{10}$.

(*d.*) In reducing compound fractions to simple ones, it is important to notice which fraction is made the unit of the other, as that is the one on which the operation is to be performed.

1. What is $\frac{8}{9}$ of $\frac{5747}{9436}$?

Solution. $\frac{8}{9}$ of $\frac{5747}{9436} = 8$ times $\frac{1}{9}$ of $\frac{5747}{9436}$; $\frac{1}{9}$ of $\frac{5747}{9436}$, found by dividing 5747 by 9, is $\frac{638\frac{5}{9}}{9436}$, and 8 times this result, found by multiplying $638\frac{5}{9}$ by 8, is $\dfrac{5108\frac{4}{9}}{9436}$. Hence the following forms of written work.

First Form.

$$a = 5747$$
$$\tfrac{1}{9} \text{ of } a = b = 638\tfrac{5}{9}$$
$$8 \text{ times } b = 5108\tfrac{4}{9} = \tfrac{8}{9} \text{ of } a.$$

Hence, $\frac{8}{9}$ of $\frac{5747}{9436} = \frac{5108\frac{4}{9}}{9436}$

Second Form.

$$\begin{array}{r} 9\ )\ 5747 \\ \hline 638\frac{5}{9} \\ 8 \\ \hline 5108\frac{4}{9} \end{array}$$

Hence, $\frac{8}{9}$ of $\frac{5747}{9436} = \frac{5108\frac{4}{9}}{9436}$

2. What is $\frac{5}{11}$ of $\frac{274}{527}$?
3. What is $\frac{4}{5}$ of $\frac{9645}{13724}$?
4. What is $\frac{6}{13}$ of $\frac{4387}{6274}$?
5. What is $\frac{5}{8}$ of $\frac{136}{879}$?
6. What is $\frac{17}{24}$ of $\frac{46252}{98325}$?
7. What is $\frac{29}{36}$ of $\frac{4932}{8575}$?

8. What is $\frac{1}{8}$ of $437\frac{5}{7}$?

Solution. $\frac{1}{8}$ of $437\frac{5}{7} = 54$, with a remainder of $5\frac{5}{7}$, which, reduced to sevenths, $= \frac{40}{7}$; $\frac{1}{8}$ of $\frac{40}{7} = \frac{5}{7}$. Hence, $\frac{1}{8}$ of $437\frac{5}{7} = 54\frac{5}{7}$.

NOTE. — Compare the above with "What is $\frac{1}{8}$ of 437 weeks and 5 days?"

9. What is $\frac{1}{5}$ of $8539\frac{4}{11}$?
10. What is $\frac{3}{4}$ of $827\frac{5}{8}$?
11. What is $\frac{6}{7}$ of $5827\frac{1}{2}$?
12. What is $\frac{4}{5}$ of $2793\frac{1}{8}$?
13. What is $\frac{7}{8}$ of $1397\frac{1}{11}$?
14. What is $\frac{5}{8}$ of $4355\frac{146}{173}$?

Second Method of Reduction.

(*e.*) When the numerator of the *unit* of the compound fraction is not a multiple of the denominator of the *other* fraction, the preceding solution will give a fraction in the numerator of the result. In all such cases the following solution should be adopted. Indeed, applying, as it does, the principles of cancellation, it really includes the preceding.

15. Reduce $\frac{3}{8}$ of $\frac{4}{9}$ to a simple fraction.

First Solution. — Since $\frac{3}{8}$ of any fraction must equal 3 times as many parts, each $\frac{1}{8}$ as large as before, $\frac{3}{8}$ of $\frac{4}{9}$ may be found by multiplying the denominator of $\frac{4}{9}$ by 8, and the numerator by 3. This gives the following written work.

$$\frac{3}{8} \text{ of } \frac{4}{9} = \frac{\cancel{4} \times \cancel{3}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}}} = \frac{1}{6}$$

Second Solution. $\frac{1}{8}$ of $\frac{4}{9}$ may be expressed by making 8 a factor of the denominator, and $\frac{3}{8}$ of $\frac{4}{9}$ must be 3 times this result, which may be expressed by making 3 a factor of the numerator. This gives the same written work as before.

NOTE. — In writing the work of such examples as the above, the fraction on which the operation is to be performed should always be written first.

16. Reduce $\frac{8}{9}$ of $\frac{15}{28}$ of $3\frac{2}{11}$ to a simple fraction.

First Solution. $3\frac{2}{11}$, or $\frac{35}{11}$, being the number on which the operation is to be performed, should be written first. $\frac{15}{28}$ of this must equal 15 times as many parts, each $\frac{1}{28}$ as large, and may be expressed by making 15 a factor of the numerator, and 28 a factor of the denominator. $\frac{8}{9}$ of this must equal 8 times as many parts, each $\frac{1}{9}$ as large, and hence may be expressed by making 8 a factor of the numerator, and 9 a factor of the denominator. This would give the following work.

$$\frac{8}{9} \text{ of } \frac{15}{28} \text{ of } 3\frac{2}{11} = \frac{\overset{5}{\cancel{35}} \times \overset{5}{\cancel{15}} \times \overset{2}{\cancel{8}}}{11 \times \underset{4}{\cancel{28}} \times \underset{3}{\cancel{9}}} = \frac{50}{33} = 1\frac{17}{33}$$

Second Solution. $3\frac{2}{11}$, or $\frac{35}{11}$, is the number on which the operation is to be performed, and should therefore be written first. $\frac{1}{28}$ of $\frac{35}{11}$ may be expressed by making 28 a factor of the denominator, and $\frac{15}{28}$ of $\frac{35}{11}$ must be 15 times this result, which may be expressed by making 15 a factor of the numerator. $\frac{1}{9}$ of this result may be expressed by making 9 a factor of the denominator, and $\frac{8}{9}$, or 8 times the last result, by making 8 a factor of the numerator. This would give the same written work as before.

NOTE. — It will be seen that both of the above solutions give the same numerical process, viz., to make all the numerators of the compound fraction factors of the new numerator, and all the denominators factors of the new denominator, and then cancel and reduce.

Reduce each of the following fractions to simple ones.

17. $\frac{7}{8}$ of $\frac{20}{21}$.
18. $\frac{13}{27}$ of $\frac{15}{26}$.
19. $\frac{5}{6}$ of $\frac{18}{25}$.
20. $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{4}{9}$.
21. $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{12}{25}$.
22. $\frac{5}{12}$ of $\frac{8}{15}$ of $13\frac{1}{2}$.
23. $\frac{3}{4}$ of $\frac{5}{6}$ of $3\frac{3}{4}$.
24. $\frac{5}{7}$ of $\frac{9}{11}$ of $\frac{77}{90}$.
25. $\frac{15}{16}$ of $\frac{18}{19}$ of $4\frac{25}{27}$.
26. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{6}{7}$ of $\frac{1}{4}$ of $\frac{12}{25}$ of $\frac{35}{36}$.
27. $\frac{7}{12}$ of $\frac{5}{6}$ of $\frac{9}{35}$ of $\frac{8}{11}$ of $9\frac{1}{6}$.
28. $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{24}{25}$ of $\frac{55}{72}$ of $\frac{25}{27}$.
29. $\frac{5}{9}$ of $\frac{7}{8}$ of $\frac{24}{49}$ of $\frac{22}{25}$ of $13\frac{1}{3}$.
30. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{8}{9}$ of $\frac{3}{16}$ of $\frac{21}{22}$ of $\frac{14}{19}$ of $\frac{22}{23}$ of $62\frac{5}{7}$.

142. *Multiplication of Fractions.*

1. What is the product of $\frac{4}{9} \times \frac{3}{8}$?

Solution. $\frac{4}{9}$ multiplied by $\frac{3}{8} = \frac{3}{8}$ of $\frac{4}{9}$, which, found by multiplying the numerator of $\frac{4}{9}$ by 3 and the denominator by 8, gives the following written work.

$$\frac{4}{9} \times \frac{3}{8} = \frac{\cancel{4} \times \cancel{3}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}}} = \frac{1}{6}$$

(*a.*) By reading the sign $\times$ as *times*, we have —

$\frac{4}{9}$ times $\frac{3}{8}$ = $\frac{4}{9}$ of $\frac{3}{8}$, which, found by multiplying the numerator of $\frac{3}{8}$ by 4 and the denominator by 9, gives the following written work.

$$\frac{4}{9} \times \frac{3}{8} = \frac{\cancel{3} \times \cancel{4}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}}} = \frac{1}{6}$$

What is the product of —

2. $\frac{8}{9} \times \frac{15}{28}$?
3. $\frac{48}{245} \times \frac{49}{72}$?
4. $\frac{4}{7} \times \frac{11}{12} \times \frac{9}{22}$?
5. $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$?
6. $4\frac{1}{2} \times 4\frac{2}{7} \times 8\frac{1}{6}$?
7. $3\frac{1}{5} \times 6\frac{1}{4} \times \frac{7}{10}$?
8. $13\frac{5}{7} \times 23\frac{7}{11} \times 3\frac{73}{104}$?
9. $5\frac{1}{2} \times \frac{7}{8}$ of $5\frac{9}{11}$?
10. $\frac{4}{5} \times \frac{3}{7} \times \frac{11}{12} \times \frac{25}{33}$?

(*b.*) The following form of explanation may be adopted when a very thorough analysis is required.

11. What is the product of $\frac{49}{54} \times \frac{81}{98}$?

Solution. — First write $\frac{49}{54}$ as the number on which the operation is to be performed; we then have $\frac{49}{54}$ multiplied by 1 equals $\frac{49}{54}$, and multiplied by $\frac{1}{98}$ will equal $\frac{1}{98}$ of this result, which may be expressed by making 98 a factor of the denominator. If multiplying by $\frac{1}{98}$ gives this result, multiplying by $\frac{81}{98}$ must give 81 times this result, which may be expressed by making 81 a factor of the numerator. Hence, —

$$\frac{49}{54} \times \frac{81}{98} = \frac{\cancel{49} \times \overset{3}{\cancel{81}}}{\underset{2}{\cancel{54}} \times \underset{2}{\cancel{98}}} = \frac{3}{4}$$

NOTE. — It will be seen that all the forms of solution give similar forms of written work, the numerators of the several fractions being in all cases factors of the numerator of the product, and the denomipators factors of the denominator.

What is the product of —

12. $\frac{8}{9} \times \frac{7}{12} \times \frac{27}{35}$?
13. $\frac{35}{36} \times \frac{30}{77} \times \frac{144}{175}$?
14. $\frac{17}{91} \times \frac{3}{11} \times \frac{13}{19}$?
15. $\frac{83}{100} \times \frac{7}{1000} \times \frac{49}{100}$?
16. .83 $\times$ .007 $\times$ .49 ? *
17. 8.7 $\times$ .43 $\times$ .006 ? *

* For models of written work, see 171st page, solution to 8th example. and 172d page, Note.

18. $4\frac{6}{7} \times 2\frac{7}{8} \times 9\frac{3}{5}$?

19. 43.79×25.7?

20. $.824 \times 4.3$?

21. $6750 \times .6750$?

22. $5.06 \times 300 \times .2 \times .0004$?

23. $.4 \times .3 \times .2 \times .6 \times .2$?

24. 4.974×1.0007?

143. *Reduction of a Vulgar Fraction to a Decimal Form*

1. Reduce $\frac{7}{8}$ to a decimal fraction.

Solution. $\frac{7}{8}$, or $\frac{7}{8}$ of 1, $= \frac{1}{8}$ of $7 = \frac{1}{8}$ of 70 tenths $=$ 8 tenths, with a remainder of 6 tenths. But 6 tenths $=$ 60 hundredths, and $\frac{1}{8}$ of 60 hundredths $=$ 7 hundredths, with a remainder of 4 hundredths. But 4 hundredths $=$ 40 thousandths, and $\frac{1}{8}$ of 40 thousandths $=$ 5 thousandths. Hence, $\frac{7}{8} = .8 + .07 + .005 = .875$

The work may be written thus: —

```
8 ) 7.000
    .875
```

Proof. $.875 = \frac{875}{1000} = \frac{7}{8}$.

2. Reduce $\frac{25}{28}$ to a decimal fraction.

Solution. $\frac{25}{28}$, or $\frac{25}{28}$ of 1, $= \frac{1}{28}$ of $25 = \frac{1}{28}$ of 250 tenths $= 8$ tenths, with a remainder of 26 tenths. But 26 tenths $=$ 260 hundredths, and $\frac{1}{28}$ of 260 hundredths $=$; &c. Hence the following written work.

```
28 ) 25.00
     2.60
      .080
      .0240
      .00160
      .000200
      .000004
```

$.892857\frac{4}{28} =$ *Ans.*, or dropping the $\frac{4}{28}$, or $\frac{1}{7}$ of a millionth, we shall have the approximate value of $\frac{25}{28} = .892857$

Proof. $.892857\frac{1}{7} = 892857\frac{1}{7}$ times $\frac{1}{1000000} = \frac{6250000}{7}$ of $\frac{1}{7000000} = \frac{25}{28}$.

Note. — Compare the above solutions with **87,** (*c.*)

(*b.*) The fractions may also be reduced by the following solutions: —

$\frac{7}{8}$, or $\frac{7}{8}$ units, $=$ 10 times as many tenths $= \frac{7 \times 10}{8}$ tenths $=$ 10 times as many hundredths as tenths $= \frac{7 \times 10 \times 10}{8}$ hundredths $=$ 10

times as many thousandths as hundredths $= \frac{7 \times 10 \times 10 \times 10}{8}$ **thousandths.**

By cancelling the last, we have, —

$$\frac{7}{8} = \frac{7 \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{8}}} \text{ thousandths} = .875$$

Note. — The above is equivalent to the following: —

$$\frac{7}{8} = \frac{7 \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{8}} \times 10 \times 10 \times 10} = \frac{875}{1000} = .875$$

So, $\frac{25}{28}$, **or** $\frac{25}{28}$ **units,** $=$ **10 times as many tenths** $= \frac{25 \times 10}{28}$ **tenths,** $=$ **10 times as many hundredths as tenths** $= \frac{25 \times 10 \times 10}{28}$ **hundredths, &c.**

$$\text{Hence, } \frac{25}{28} = \frac{25 \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times 10 \times 10 \times 10 \times 10}{\underset{\underset{7}{\cancel{14}}}{\cancel{28}}}$$

$$\text{millionths} = \frac{6250000}{7} \text{ millionths} = .892857\tfrac{1}{7}$$

Note. — The above is equivalent to

$$\frac{25}{28} = \frac{25 \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times 10 \times 10 \times 10 \times 10}{\underset{\underset{7}{\cancel{14}}}{\cancel{28}} \times 10 \times 10 \times 10 \times 10 \times 10 \times 10} =$$

$$\frac{6250000}{7000000} = \frac{6250000}{7} \text{ of } .000001 = .892857\tfrac{1}{7}$$

(*c.*) It is obvious that $\frac{25}{28}$ cannot be reduced to an exactly equivalent decimal form, for introducing into the numerator

any number of factors each equal to 10 will not enable us to cancel the factor 7 from the denominator. As the same principles apply to any other fraction, it follows that, —

(*d.*) *No vulgar fraction can be reduced to an exactly equivalent decimal form, if, when reduced to its lowest terms, its denominator contains other factors than such as are found in* 10, i. e., *other than 2's or 5's; and conversely, that,* —

(*e.*) *Every vulgar fraction which, when reduced to its lowest terms, contains in its denominator only such factors as are found in* 10, *can be reduced to an exactly equivalent decimal form, and will contain as many decimal places as there are 2's or 5's to be cancelled from the denominator.*

Illustrations. $\frac{15}{16} = \frac{15}{2^4}$, and hence can be reduced to an exactly equivalent decimal. Moreover, it will contain four decimal places; for 10 must be introduced 4 times as a factor into the numerator, to cancel 2^4 from the denominator.

Again. $\frac{13}{250} = \frac{13}{2 \times 5^3}$, and hence can be reduced to an exactly equivalent decimal. Moreover, it will contain 3 decimal places, for 10 must be introduced 3 times as a factor in the numerator, to cancel the factors of the denominator.

Again. $\frac{27}{52} = \frac{27}{2^2 \times 13}$, and hence cannot be reduced to an exactly equivalent decimal form.

(*f.*) Vulgar fractions which cannot be exactly reduced give rise to REPEATING or CIRCULATING DECIMALS.

Reduce each of the following to a decimal form, carrying the division, when only approximate values can be obtained, to six decimal places.

3. $\frac{347}{625}$.	7. $\frac{4}{13}$.	11. $\frac{28}{33}$.
4. $\frac{17}{36}$.	8. $\frac{19}{32}$.	12. $\frac{73}{75}$.
5. $\frac{17}{24}$.	9. $\frac{37}{125}$.	13. $\frac{13}{17}$.
6. $\frac{15}{17}$.	10. $\frac{6}{7}$.	14. $\frac{11}{40}$.

144. *Fractional Parts of Denominate Numbers.*

(*a.*) What is the value of $\frac{5}{8}$ of a mile in whole numbers of lower denominations, i. e., in furlongs, rods, yards, &c.?

First Solution. — This example may be solved by the common process of compound division, thus: $\frac{8}{9}$ of a mile $= \frac{1}{9}$ of 8 miles $= 0$ miles, with 8 miles remaining. But 8 miles $= 64$ furlongs, and $\frac{1}{9}$ of 64 furlongs $= 7$ furlongs, with, &c.

Second Solution. — Since 1 m. $=$ 8 fur., $\frac{8}{9}$ of a mile must equal $\frac{8}{9}$ of 8 fur., which is $7\frac{1}{9}$ fur. Since 1 fur. $=$ 40 rods, $\frac{1}{9}$ of a fur. must equal $\frac{1}{9}$ of 40 rods, which is $4\frac{4}{9}$ rods. Since 1 rod $= 5\frac{1}{2}$, or $\frac{11}{2}$ yds., $\frac{4}{9}$ of a rod must equal $\frac{4}{9}$ of $\frac{11}{2}$ yds., which is $2\frac{4}{9}$ yds. Since 1 yd. $=$ 3 ft., $\frac{4}{9}$ of a yd. must equal $\frac{4}{9}$ of 3 ft., which is $1\frac{1}{3}$ ft. Since 1 ft. $=$ 12 in., $\frac{1}{3}$ of a ft. must equal $\frac{1}{3}$ of 12 in., which is 4 in. Therefore, $\frac{8}{9}$ of a mile $=$ 7 fur., 4 rd., 2 yd., 1 ft., 4 in. The work may be written thus: —

$$\overset{\text{m.}}{\frac{8}{9}} = \overset{\text{fur.}}{\frac{8 \times 8}{9}} = \overset{\text{fur.}}{\frac{64}{9}}, \text{ or } 7\frac{1}{9} \qquad \overset{\text{fur.}}{\frac{1}{9}} = \overset{\text{rd.}}{\frac{1 \times 40}{9}} = \overset{\text{rd.}}{\frac{40}{9}}, \text{ or } 4\frac{4}{9}$$

$$\overset{\text{rd.}}{\frac{4}{9}} = \overset{\text{yd.}}{\frac{4 \times 11}{9 \times 2}} = \overset{\text{yd.}}{\frac{22}{9}}, \text{ or } 2\frac{4}{9} \qquad \overset{\text{yd.}}{\frac{4}{9}} = \overset{\text{ft.}}{\frac{4 \times 3}{9}} = \overset{\text{ft.}}{\frac{4}{3}}, \text{ or } 1\frac{1}{3}$$

$$\overset{\text{ft.}}{\frac{1}{3}} = \overset{\text{in.}}{\frac{1 \times 12}{3}} = 4 \text{ in.} \quad \text{Hence, } \tfrac{8}{9} \text{ m.} = 7 \text{ fur. } 4 \text{ rd. } 2 \text{ yd. } 1 \text{ ft. } 4 \text{ in.}$$

Third Solution. — Since there are 8 furlongs for every mile, there must be 8 times as large a part of a furlong as of a mile, or, in this instance, 8 times $\frac{8}{9}$ of a furlong, which is $7\frac{1}{9}$ furlongs. Since there are 40 rods for every furlong, there must be 40 times as large a part of a rod as of a furlong, or, in this instance, 40 times $\frac{1}{9}$ of a rod, which is $4\frac{4}{9}$ rods. Since for every rod there are $5\frac{1}{2}$, or $\frac{11}{2}$ yards, there must be $\frac{11}{2}$ as large a part of a yard as of a rod, or, in this instance, $\frac{11}{2}$ of $\frac{4}{9}$ yard, which is, &c. The written work would be the same as before.

(*b.*) The only difference between the processes of this and the previous article is the difference between decimal and denominate numbers. In the former, a unit of any denomination equals 10 of the next lower, while in the latter the number of units of a lower denomination to which any unit is equal, varies with the denomination of the unit considered.

What is the value of each of the following in whole numbers of lower denominations?

1. $\frac{3}{7}$ of a £.
2. $\frac{11}{12}$ of a gal.
3. $\frac{17}{28}$ of a bu.
4. $\frac{49}{55}$ of a week.
5. $\frac{27}{40}$ of a Cd. ft.
6. $\frac{8}{11}$ of an acre.
7. $\frac{15}{16}$ of a mile.
8. $\frac{7}{13}$ of a ton.
9. $\frac{8}{21}$ of a lb. T.

(*c.*) Reduce .7925 of a £ to whole numbers of lower denominations.

First Solution. — Since £1 = 20 s., .7925 of a £ must equal .7925 of 20 s. = 20 times .7925 s. = 15.8500 s. Since 1 s. = 12 d., .85 of a s. must equal .85 of 12 d. = 12 times .85 d. = 10.20 d. Since 1 d. = 4 qr., .2 of a penny must equal .2 of 4 qr. = .8 qr. Hence, .7925 of a £ = 15 s., 10 d., .8 qr.

WRITTEN WORK.

$$\begin{array}{r} .7925\ £. \\ 20 \\ \hline 15.8500\ s. \\ 12 \\ \hline 10.20\ d. \\ 4 \\ \hline .80 \end{array}$$

Or, by omitting to write the multipliers, we have, —

$$\begin{array}{r} .7925\ £. \\ \hline 15.8500\ s. \\ \hline 10.20\ d. \\ \hline .8\ qr. \end{array}$$

Hence, .7925 of a £ = 15 s. 10 d. 8 qr.

Second Solution. — Since there are 20 s. for every pound, there must be 20 times as large a part of a shilling as of a pound, or, in this case, 20 times .7925 s. = 15.85 s. Since there are 12 d. for every shilling, there must be 12 times as large a part of a penny as of a shilling, or, in this case, 12 times .85 s., &c. The written work is the same as in the last solution.

NOTE. — The student should be careful to multiply only the fractional part of each number.

What is the value of —

1. .345 of a ton?
2. .6375 of a mile?
3. .3426 of a lb.?
4. .754 of a £.?
5. .2376 of a gal.?
6. .625 of a bu.?
7. .317 of a cu. yd.?
8. .2569 of a cwt.?
9. What part of 1 mile is 7 fur. 4 rd. 2 yd. 1 ft. 4 in.?

Solution. — Since 1 in. = $\frac{1}{12}$ of a foot, 4 in. must equal $\frac{4}{12}$, or $\frac{1}{3}$ of a foot, to which adding the 1 foot gives $1\frac{1}{3}$ ft. = $\frac{4}{3}$ of a foot. Since 1 ft. = $\frac{1}{3}$ of a yd., $\frac{4}{3}$ of a foot must equal $\frac{4}{3}$ of $\frac{1}{3}$ of a yd., or $\frac{4}{9}$ of a yd., to which adding the 2 yards gives $2\frac{4}{9}$ yd., or $\frac{22}{9}$ yd. Since 1 yd. = $\frac{2}{11}$ of a rd., $\frac{22}{9}$ of a yd. must equal $\frac{22}{9}$ of $\frac{2}{11}$ of a rd., or $\frac{4}{9}$ of a rd., to which adding the 4 rd. gives, &c.

When the work is written, the following form may be adopted: —

$$4\text{ in.} = \overbrace{\frac{4}{12} = \frac{1}{3}}^{\text{ft.}} \qquad 1\frac{1}{3} = \overbrace{\frac{4}{3}}^{\text{ft.}} = \overbrace{\frac{4}{3}\text{ of }\frac{1}{3} = \frac{4}{9}}^{\text{yd.}}$$

$$\overbrace{2\frac{4}{9} = \frac{22}{9}}^{\text{yd.}} = \overbrace{\frac{\cancel{22}^{\,2}}{9}\text{ of }\frac{2}{\cancel{11}} = \frac{4}{9}}^{\text{rd.}} \qquad \overbrace{4\frac{4}{9} = \frac{40}{9}}^{\text{rd.}} = \overbrace{\frac{\cancel{40}}{9}\text{ of }\frac{1}{\cancel{40}} = \frac{1}{9}}^{\text{fur.}}$$

$$\overbrace{7\frac{1}{9} = \frac{64}{9}}^{\text{fur.}} = \overbrace{\frac{\cancel{64}^{\,8}}{9}\text{ of }\frac{1}{\cancel{8}} = \frac{8}{9}}^{\text{m.}}$$

Hence, 7 fur. 4 rd. 2 yd. 1 ft. 4 in. = $\frac{8}{9}$ of a mile.

Proof. — Reduce $\frac{8}{9}$ of a mile to fur., rd., &c.

Second Solution. — Since there is $\frac{1}{12}$ of a foot for each inch, there must be $\frac{1}{12}$ as many feet as inches, or, in this case, $\frac{1}{12}$ of 4 ft. = $\frac{4}{12}$, or $\frac{1}{3}$ of a ft. Since there is $\frac{1}{3}$ of a yard for each foot, there must be $\frac{1}{3}$ as many yards as feet, or, in this case, $\frac{1}{3}$ of $\frac{4}{3}$, or $\frac{4}{9}$ yd. =, &c.

The written work is the same as in the last solution.

Note. — The method here given is usually preferable to that of **129**, solution of 72d example, inasmuch as it keeps all the fractions reduced to their lowest terms, and enables us to perform very many, if not most, such examples without writing any figures.

What part —

10. Of 1 A. is 2 R. 36 sq. rd. 8 sq. yd. 2 sq. ft. 36 sq. in.?
11. Of 1 gal. is 3 qt. 1 pt. $1\frac{1}{3}$ gi.?
12. Of 1 Cd. ft. is 10 cu. ft. $1382\frac{2}{5}$ cu. in.?
13. Of 1 T. is 17 cwt. 3 qr. 2 lb. 12 oz. $7\frac{1}{9}$ dr.
14. Of 1 lb. is 6 oz. 13 dwt. 8 gr.?
15. Of 1 £ is 9 s. 5 d. $1\frac{1}{3}$ qr.?
16. Of 1 circumference is $155° 4' 36\frac{12}{13}''$?
17. Of 1 ℔ is 5 ℥ 6 ʒ 0 ℈ $6\frac{2}{3}$ gr.?
18. Of 1 w. is 3 da. 10 h. 17 m. $8\frac{4}{7}$ sec.?

(*d.*) Should it be required to give the answers to such questions as the above in a decimal form, it will only be necessary to reduce the vulgar fractions obtained by the preceding process to equivalent decimals.

(*e.*) The following process may also be applied :—

19. What part of a pound is 13 s. 7 d. 2 qr.?

Solution.—Since for every farthing there is $\frac{1}{4}$ of a penny, there must

be $\frac{1}{4}$ as many pence as farthings, or, in this case, $\frac{1}{4}$ of 2 d. = .5 of a penny, to which adding the 7 d. gives 7.5 d. Since for every penny there is $\frac{1}{12}$ of a shilling, there must be $\frac{1}{12}$ as many shillings as pence, or, in this case, $\frac{1}{12}$ of 7.5 s. =, &c.

The most convenient form of writing the work is to arrange the numbers expressing the various denominations in a vertical column, thus:—

$$\begin{array}{r|l} 4\ ) & \underline{2.0\text{ qr.}} \\ 12\ ) & \underline{7.500\text{ d.}} = 7\text{ d. }2\text{ qr.} \\ 20\ ) & \underline{13.6250\text{ s.}} = 13\text{ s. }7\text{ d. }2\text{ qr.} \\ & .68125\text{ £} = 13\text{ s. }7\text{ d. }2\text{ qr.} = \textit{Ans.} \end{array}$$

In like manner perform the following:—

20. What part of 1 bu. is 5 pk. 3 qt. 1 pt.?
21. What part of 1 lb. is 6 oz. 13 dwt. 8 gr.?
22. What part of 1 gal. is 3 qt. 1 pt. 3 gi.?
23. What part of 1 Cd. ft. is 10 cu. ft. $1382\frac{2}{5}$ cu. in.?

145. *To find a Number from a Fractional Part of it.*

1. 5861 = $\frac{1}{7}$ of what number?

First Solution. 5861 = $\frac{1}{7}$ of 7 times 5861, which is 41027.

Second Solution. If 5861 = $\frac{1}{7}$ of some number, $\frac{7}{7}$, or the number itself, must equal 7 times 5861, which is 41027.

Proof. $\frac{1}{7}$ of 41027 = 5861.

Of what number—

2. Does 3498 = $\frac{1}{8}$?
3. Does 59387 = $\frac{1}{9}$?
4. Does $\frac{137}{498}$ = $\frac{1}{6}$?
5. Does $\frac{341}{438}$ = $\frac{1}{9}$?
6. Does 58.46 = .1?
7. Does 327.93 = .0001?

8. 3476 = $\frac{8}{9}$ of what number?

Solution.—If 3476 = $\frac{8}{9}$ of some number, $\frac{1}{9}$ of that number must be $\frac{1}{8}$ of 3476, which is $\frac{3476}{8}$, and $\frac{9}{9}$, or the number, must be 9 times this result, and may be expressed by multiplying the numerator by 9. Hence we have the following written work:—

$$3476 = \frac{8}{9} \text{ of } \frac{\overset{869}{\cancel{3476}} \times 9}{\underset{2}{\cancel{8}}} = \frac{8}{9} \text{ of } \frac{7821}{2} = \frac{8}{9} \text{ of } 3910\tfrac{1}{2}$$

= *Answer.*

Proof $\frac{8}{9}$ of $3910\frac{1}{2}$ = 3476.

9. $\frac{49}{54} = \frac{35}{36}$ of what number?

Solution. — If $\frac{49}{54} = \frac{35}{36}$ of some number, $\frac{1}{35}$ of that number must equal $\frac{1}{35}$ of $\frac{49}{54}$, which may be expressed by making 35 a factor of the denominator, and $\frac{36}{36}$, or the number, must be 36 times the last product, which may be expressed by making 36 a factor of the numerator. This gives the following written work: —

$$\frac{\overset{7}{\cancel{49}} \times \overset{2}{\cancel{36}}}{\underset{3}{\cancel{54}} \times \underset{5}{\cancel{35}}} = \frac{14}{15} = Ans.$$

Proof. — See if $\frac{35}{36}$ of $\frac{14}{15}$ is equal to $\frac{49}{54}$.

Of what number —

10. Does 5496 = $\frac{6}{7}$?
11. Does 23584 = $\frac{33}{34}$?
12. Does 16875 = $\frac{12}{13}$?
13. Does $\frac{3}{4} = \frac{2}{7}$?
14. Does $\frac{9}{16} = \frac{3}{20}$?
15. Does $\frac{45}{49} = \frac{75}{91}$?

16. 8372 = .07 of what number?

Solution. — Since 8372 = .07 of some number, .01 of that number must equal $\frac{1}{7}$ of 8372, which is 1196, and $\frac{100}{100}$, or the number, must equal 100 times this result, which, found by removing the point two places towards the right, is 119600.

Of what number —

17. Does 5987 = .9?
18. Does 2.475 = .03?
19. Does 79.84 = .004?
20. Does .58674 = .0011?

146. *Practical Problems.*

1. If $\frac{2}{3}$ of a yard of cloth cost \$3.74, how much will 1 yard cost?

Solution. — If $\frac{2}{3}$ of a yard of cloth cost \$3.74, $\frac{1}{3}$ of a yard will cost $\frac{1}{2}$ of \$3.74, which is \$1.87, and $\frac{3}{3}$, or a yard, will cost 3 times \$1.87, which is \$5.61 = *Ans.*

The work may be written in either of the following forms: —

First Form.

$$\frac{\overset{1.87}{\cancel{\$3.74}} \times 3}{\cancel{2}} = \$5.61$$

Second Form.

$$\begin{array}{r} 2\;)\;\$3.74 \\ \hline \$1.87 \\ 3 \\ \hline \$5.61 \end{array}$$

Proof — If 1 yard of cloth costs \$5.61, $\frac{1}{3}$ of a yard will cost $\frac{1}{3}$ of \$5.61, which is \$1.87, and $\frac{2}{3}$ of a yard will cost two times \$1 87, which is \$3.74.

2. If $\frac{7}{8}$ of an acre of land cost \$8.54, what will an acre cost?

3. If $\frac{5}{8}$ of a vessel cost \$17645, what will the vessel cost?

4. If $\frac{7}{11}$ of a cask of oil is 143 gallons, how many gallons are there in the cask?

5. If .09 of a lot is worth \$594.72, how much is the lot worth?

6. If .13 of a cargo of coal is worth \$213.46, how much is the cargo worth?

7. If $\frac{5}{7}$ of a pound of tea is worth $\frac{25}{27}$ of a dollar, what is a pound of tea worth?

Solution. — Since the answer is to be in dollars, we write $\frac{25}{27}$ of a dollar as the number to be operated upon. If $\frac{5}{7}$ of a pound of tea cost so much, $\frac{1}{7}$ of a pound will cost $\frac{1}{5}$ of this, which may be expressed by making 5 a factor of the denominator; and $\frac{7}{7}$, or a pound, will cost 7 times this result, which may be expressed by making 7 a factor of the numerator.

This gives the following written work. —

$$\$\frac{\overset{5}{\cancel{25}} \times 7}{27 \times \cancel{5}} = \$\frac{35}{27} = \$1\frac{8}{27} = \$1.296$$

8. If $\frac{24}{25}$ of a bale of cotton weighs $\frac{16}{49}$ of a ton, how much will the bale weigh?

9. Bought $\frac{27}{32}$ of a gallon of oil for $\frac{63}{72}$ of a dollar. How much would a gallon have cost at the same rate?

10. If $\frac{8}{9}$ of a yard of silk cost \$1.572, what will $\frac{20}{27}$ of a yard cost?

Solution. — Since the answer is to be in dollars, we first write \$1.572 as the number to be operated upon. If $\frac{8}{9}$ of a yard cost \$1.572, $\frac{1}{9}$ of a yard will cost $\frac{1}{8}$ of \$1.572, which may be expressed by writing 8 under

\$1.572 as a denominator, and $\frac{9}{9}$, or a yard, will cost 9 times this result, expressed by making 9 a factor of the numerator. If 1 yard costs so much, $\frac{20}{27}$ of a yard will cost $\frac{20}{27}$ of this, found by making 20 a factor of the numerator, and 27 a factor of the denominator. This gives the following written work:—

$$\$\frac{\overset{\overset{.131}{\cancel{.393}}}{\cancel{1.572}} \times \cancel{9} \times \overset{10}{\cancel{20}}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{27}}} = \$1.310 = \textit{Ans.}$$

NOTE.—Questions like the above can always be resolved into two or more simple ones. Thus: If $\frac{8}{9}$ of a yard of silk costs \$1.572, what will 1 yard cost? If 1 yard costs the last result, what will $\frac{20}{27}$ of a yard cost?

11. How much will $\frac{7}{11}$ of a cord of wood cost, if $\frac{5}{8}$ of a cord cost \$3.85?

12. If a ship sails 157 miles in $\frac{7}{12}$ of a day, how many miles will she sail in $8\frac{3}{4}$ days?

13. A man sold $8\frac{4}{7}$ tons of iron for \$384, and afterwards sold $9\frac{3}{8}$ tons at the same rate. How much did he receive for the last lot?

Suggestion.—Reduce the mixed numbers to improper fractions. The question then will read,—A man sold $\frac{60}{7}$ of a ton of iron for \$384, and afterwards sold $\frac{75}{8}$ of a ton, &c.

14. How much must be paid for $5\frac{5}{6}$ acres of land, when \$868 are paid for $2\frac{2}{3}$ acres?

15. If $6\frac{3}{8}$ barrels of flour cost \$38.25, how much will $14\frac{5}{8}$ barrels cost?

16. If $\frac{7}{8}$ of a dollar will purchase $7\frac{3}{11}$ lb. of coffee, how many pounds can be purchased for $\$5\frac{1}{4}$?

Solution.—Since the answer is to be in pounds, we write $7\frac{3}{11}$, or $\frac{80}{11}$, as the number to be operated on. If $\frac{7}{8}$ of a dollar will purchase $\frac{80}{11}$ lb. of coffee, $\frac{1}{8}$ of a dollar will purchase $\frac{1}{7}$ of $\frac{80}{11}$ lb., and $\frac{8}{8}$, or a dollar, will purchase 8 times this result, and $\$5\frac{1}{4}$, or $\$\frac{21}{4}$, will purchase $\frac{21}{4}$ of this last result. The work, then, would be written thus:—

$$\text{lb.}\frac{80 \times \overset{2}{\cancel{8}} \times \overset{3}{\cancel{21}}}{11 \times \cancel{7} \times \cancel{4}} = \frac{480}{11} = 43\frac{7}{11}\text{lb.} = \textit{Ans.}$$

17. If $12\frac{4}{5}$ yards of calico are given for $17\frac{1}{3}$ yards of sheeting, how many yards of calico must be given for $25\frac{1}{13}$ yards of sheeting?

Suggestion. — Observe that as the answer is to be *yards of calico*, the number of yards of *calico* is that to be operated on.

18. If $1\frac{7}{11}$ tons of hay cost $\$22\frac{7}{8}$, how much will $9\frac{3}{5}$ tons cost?

19. If $13\frac{1}{2}$ lb. coffee are worth as much as $3\frac{3}{5}$ lb. of tea, how many pounds of coffee are worth as much as $7\frac{1}{3}$ lb. of tea?

20. If $8\frac{1}{3}$ yards of silk are worth as much as $24\frac{4}{9}$ yards of gingham, how many yards of gingham can be obtained for $57\frac{1}{7}$ yards of silk?

21. When $3\frac{1}{7}$ yards of silk, worth $\$1\frac{1}{4}$ per yard, are given for $1\frac{2}{3}$ yards of broadcloth, how many dollars should be given for $1\frac{10}{11}$ yards of broadcloth?

147. *Division by Fractions.*

(*a.*) Since $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$, &c., it follows that there must be two times as many halves, three times as many thirds, four times as many fourths, &c., as there are ones in any number.

Or, which is the same thing, —

(*b.*) Since $1 = 2 \times \frac{1}{2}, = 3 \times \frac{1}{3}, = 4 \times \frac{1}{4}$, &c., it follows that there must be twice as many times $\frac{1}{2}$, three times as many times $\frac{1}{3}$, four times as many times $\frac{1}{4}$, five times as many times $\frac{1}{5}$, &c., as there are times 1 in any number.

(*c.*) But the quotient of a number divided by 1 equals the number itself; hence, the quotient of a number divided by $\frac{1}{2}$ must equal twice the number; divided by $\frac{1}{3}$ must equal three times the number; by $\frac{1}{4}$, four times the number; &c.

NOTE. — This is but an application of the principle, that if one number contains another a certain number of times, it will contain half that number twice as many times; $\frac{1}{3}$ of it three times as many times $\frac{1}{4}$ of it four times as many times; &c.

For example, —

$24 \div 12 = 2$; $24 \div \frac{1}{2}$ of 12, or $6, = 2 \times 2$, or 4.

$24 \div \frac{1}{3}$ of 12, or 4, $= 3 \times 2$, or 6;
$24 \div \frac{1}{4}$ of 12, or 3, $= 4 \times 2$, or 8.
In like manner, —
$1 \div 1 = 1$; $1 \div \frac{1}{2}$ of 1, or $\frac{1}{2}$, $= 2 \times 1$, or 2.
$1 \div \frac{1}{3}$ of 1, or $\frac{1}{3}$, $= 3$ times 1, or 3;
$1 \div \frac{1}{4}$ of 1, or $\frac{1}{4}$, $= 4 \times 1$, or 4.
In like manner, —
$7 \div 1 = 7$; $7 \div \frac{1}{2}$ of 1, or $\frac{1}{2}$, $= 2 \times 7$, or 14.
$7 \div \frac{1}{5}$ of 1, or $\frac{1}{5}$, $= 5 \times 7$, or 35;
$7 \div \frac{1}{9}$ of 1, or $\frac{1}{9}$, $= 9 \times 7$, or 63.

1. What is the quotient of $9 \div \frac{1}{4}$?*

First Solution. 9 divided by $1 = 9$, hence, divided by $\frac{1}{4}$, it must equal 4 times 9, or 36.

Second Solution. Since 9 contains 1, 9 times, it must contain $\frac{1}{4}$, 4 times 9 times, or 36 times.

What is the quotient —

2. Of $4 \div \frac{1}{3}$?*	7. Of $4.736 \div .01$?
3. Of $311 \div \frac{1}{5}$?	8. Of $.75 \div .0001$?
4. Of $287 \div \frac{1}{8}$?	9. Of $9000. \div .01$?
5. Of $347 \div \frac{1}{10}$?	10. Of $.0007 \div .001$?
6. Of $347 \div .1$?	11. Of $864 \div .0001$?

12. What is the quotient of $\frac{7}{8} \div \frac{1}{6}$?

First Solution. $\frac{7}{8}$ divided by $1 = \frac{7}{8}$, and divided by $\frac{1}{6}$ must equal 6 times $\frac{7}{8}$, which, by cancelling, $= 3$ times $\frac{7}{4} = \frac{21}{4}$, $= 5\frac{1}{4}$.

Second Solution. Since $\frac{7}{8}$ contains 1, $\frac{7}{8}$ times, it must contain $\frac{1}{6}$, 6 times $\frac{7}{8}$ times, $= 3$ times $\frac{7}{4} = \frac{21}{4} = 5\frac{1}{4}$ times.

By either form of solution, the work may be written thus: —

$$\frac{7}{8} \div \frac{1}{6} = \frac{7 \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{21}{4} = 5\frac{1}{4}$$

What is the quotient —

13. Of $\frac{5}{9} \div \frac{1}{9}$?	16. Of $\frac{15}{16} \div \frac{1}{48}$?
14. Of $\frac{8}{9} \div \frac{1}{12}$?	17. Of $\frac{157}{168} \div \frac{1}{72}$?
15. Of $\frac{17}{24} \div \frac{1}{18}$?	18. Of $\frac{895}{1728} \div \frac{1}{720}$?

* These questions are really equivalent to, "9 = how many fourths?" "4 = how many thirds?" (See **126**, 16th Example.)

19. What is the quotient of $\frac{16}{21} \div \frac{8}{9}$?

First Solution. $\frac{16}{21}$ divided by $1 = \frac{16}{21}$, and by $\frac{1}{9}$ must equal 9 times $\frac{16}{21}$; if it contains $\frac{1}{9}$ so many times, it must contain $\frac{8}{9}$ only $\frac{1}{8}$ as many times.* Hence, —

$$\frac{16}{21} \div \frac{8}{9} = \frac{\overset{2}{\cancel{16}} \times \overset{3}{\cancel{9}}}{\underset{7}{\cancel{21}} \times \cancel{8}} = \frac{6}{7}$$

Second Solution. $\frac{16}{21}$ divided by 1 equals $\frac{16}{21}$, and divided by $\frac{1}{9}$ must equal 9 times $\frac{16}{21}$; if the quotient by $\frac{1}{9}$ equals so much, the quotient by $\frac{8}{9}$ must equal $\frac{1}{8}$ of this result.* Hence, —

$$\frac{16}{21} \div \frac{8}{9} = \frac{\overset{2}{\cancel{16}} \times \overset{3}{\cancel{9}}}{\underset{7}{\cancel{21}} \times \cancel{8}} = \frac{6}{7}, \text{ as before.}$$

What is the quotient —

20. Of $\frac{5}{9} \div \frac{7}{12}$?
21. Of $\frac{18}{25} \div \frac{14}{15}$?
22. Of $8\frac{5}{9}$† $\div \frac{7}{8}$?
23. Of $2\frac{28}{33} \div \frac{7}{11}$?
24. Of $\frac{196}{225} \div \frac{224}{279}$?
25. Of $\frac{486}{675} \div \frac{11}{18}$?
26. Of $824 \div \frac{8}{9}$?
27. Of $617 \div \frac{5}{12}$?
28. Of $675 \div \frac{9}{10}$?

29. What is the quotient of $675 \div .9$?

The work may be written thus: —

$$\begin{aligned} a &= 675 = \text{dividend} \\ 10 \times a = b &= 6750 = \text{quotient by } .1 \\ \tfrac{1}{9} \text{ of } b &= 750 = \text{quotient by } .9 \end{aligned}$$

What is the quotient —

30. Of $8.47 \div .07$?
31. Of $75. \div .05$?
32. Of $75. \div .005$?
33. Of $.75 \div .05$?
34. Of $.075 \div .005$?
35. Of $.00084 \div .0012$?

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* In accordance with the principle, that if a number contains another a certain number of times, it will contain 8 times that number only $\frac{1}{8}$ as many times; 12 times the number only $\frac{1}{12}$ as many times, &c.

Thus, $72 \div 2 = 36$, and $72 \div 6$ times 2, or 12, $= \frac{1}{6}$ of 36, or 6.

$48 \div 3 = 16$, and $48 \div 8$ times 3, or 24, $= \frac{1}{8}$ of 16, or 2.

$7 \div \frac{1}{8} = 56$, and $7 \div 5$ times $\frac{1}{8}$, or $\frac{5}{8}$, $= \frac{1}{5}$ of 56, or $\frac{56}{5} = 11\frac{1}{5}$.

$\frac{5}{9} \div \frac{1}{10} = \frac{50}{9}$, and $\frac{5}{9} \div 7$ times $\frac{1}{10}$, or $\frac{7}{10}$, $= \frac{1}{7}$ of $\frac{50}{9}$, or $\frac{50}{63}$.

† *Reduce* to an improper fraction.

148. *Process of Division generalized.*

(*a.*) Since the quotient of any number divided by $\frac{1}{5} = 5$ times the number; by $\frac{1}{9} = 9$ times the number; by $\frac{1}{13} =$ 13 times the number, &c.; and since the quotient of a number divided by $\frac{3}{5} = \frac{1}{3}$ its quotient divided by $\frac{1}{5}$; divided by $\frac{8}{9} = \frac{1}{8}$ of its quotient divided by $\frac{1}{9}$; divided by $\frac{11}{13} = \frac{1}{11}$ of its quotient divided by $\frac{1}{13}$, &c., it follows that the quotient of a number divided by —

$\frac{3}{5} = \frac{1}{3}$ of 5 times the number, $= \frac{5}{3}$ of the number;
$\frac{8}{9} = \frac{1}{8}$ of 9 times the number, $= \frac{9}{8}$ of the number;
$\frac{11}{13} = \frac{1}{11}$ of 13 times the number, $= \frac{13}{11}$ of the number;
$.07 = \frac{1}{7}$ of 100 times the number $= \frac{100}{7}$ of the number;

and, universally, that —

The quotient of any number divided by a fraction, is equal to the product of that number multiplied by the fraction inverted.

Illustrations. — 1. To divide by $\frac{8}{9}$, we have only to multiply by 9 and divide by 8.

2. To divide by $\frac{3}{17}$, we have only to multiply by 17 and divide by 3.

3. To divide by .03, we have only to multiply by 100 and divide by 3, or, which is the same thing, to divide by 3 and remove the point 2 places towards the right.

4. To divide by .000037, we have only to multiply by 1000000, and divide by 37, or, which is the same thing, to divide by 37, and remove the point 6 places towards the right.

1. What is the quotient of $\frac{5}{11} \div \frac{10}{13}$?

$$\textit{Ans.}\quad \frac{5}{11} \div \frac{10}{13} = \frac{\cancel{5} \times 13}{11 \times \underset{2}{\cancel{10}}} = \frac{13}{22}$$

2. What is the quotient of $520.6 \div .011$?

Ans. — The quotient of $520.6 \div .011$ may be obtained by dividing 520.6 by 11 and removing the point three places towards the right, thus: —

First Form.	*Second Form.*
.011) 520.6	.011) 520.600
$47327\frac{3}{11}$	$47327\frac{3}{11}$

NOTE. — The second form of writing the work differs from the first only in this, — that in it as many zeros are annexed to the dividend as would be necessary were the point actually changed before performing the division. Great care is necessary, by either of these forms, to insure that the point is placed correctly in the quotient; and if in any case there is a doubt as to its true position, the work should be written in full, as in the model given after example 29th, **147.**

3. What is the quotient of $\frac{4}{5}$ of $\frac{7}{9}$ of $\frac{8}{11} \div \frac{8}{9}$ of $\frac{12}{25}$ of $\frac{21}{22}$?

Solution. —

$$\frac{4}{5} \text{ of } \frac{7}{9} \text{ of } \frac{8}{11} \div \frac{8}{9} \text{ of } \frac{12}{25} \text{ of } \frac{21}{22} = \left(\frac{4 \times 7 \times 8}{5 \times 9 \times 11} \div \frac{8 \times 12 \times 21}{9 \times 25 \times 22}\right)^* = \frac{\cancel{4} \times \cancel{7} \times \cancel{8} \times \cancel{9} \times \overset{5}{\cancel{25}} \times \overset{2}{\cancel{22}}}{\cancel{5} \times \cancel{9} \times \cancel{11} \times \cancel{8} \times \underset{3}{\cancel{12}} \times \underset{3}{\cancel{21}}} = \frac{10}{9} = 1\frac{1}{9}$$

NOTE. — The more full and analytical explanation would be the following: $\frac{8}{11}$ is the number to be operated upon. $\frac{7}{9}$ of $\frac{8}{11}$ may be expressed by making 7 a factor of the numerator, and 9 a factor of the denominator. $\frac{4}{5}$ of this may be expressed by making 4 a factor of the numerator, and 5 a factor of the denominator. The quotient of this quantity divided by $\frac{21}{22}$ will be $\frac{22}{21}$ of it, and may be expressed by making 22 a factor of the numerator, and 21 a factor of the denominator. If $\frac{21}{22}$ is contained so many times, $\frac{12}{25}$ of $\frac{21}{22}$ must be contained $\frac{25}{12}$ as many times, expressed by making 25 a factor of the numerator, and 12 a factor of the denominator. If this divisor ($\frac{12}{25}$ of $\frac{21}{22}$) is contained so many times, $\frac{8}{9}$ of it must be contained $\frac{9}{8}$ as many times, expressed by making 9 a factor of the numerator, and 8 a factor of the denominator. Hence, —

$$\frac{4}{5} \text{ of } \frac{7}{9} \text{ of } \frac{8}{11} \div \frac{8}{9} \text{ of } \frac{12}{25} \text{ of } \frac{21}{22} =$$

$$\frac{\cancel{8} \times \cancel{7} \times \cancel{4} \times \overset{2}{\cancel{22}} \times \overset{5}{\cancel{25}} \times \cancel{9}}{\cancel{11} \times \cancel{9} \times \cancel{5} \times \underset{3}{\cancel{21}} \times \underset{3}{\cancel{12}} \times \cancel{8}} = \frac{10}{9} = 1\frac{1}{9}$$

* The equation in the parenthesis may be omitted in practical operations

What is the quotient —

4. Of $13\frac{1}{7} \div 8\frac{13}{30}$?	7. Of $.004 \div 25$?
5. Of $11\frac{2}{7} \div 4\frac{1}{2}$?	8. Of $.067 \div .02$?
6. Of $16\frac{2}{3} \div 28\frac{1}{7}$?	9. Of $3287 \div .0004$?

10. Of $\frac{5}{7}$ of $\frac{11}{12}$ of $\frac{25}{36} \div \frac{15}{26}$ of $\frac{55}{108}$ of $\frac{8}{35}$?
11. Of $\frac{3}{8}$ of $\frac{12}{7}$ of $\frac{8}{16} \div \frac{4}{7}$ of $\frac{9}{10}$ of $6\frac{1}{4}$?
12. Of $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7} \div \frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$?
13. Of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{8}{9} \div \frac{7}{10}$ of $\frac{12}{13}$ of $1\frac{1}{4}$?

149. *Complex Fractions.*

(*a.*) A COMPLEX FRACTION is one having a fractión in either numerator or denominator, or in both; as, $\frac{7}{3\frac{2}{3}}$, $\frac{2\frac{1}{2}}{4\frac{1}{6}}$, $\frac{\frac{2}{3}}{7\frac{3}{5}}$.

NOTE. — Complex fractions are usually considered as expressions of unexecuted divisions, and are read accordingly. Thus, —

$$\frac{7}{3\frac{2}{3}} = 7 \div 3\tfrac{2}{3}; \quad \frac{2\frac{1}{2}}{4\frac{1}{6}} = 2\tfrac{1}{2} \div 4\tfrac{1}{6}; \quad \frac{\frac{2}{3}}{7\frac{3}{5}} = \tfrac{2}{3} \div 7\tfrac{3}{5}$$

(*b.*) To show their similarity to other fractions, we may explain them thus: —

$\frac{7}{3\frac{2}{3}}$ = 7 parts of such kind that $3\frac{2}{3}$ of them would equal a unit.

(*c.*) Complex fractions can be reduced to simple fractions by the ordinary process of division.

1. Reduce $\frac{5\frac{1}{7}}{9\frac{1}{4}}$ to a simple fraction.

Solution. $\frac{5\frac{1}{7}}{9\frac{1}{4}} = \frac{36}{7}$* $\div \frac{37}{4} = \frac{36 \times 4}{7 \times 37} = \frac{144}{259}$.

Reduce each of the following to simple fractions.

2. $\frac{7\frac{1}{2}}{11\frac{2}{3}}$.	5. $\frac{8\frac{1}{6}}{12\frac{1}{4}}$.	8. $\frac{7\frac{1}{2}}{8}$.
3. $\frac{2\frac{1}{4}}{4\frac{4}{5}}$.	6. $\frac{6\frac{2}{3}}{4\frac{2}{7}}$.	9. $\frac{9\frac{1}{6}}{11}$.
4. $\frac{4\frac{7}{8}}{7\frac{2}{9}}$.	7. $\frac{13\frac{1}{2}}{11\frac{1}{4}}$.	10. $\frac{4\frac{2}{3}}{5\frac{3}{5}}$.

* By reducing $5\frac{1}{7}$ to sevenths, and $9\frac{1}{4}$ to fourths.

(*d.*) Complex fractions may often be reduced to simple ones, by reducing them to their lowest terms, —

Thus: Dividing both terms of $\frac{3}{4\frac{1}{2}}$ by $1\frac{1}{2}$ gives $\frac{3}{4\frac{1}{2}} = \frac{2}{3}$. Dividing both terms of $\frac{1\frac{2}{3}}{5}$ by $1\frac{2}{3}$ gives $\frac{1\frac{2}{3}}{5} = \frac{1}{3}$.

Reduce each of the following in the same manner: —

11. $\frac{2\frac{1}{2}}{5}$.	14. $\frac{8}{2\frac{2}{3}}$.	17. $\frac{6\frac{2}{3}}{16\frac{2}{3}}$.
12. $\frac{1\frac{6}{7}}{6\frac{2}{7}}$.	15. $\frac{1\frac{4}{5}}{9}$.	18. $\frac{8\frac{2}{5}}{9\frac{4}{5}}$.
13. $\frac{7\frac{1}{2}}{2\frac{1}{2}}$.	16. $\frac{3\frac{3}{4}}{6\frac{1}{4}}$.	19. $\frac{4\frac{2}{7}}{7\frac{1}{2}}$.

(*e.*) Complex fractions may also be reduced to simple ones, by multiplying both numerator and denominator by such a number as will give a whole number in place of each.

20. Reduce $\frac{4\frac{2}{3}}{10\frac{1}{2}}$ to a simple fraction.

Solution. — If $4\frac{2}{3}$ be multiplied by 3, or some multiple of 3, and $10\frac{1}{2}$ be multiplied by 2, or some multiple of 2, the result will in each case be a whole number. Hence, if both terms of the fraction $\frac{4\frac{2}{3}}{10\frac{1}{2}}$ be multiplied by some multiple of both 2 and 3, the resulting fraction will be a simple one. Multiplying by 6 gives $\frac{4\frac{2}{3}}{10\frac{1}{2}} = \frac{28}{63} = \frac{4}{9}$.

In the same way reduce each of the following complex fractions to simple ones: —

21. $\frac{3\frac{1}{2}}{5\frac{5}{6}}$.	24. $\frac{7\frac{1}{5}}{13\frac{1}{2}}$.	27. $\frac{10\frac{5}{7}}{83\frac{1}{4}}$.
22. $\frac{2\frac{1}{4}}{7\frac{1}{3}}$.	25. $\frac{435\frac{2}{3}}{627\frac{3}{4}}$.	28. $\frac{163\frac{5}{6}}{847}$.
23. $\frac{4\frac{1}{6}}{8\frac{1}{3}}$.	26. $\frac{48\frac{1}{3}}{64\frac{3}{7}}$.	29. $\frac{47\frac{3}{4}}{28\frac{5}{6}}$.

150. *Other Changes in the Terms of a Fraction.*

1. Reduce $\frac{3}{4}$ to an equivalent fraction, having 6 for a numerator.

Solution. — Observing that the *proposed numerator*, 6, is two times the *given numerator*, 3, we multiply both terms by 2, which gives $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$, or we may at once write $\frac{3}{4} = \frac{6}{8}$.

2. Reduce $\frac{8}{9}$ to an equivalent fraction, having 10 for its numerator.

Solution. — Observing that the *proposed* numerator, 10, is $\frac{10}{8}$, or $\frac{5}{4}$, of the *given* numerator, 8, we multiply both terms by $\frac{5}{4}$, or by $1\frac{1}{4}$, which gives $\frac{8}{9} = \frac{8 \times 1\frac{1}{4}}{9 \times 1\frac{1}{4}} = \frac{10}{11\frac{1}{4}}$, or $\frac{8}{9} = \frac{10}{11\frac{1}{4}}$.

3. Reduce $\frac{5}{7}$ to an equivalent fraction, having 10 for its numerator.

4. Reduce $\frac{8}{12}$ to an equivalent fraction, having 9 for its numerator.

5. Reduce $\frac{8}{10}$ to an equivalent fraction, having 6 for its numerator.

6. Reduce $\frac{2}{7}$ to twenty-firsts.

Solution. — Observing that the proposed denominator, 21, is three times the given denominator, 7, we have only to multiply both terms by 3, which gives $\frac{2}{7} = \frac{2 \times 3}{7 \times 3} = \frac{6}{21}$, or, omitting to write the intermediate work, we have $\frac{2}{7} = \frac{6}{21}$.

Note. — The same result might have been obtained thus: — $1 = \frac{21}{21}$, hence $\frac{2}{7}$ of 1 must equal $\frac{2}{7}$ of $\frac{21}{21}$; $\frac{1}{7}$ of $\frac{21}{21}$ is $\frac{3}{21}$, and two times $\frac{3}{21} = \frac{6}{21}$; but, in practice, the first form will usually be found most convenient.

7. Reduce $\frac{2}{7}$ to fifteenths.

Solution. — Observing that the proposed denominator, 15, is $\frac{15}{7}$ of the given denominator, 7, we have only to multiply both terms by $\frac{15}{7}$, or $2\frac{1}{7}$.

Hence, $\frac{2}{7} = \frac{2 \times 2\frac{1}{7}}{7 \times 2\frac{1}{7}} = \frac{4\frac{2}{7}}{15}$, or $\frac{2}{7} = \frac{4\frac{2}{7}}{15}$.

8. Reduce $\frac{5}{9}$ to halves.

Solution. — Observing that the proposed denominator, 2, is $\frac{2}{9}$ of the given denominator, 9, we have only to multiply both terms by $\frac{2}{9}$. Hence, $\frac{5}{9} = \frac{5 \times \frac{2}{9}}{9 \times \frac{2}{9}} = \frac{1\frac{1}{9}}{2}$, or $\frac{5}{9} = \frac{1\frac{1}{9}}{2}$.

Reduce —

9. $\frac{2}{3}$ to ninths.
10. $\frac{7}{12}$ to fourths.
11. $\frac{8}{15}$ to forty-fifths.
12. $\frac{5}{8}$ to twelfths.
13. $\frac{6}{7}$ to forty-ninths.
14. $\frac{4}{5}$ to thirtieths.
15. $\frac{7}{9}$ to twenty-sevenths.
16. $\frac{5}{11}$ to ninths.
17. $\frac{4}{13}$ to sixty-fifths.
18. $\frac{7}{11}$ to twenty-seconds.

151. *Reduction to a Common Denominator.*

(*a.*) Fractions having their denominators alike are said to have a COMMON DENOMINATOR.

Thus, $\frac{4}{7}$, $\frac{3}{7}$, $\frac{6}{7}$, and $\frac{1}{7}$ have a common denominator, but $\frac{4}{7}$ and $\frac{2}{3}$ have not.

(*b.*) In reducing fractions having different denominators to a common denominator, (i. e., to equivalent ones having the same denominator,) we first select a convenient number for the common denominator, and then make the reductions as in the last article.

(*c.*) As far as the denominator is concerned, one number may as well be selected for a common denominator as another; but unless the number selected is a common multiple of all the given denominators, one or more of the resulting numerators will be likely to contain a fraction.*

(*d.*) To avoid such an inconvenience, and at the same time to avoid as far as possible the use of large numbers, it will usually be best to select the least common multiple of the given denominators for a common denominator.

1. Reduce $\frac{7}{8}$, $\frac{5}{12}$, $\frac{4}{9}$, and $\frac{13}{24}$ to a common denominator.

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* For it will be the product of a whole number multiplied by a fractional quantity.

Solution. — We select 72 for the least common denominator, because it is the least common multiple of the given denominators.

Then, since 72 = 9 times 8, we multiply both terms of the fraction $\frac{7}{8}$ by 9. which gives $\frac{7}{8} = \frac{63}{72}$. Since 72 = 6 times 12, we multiply both terms of the fraction $\frac{5}{12}$ by 6, which gives, &c.

Hence, $\frac{7}{8} = \frac{63}{72}$; $\frac{5}{12} = \frac{30}{72}$; $\frac{4}{9} = \frac{32}{72}$; and $\frac{13}{24} = \frac{39}{72}$.

(*e*.) Many adopt it as a general rule, to select the product of the given denominators for a common denominator; but it usually involves larger numbers than the preceding method, and is hence much less convenient. The following illustrates it.

Solution to preceding Example. — The product of 8, 9, 12, and 24, the given denominators, is 20736, which we select for the common denominator. To get this, we multiplied 8, the denominator of $\frac{7}{8}$, by 9 $\times$ 12 $\times$ 24, and therefore we multiply the numerator, 7, by the same numbers, which gives $\frac{7}{8} = \frac{18144}{20736}$. To obtain 20736, we multiplied 12, the denominator of $\frac{5}{12}$, by 8 $\times$ 9 $\times$ 24, the product of the other denominators, and therefore we multiply the numerator, 5, by the same numbers, which gives $\frac{5}{12} = \frac{8640}{20736}$. To obtain 20736, we multiplied 9, the denominator of $\frac{4}{9}$, by, &c.

(*f*.) When any of the fractions to be reduced are compound or complex, they must first be reduced to simple ones, and the simple fractions should be reduced to their lowest terms, except when to do it would increase the labor of reducing to a common denominator.

(*g*.) Reduce the fractions in each of the following examples to a common denominator: —

2. $\frac{4}{5}, \frac{2}{3}, \frac{5}{6},$ and $\frac{7}{15}$.

3. $\frac{3}{7}, \frac{5}{9}, \frac{1}{2}, \frac{1}{6},$ and $\frac{4}{21}$.

4. $\frac{19}{24}, \frac{23}{36}, \frac{41}{72}, \frac{37}{108},$ and $\frac{49}{216}$.

5. $\frac{24}{25}, \frac{7}{9}, \frac{14}{15}, \frac{37}{225},$ and $\frac{44}{75}$.

6. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6},$ and $\frac{1}{12}$.

7. $\frac{5}{7}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{1}{2}$.

8. $\frac{13}{14}$, $\frac{13}{21}$, $\frac{13}{28}$, and $\frac{13}{25}$.

9. $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{5}{6}$ of $\frac{8}{15}$, and $\frac{2}{9}$ of $\frac{12}{16}$.

10. $\frac{3\frac{12}{17}}{8\frac{2}{5}}$, $\frac{7}{11}$ of $7\frac{1}{3}$, and $\frac{7}{5\frac{1}{2}}$.

11. $\frac{3\frac{1}{2}}{4\frac{2}{3}}$, $\frac{5}{6}$, $\frac{8\frac{2}{5}}{9\frac{5}{8}}$, and $\frac{7}{9}$.

12. $\frac{4}{5}$ of $\frac{15}{16}$, $\frac{8}{32}$, $\frac{5\frac{1}{2}}{16\frac{1}{2}}$, $\frac{8\frac{4}{5}}{7\frac{1}{9}}$, and $\frac{16}{32}$.

13. $\frac{7}{11}$, $\frac{12}{16}$, $\frac{13}{22}$, $\frac{8}{9}$ of $\frac{21}{22}$, $\frac{4\frac{1}{2}}{9}$, and $\frac{16\frac{2}{3}}{100}$.

152. *Addition and Subtraction of Fractions.*

(*a.*) Fractions, like other numbers, must be of the same denomination, in order to be added or subtracted.

(*b.*) To be of the same denomination, they must (**121**, *e.*) be fractions of the same unit, and also have a common denominator.

$\frac{3}{7} + \frac{4}{9}$ cannot be added in their present form, any more than can 3 pounds and 4 ounces.

$\frac{3}{7}$ of a yard and $\frac{3}{7}$ of an inch cannot be added in their present form, any more than can 3 yards and 3 inches.

(*c.*) When abstract fractions are given, (i. e., fractions of the abstract unit, as $\frac{3}{4}$, $\frac{5}{7}$,) they may, if simple, be reduced at once to a common denominator; but compound and complex fractions must be reduced to simple ones, and fractional parts of denominate numbers, as $\frac{3}{7}$ of a yard, $\frac{5}{9}$ of a foot, must be reduced to fractions of the same unit, before they are reduced to a common denominator.

1. What is the sum of $\frac{7}{8} + \frac{5}{6} + \frac{2}{3} + \frac{11}{12} + \frac{3}{4}$?

Solution. — By reducing to a common denominator, we have —

$$\frac{7}{8}+\frac{5}{6}+\frac{2}{3}+\frac{11}{12}+\frac{3}{4}=\frac{21}{24}+\frac{20}{24}+\frac{16}{24}+\frac{22}{24}+\frac{18}{24}=$$

$$\frac{21+20+16+22+18}{24}=4\tfrac{1}{24}.$$

2. What is the sum of $87\frac{1}{2}+13\frac{5}{6}+27\frac{4}{9}+37\frac{5}{18}+43\frac{7}{12}$?

Solution. — The sum of the whole numbers is 212. Reducing the fractions to a common denominator, we have $\frac{1}{2}+\frac{5}{6}+\frac{4}{9}+\frac{5}{18}+\frac{7}{12}=\frac{18}{36}+\frac{30}{36}+\frac{16}{36}+\frac{10}{36}+\frac{21}{36}=2\frac{23}{36}$, which added to $212=214\frac{23}{36}$.

The following arrangement of the numbers shows the resemblance of the work to compound addition: —

		Ones.	36ths.
$87\frac{1}{2}$	=	87	18
$13\frac{5}{6}$	=	13	30
$27\frac{4}{9}$	=	27	16
$37\frac{5}{18}$	=	37	10
$48\frac{7}{12}$	=	48	21
		214	23 $= 214\frac{23}{36}$.

3. What is the sum of $\frac{7}{12}$ of a ton $+\ \frac{5}{6}$ of a quarter?

Solution. — By reducing the $\frac{7}{12}$ of a ton to cwt. and qrs., and adding the $\frac{5}{6}$ of a quarter, we should have the following written work: —

$\frac{7}{12}$ T. $+\ \frac{5}{6}$ qr. $=$ 11 cwt. $2\frac{2}{3}$ qr. $+\ \frac{5}{6}$ qr. $=$ 11 cwt. $3\frac{1}{2}$ qr. $=$ 11 cwt. 3 qr. 12 lb. 8 oz.

4. What is the value of $343\frac{2}{9}-138\frac{23}{24}$?

Solution. — By reducing the fractions to a common denominator, we have $343\frac{2}{9}-138\frac{23}{24}=343\frac{16}{72}-138\frac{69}{72}=204\frac{19}{72}$.

Or the work may be written thus: —

		Ones.	72ds.	
$343\frac{2}{9}$	=	343	16	= Min.
$138\frac{23}{24}$	=	138	69	= Sub.
		204	19	$= 204\frac{19}{72}$ = Rem.

(*d.*) When several fractions are to be added, it will oftentimes be a saving of labor to consider at first only two of them, and then a third with the sum of these two, and then a fourth with the sum of these, and so on, till all are added. Care should be taken to couple them in such a way as to make the reductions easy.

Thus, in the 24th example below, $\frac{5}{6}+\frac{2}{3}=1\frac{1}{2}$; $1\frac{1}{2}+\frac{1}{2}=2$; and $2+\frac{3}{4}=2\frac{3}{4}=$ *Ans.*

Perform the following examples: —

5. $\frac{7}{8}+\frac{9}{10}$.

6. $\frac{6}{7}+\frac{3}{9}$.

7. $\frac{4}{8}+\frac{8}{12}$.

8. $4\frac{1}{2}+2\frac{2}{3}$.

9. $3\frac{2}{9}+1\frac{5}{6}$.

10. $\frac{1}{2}-\frac{1}{3}$.

11. $\frac{3}{4}-\frac{2}{3}$.

12. $\frac{7}{8}-\frac{7}{9}$.

13. $3\frac{1}{5}-2\frac{4}{7}$.

14. $5\frac{3}{4}-3\frac{5}{6}$.

15. $\frac{1}{4}+\frac{1}{5}$.

16. $\frac{1}{4}-\frac{1}{5}$.

17. $\frac{35}{52}-\frac{35}{78}$.

18. $2\frac{4\frac{1}{2}}{5\frac{1}{3}}-\frac{15}{16}$.

19. $8\frac{14}{17\frac{1}{9}}-4\frac{10}{11}$.

20. $\frac{18}{25}+\frac{7}{15}$.

21. $\frac{13}{18}-\frac{1}{4}$.

22. $\frac{27}{30}-\frac{9}{10}$.

23. $\frac{3\frac{4}{7}}{6\frac{1}{2}}+\frac{5\frac{1}{6}}{7\frac{4}{9}}$.

24. $\frac{5}{6}+\frac{3}{4}+\frac{2}{3}+\frac{1}{2}$.

25. $9\frac{7}{8}-8\frac{8}{9}$.

26. $\frac{5}{6}+3\frac{2}{9}+7\frac{1}{2}+\frac{7}{12}$.

27. $\frac{7}{9}+\frac{3}{8}+\frac{5}{12}+\frac{17}{36}$.

28. $9\frac{3}{5}+\frac{1}{9}+4\frac{17}{25}+\frac{49}{75}$.

29. $\frac{5}{12}+\frac{5}{6}+\frac{3}{4}+\frac{2}{3}$.

30. $5\frac{5}{9}+4\frac{6}{13}+\frac{5}{18}+\frac{2}{3}$.

31. $2\frac{1}{2}+3\frac{1}{3}-4\frac{1}{4}+5\frac{1}{6}-\frac{3}{8}$.

32. $\frac{11}{12}$ of $\frac{8}{9}$ of $\frac{1}{2}+4\frac{3}{4}+1\frac{5}{12}+7$.

33. $\frac{17}{24}+\frac{95}{96}-\frac{4}{9}$ of $\frac{7}{8}$ of $\frac{4\frac{1}{2}}{8}$ of $\frac{2}{3}$.

34. $\frac{2}{3}$ of $\frac{9}{10}+\frac{6}{7}$ of $\frac{2}{3}$ of $4\frac{2}{3}-\frac{4}{5}$ of $\frac{7}{8}$.

35. $\frac{357}{853}+\frac{294}{671}+\frac{423}{849}$.

36. $3\frac{6\frac{2}{3}}{11\frac{3}{7}}+\frac{2\frac{7}{9}}{14\frac{2}{7}}-\frac{5}{6}-\frac{2\frac{1}{2}}{12\frac{1}{2}}$ of $\frac{5}{4\frac{4}{5}}$.

37 $\frac{47}{91} + \frac{83}{77} + \frac{79}{143} + \frac{3\frac{1}{2}}{38\frac{1}{2}} + \frac{8\frac{5}{7}}{61} + \frac{4\frac{2}{13}}{54} + \frac{5}{8}$ of $\frac{12}{13}$ of $\frac{10\frac{3}{7}}{73}$ of $\frac{2}{3}$.

38. $\frac{6}{7}$ of a bu. + $\frac{1}{2}$ of a peck.

39. $\frac{8}{11}$ of a m. + $\frac{5}{6}$ of a furlong.

40. $\frac{17}{32}$ of a £ — $\frac{2}{3}$ of a shilling.

41. $\frac{7}{9}$ of a lb. + $\frac{3}{4}$ of a dwt. + $\frac{1}{2}$ of a grain.

42. $\frac{4}{5}$ of a bu. + $\frac{5}{9}$ of a pk. — $\frac{12\frac{1}{4}}{22\frac{1}{2}}$ of a quart.

43. $\frac{5}{6}$ of a yd. + $\frac{2}{3}$ of a qr. + $\frac{2}{3}$ of a nail.

SECTION XI.

APPLICATIONS OF FOREGOING PRINCIPLES.

153. *Introductory Note.*

In performing the examples of this section, as of every other, the student should observe that the solutions and models of writing the work are designed to suggest methods of applying principles, and are not to be regarded as forms which must be rigidly followed. They should be deviated from when better forms can be discovered, or will apply. Indeed, a habit of examining each question carefully before attempting its numerical solution is one of the most valuable a pupil can acquire. A distinguished teacher once said, "If I were required, on peril of my life, to perform a complicated problem in two minutes, I would spend the first minute in considering how to do it."

154. *Miscellaneous Problems.*

1. How much will $7\frac{7}{8}$ yards of silk cost at $\$1\frac{7}{9}$ per yard?

2. If $7\frac{7}{8}$ yards of silk cost \$14, how much will 1 yard cost?

3. How many yards of silk, at $\$1\frac{7}{9}$ per yard, can be bought for \$14?

4. How much will a lot of land $28\frac{3}{4}$ rods long and $23\frac{1}{5}$ rods wide cost, at $\$2\frac{8}{23}$ per acre?

5. How many pounds of coffee, at $15\frac{3}{4}$ cents per lb., can be bought for \$8.40?

6. How many square feet and inches are there in a board $13\frac{1}{3}$ feet long and $2\frac{5}{12}$ feet wide?

7. How many square feet in the four walls of a room, $15\frac{1}{3}$ feet long, $12\frac{3}{4}$ feet wide, and $8\frac{1}{2}$ feet high?

8. I bought a cask, containing $94\frac{1}{2}$ gallons of oil, at \$1.375 per gallon; $\frac{3}{7}$ of it leaked out, and I sold the remainder at \$1.50 per gallon. How much did I lose by the transaction?

9. Mr. Whitney is worth \$1473.21 more than Mr. Whipple, and Mr. Whipple is worth just $\frac{3}{4}$ as much as Mr. Whitney. How many dollars is each of them worth?

Suggestion. — If Mr. Whipple is worth only $\frac{3}{4}$ as much as Mr. Whitney, the difference between the values of their estates must equal $\frac{1}{4}$ of Mr. Whitney's estate.

10. What is the value of $\frac{5}{7}$ of $\frac{8\frac{1}{2}}{9\frac{2}{3}}$ of $\frac{3\frac{2}{9}}{11} \div \frac{2\frac{3}{7}}{5\frac{1}{2}}$ of $\frac{3}{4}$ of $\frac{2\frac{1}{11}}{6\frac{3}{11}}$ of $\frac{6\frac{2}{3}}{1\frac{2}{3}}$?

11. What is the value of $\frac{4}{5} + \frac{7}{8} - \frac{9}{10} + 3\frac{1}{2} + \frac{2}{3} - \frac{3}{5} + \frac{3}{4} + \frac{4}{15}$?

12. If $8\frac{1}{2}$ yards of broadcloth cost $\$29\frac{3}{8}$, how much will $10\frac{1}{5}$ yards cost?

13. If $8\frac{1}{2}$ yards of broadcloth can be purchased for $\$29\frac{3}{8}$, how many yards can be purchased for $\$35\frac{1}{4}$?

14. If $10\frac{1}{5}$ yards of broadcloth cost $\$35\frac{1}{4}$, how much will $8\frac{1}{2}$ yards cost?

15. If $\$35\frac{1}{4}$ are paid for $10\frac{1}{5}$ yards of broadcloth, for how many yards should $\$29\frac{3}{8}$ be paid?

16. If $\frac{11}{12}$ of a ton of hay costs $17.50, how much will two loads cost, one weighing $\frac{5}{6}$ of a ton, and the other $\frac{13}{24}$ of a ton?

17. If 22 horses eat $41\frac{1}{4}$ bushels of grain in $10\frac{1}{2}$ days, how many bushels will 21 horses eat in $9\frac{1}{3}$ days?

Solution. — As the answer is to be in bushels of grain, we write $41\frac{1}{4}$, or $\frac{165}{4}$, as the number to be operated on. If 22 horses eat $\frac{165}{4}$ bushels, 1 horse will eat $\frac{1}{22}$ as much in the same time, which may be expressed by making 22 a factor of the denominator; thus, $\frac{165}{4 \times 22}$. If 1 horse will eat this quantity in $10\frac{1}{2}$, or $\frac{21}{2}$, days, in $\frac{1}{2}$ day he will eat $\frac{1}{21}$ of this, and in 2 half-days, or a day, he will eat twice this result, expressed by making 21 a factor of the denominator, and 2 a factor of the numerator; thus, $\frac{165 \times 2}{4 \times 22 \times 21}$.* If 1 horse eats this quantity, 21 horses will in the same time eat 21 times this, expressed by making 21 a factor of the numerator; thus, $\frac{165 \times 2 \times 21}{4 \times 22 \times 21}$.† If 21 horses eat so much in 1 day, in $9\frac{1}{3}$, or $\frac{28}{3}$, days, they will eat $\frac{28}{3}$ of this, which may be expressed by making 28 a factor of the numerator, and 3 a factor of the denominator; thus, $\frac{165 \times 2 \times 21 \times 28}{4 \times 22 \times 21 \times 3}$.‡

Hence the written work and answer are as follows: —

$$\text{bu.}\ \frac{\overset{\overset{5}{\cancel{55}}}{\cancel{165}} \times \cancel{2} \times \cancel{21} \times \overset{7}{\cancel{28}}}{\cancel{4} \times \underset{\cancel{2}}{\cancel{22}} \times \cancel{21} \times \cancel{3}} = 35 \text{ bu}$$

NOTE. — It will be seen that in the solution, each condition of the question was considered by itself, without reference to the other conditions, and that the problem was dealt with as though composed of the following series of simpler problems: —

1. If 22 horses eat $41\frac{1}{4}$ bushels of grain in a given time, how many bushels would 1 horse eat in the same time?

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* This shows how many bushels 1 horse will eat in 1 day.

† This shows how many bushels 21 horses will eat in 1 day.

‡ This shows how many bushels 21 horses will eat in $9\frac{1}{3}$ days.

2. If 1 horse eats the quantity obtained as the answer to the last question in $10\frac{1}{2}$ days, how many bushels will he eat in 1 day ?

3. If 1 horse eats so much (the answer last obtained) in a day, how many bushels will 21 horses eat in the same time ?

4. If 21 horses eat so much (the answer last obtained) in 1 day, how many bushels will they eat in $9\frac{1}{3}$ days ?

The student should notice that the question in each case has been, "How many bushels ?" and that the denomination of each answer has been the same as that required for the final answer.

18. If 22 horses can be kept $10\frac{1}{2}$ days on $41\frac{1}{4}$ bushels of grain, how many horses can be kept $9\frac{1}{3}$ days on 35 bushels of grain.

19. If 22 horses can be kept $10\frac{1}{2}$ days on $41\frac{1}{4}$ bushels of grain, how many days can 21 horses be kept on 35 bushels of grain ?

20. If 21 horses eat 35 bushels of grain in $9\frac{1}{3}$ days, how many bushels will 22 horses eat in $10\frac{1}{2}$ days ?

21. If 21 horses can be kept $9\frac{1}{3}$ days on 35 bushels of grain, how many days can 22 horses be kept on $41\frac{1}{4}$ bushels of grain ?

22. If 21 horses can be kept $9\frac{1}{3}$ days on 35 bushels of grain, how many horses can be kept $10\frac{1}{2}$ days on $41\frac{1}{4}$ bushels of grain ?

23. If 7 men, by working $8\frac{1}{4}$ hours per day during 6 days of the week, can earn $\$320\frac{5}{8}$ in $5\frac{1}{2}$ weeks, how many dollars would 4 men earn in $4\frac{4}{5}$ weeks, by working $9\frac{1}{3}$ hours per day, and 5 days per week ?

24. If it costs \$9.25 to dig a ditch 47 feet long, $1\frac{2}{3}$ deep, and 3 feet wide, how much will it cost to dig a ditch $70\frac{1}{2}$ feet long, $2\frac{1}{2}$ feet deep, and $4\frac{1}{2}$ feet wide ?

25. If, when flour is $\$6\frac{2}{3}$ per barrel, a six-cent loaf weighs 15 oz., how many ounces ought a ten-cent loaf to weigh when flour is $\$9\frac{3}{8}$ per barrel ?

26. If a block of oak, $3\frac{3}{4}$ feet long, 1 foot wide, and $\frac{3}{4}$ of a foot thick, weighs $120\frac{4}{5}$ lb., how much will a block of pine, 8 feet long, 2 feet wide, and $1\frac{2}{3}$ feet thick weigh, pine being only $\frac{2}{3}$ as heavy as oak ?

27. If 9 men, by working 10 hours per day during 6 days

of the week, can in 4 weeks dig a trench 450 feet long, $3\frac{1}{2}$ feet wide, and $2\frac{2}{3}$ feet deep, how many men, working $9\frac{1}{3}$ hours per day during 5 days of the week, can in 9 weeks dig a trench 539 feet long, $6\frac{1}{4}$ feet wide, and $2\frac{10}{11}$ feet deep?

28. If 12 men, by working $9\frac{1}{3}$ hours per day during 5 days of the week, can in 9 weeks dig a trench 539 feet long, $6\frac{1}{4}$ feet wide, and $2\frac{10}{11}$ feet deep, how many weeks would it take 9 men, working 10 hours per day during 6 days of the week, to dig a trench 450 feet long, $3\frac{1}{2}$ feet wide, and $2\frac{2}{3}$ feet deep?

29. If 36 lb. of sugar are worth 24 lb. of coffee, and 22 lb. of coffee are worth 55 lb. of rice, how many pounds of rice can be bought for 16 lb. of sugar?

Preliminary Explanation. — As the answer is to be pounds of rice, we must write 55, the given number of pounds of rice, as the number to be operated on, and must take care to express the values throughout in pounds of rice. We should then have the following

Solution. — Since 22 lb. of coffee are worth 55 lb. of rice, 1 lb. of coffee must be worth $\frac{1}{22}$ of 55 lb. of rice, which may be expressed by writing 22 as a denominator under the 55; and 24 lb. of coffee must be worth 24 times the last result, which may be expressed by making 24 a factor of the numerator. But as this is also the value of 36 lb. of sugar, 1 lb. of sugar must be worth $\frac{1}{36}$ of this, which may be expressed by making 36 a factor of the denominator; and 16 lb. of sugar must be worth 16 times the last result, which may be expressed by making 16 a factor of the numerator.

The work would be written thus: —

$$\text{lb. of rice } \frac{\overset{5}{\cancel{55}} \times \overset{12}{\cancel{24}} \times 16}{\underset{2}{\cancel{22}} \times \underset{3}{\cancel{36}}} = 26\tfrac{2}{3} \text{ lb. of rice } = \textit{Ans.}$$

NOTE. — Examples like the above are usually solved by a process called CONJOINED PROPORTION; but as it is much less simple and convenient than the preceding, we omit it.

30. If 40 bushels of potatoes are worth 45 bushels of corn, and 18 bushels of corn are worth 14 cwt. of hay, and 35 cwt. of hay are worth 4 barrels of flour, how many barrels of flour are 75 bushels of potatoes worth?

31. If 7 American yards are equal to 12 braces at Leghorn, and 54 braces at Leghorn are equal to 45 braces at Venice, how many braces at Venice are equal to 56 American yards?

32. If John can earn as much in 7 days as William can earn in 9 days, and William can earn as much in 15 days as Samuel can in 14, and Samuel can earn as much in 12 days as Otis can in 8, and Otis can earn as much in 24 days as Rufus can in 21, how many days will it take John to earn as much as Rufus can earn in 49 days?

33. Multiply $52\frac{1}{2}$ by $7\frac{1}{5}$, subtract $14\frac{3}{8}$ from the product, and divide by $\frac{7}{8}$.

34. A man gave $4\frac{1}{2}$ acres of land, worth $125.37 per acre, in exchange for 75 barrels of flour. He sold $\frac{2}{3}$ of the flour at $$9\frac{3}{8}$ per barrel, and the rest at $$8\frac{1}{6}$ per barrel. How much did he gain by the transaction?

35. A garrison of 2000 men had bread enough to allow each 24 oz. a day for 75 days; but being besieged, it received a reënforcement of 1600 men. How many ounces per day can each man be allowed, in order that they may hold out 60 days?

36. Half of Arthur's money equals just $\frac{2}{7}$ of William's, and William has $187.47 more than Arthur. How much money has each?

155. *Practice.*

The following examples illustrate what is sometimes called the Rule of Practice.

1. How much will 47 yd. 3 qr. 3 n. of broadcloth cost, at $3.125 per yd.?

Suggestion. — Since 47 yd. 3 qr. 3 na. lacks but 1 nail of being 48 yards, its cost must equal the cost of 48 yards, minus the cost of 1 nail

WRITTEN WORK.

$$a = \$\ 3.125$$

$48 \times a = b = \$150.000 =$ cost of 48 yds.

$\frac{1}{16}$ of $a = c = \$\ \ .195 =$ " " 1 na.

$b - c = \$149.805 =$ " " 47 yds. 3 qr. 3 na.

2. How much will 26 A. 1 R. 25 sq. rd. of land cost, at $174.25 per acre?

WRITTEN WORK.

a =	$ 174.25	= cost of 1 A.	
26 × a = b =	$4530.50	= " " 26 A.	
$\frac{1}{4}$ of a = c =	43.562	= " " 1 R.	
$\frac{1}{2}$ of c = d =	21.781	= " " 20 sq. rd.	
$\frac{1}{4}$ of d = e =	5.445	= " " 5 sq. rd.	
b + c + d + e =	$4601.288	= " " 26 A. 1 R. 25 sq. rd.	

NOTE. — In dividing, in examples like the above, it is unnecessary to carry out the work further than to mills.

3. How much will 13 lb. 4 oz. 10 dwt. 8 gr. of silver cost at $13.37 per lb.?

4. How much will 9 T. 15 cwt. 1 qr. 14 lb. of English bar iron cost, at $83.625 per ton, reckoning the quarter at 28 lbs.? (See **34**, *c*.)

5. How much will it cost to build a road 37 m. 5 fur. 30 rd. in length, at the rate of $2173.75 per mile?

WRITTEN WORK.

a =	$ 2173.75	= cost of 1 m.
38 × a = b =	$82602.50	= " " 38 m.
$\frac{1}{4}$ of a = c =	$ 543.437	= " " 2 fur.
$\frac{1}{8}$ of c = d =	$ 67.929	= " " 10 rd.
b — c — d =	$81991.134	= cost of 37 m. 5 fur. 30 rd.

6. How much will 22 hhd. 21 gal. 3 qt. 1 pt. of wine cost, at $47.875 per hhd., if each hogshead contains 63 gallons?

7. How much will 24 bu. 1 pk. 6 qt. of wheat cost, at $1.375 per bushel?

8. How much will 37 bu. 3 pk. 4 qt. of wheat cost at $1.625 per bushel?

9. How much will 14 cwt. 2 qr. 15 lb. of coffee cost, at $12.625 per cwt.?

10. A man bought a farm, containing 137 A. 3 R. 30 sq. rd., at $46.94 per acre? How much did it cost him?

11. A trader bought 184 bu. 2 pk. 6 qt. of Indian corn, at $.875 per bushel. How much did it cost him?

12. How many rods will a man travel in 13 h. 20 m. 45 sec., if he travels 1289 rods per hour?

13. What will 24 A. 1 R. 25 sq. rod cost, at $47.98 per acre?

14. What will 17 T. 15 cwt. 2 qr. 7 lb. of railroad iron cost, at $47.38 per ton?

15. How much will 4 lb. 11 oz. 19 dwt. 12 gr. of silver cost, at $13.96 per pound?

16. What will 428 bu. of oats cost, at $.49 per bushel?

Suggestion. — At 49 cents per bushel, 428 bushels will cost 428 half dollars, minus 428 cents.

17. How much will 43 volumes cost, at $1.99 per volume?

18. How much will 48 bbls. of flour cost, at $8.75 per barrel?

Suggestion. — At $8.75 per barrel, 48 barrels will cost 48 times $9 — 48 times $\frac{1}{4}$ of a dollar.

19. How much will 32 yds. of cloth cost, at $1.875 per yard?

Suggestion. — At $1.875 per yard, 32 yards will cost 32 times $2 — 32 times $\frac{1}{8}$ of a dollar.

20. How much will 747 lbs. of tea cost, at $.66$\frac{2}{3}$ per lb.?

21. How much will 8 cords of wood cost, at $4.94 per cord?

22. How much will 4 acres of land cost, at $98.75 per acre?

23. What will 1728 yd. of cloth cost, at 1 s. 8 d. per yard?

Suggestion. 1 s. 8 d. = $\frac{1}{12}$ of £1.

24. What will 857 yd. of broadcloth cost, at 16 s. 8 d. per yard?

Suggestion. 16 s. 8 d. = £1 — 3 s. 4 d. = £1 — $\frac{1}{6}$ of £1.
Or, 16 s. 8 d. = 10 s. + 6 s. 8 d. = $\frac{1}{2}$ of £1 + $\frac{1}{3}$ of £1.

25. What will 1478 reams of paper cost, at 13 s. 4 d. per ream?

Suggestion. 13 s. 4 d. = £1 — 6 s. 8 d. = £1 — $\frac{1}{3}$ of £1.

26. What will 737 yd. of black silk cost, at 17 s. 6 d. per yard?

Suggestion. 2 s. 6 d. = $\frac{1}{8}$ of £1.

27. What will 138 yd. of silk velvet cost, at £1 5 s. 6 d. per yard?

WRITTEN WORK.

a	= £138	= cost at £1 per yd.
$\frac{1}{4}$ of a = b	= £ 34 10 s.	= " " 5 s. per yd.
$\frac{1}{10}$ of b = c	= £ 3 9 s.	= " " 6 d. per yd.
a + b + c	= £175 19 s.	= cost at £1 5 s. 6 d. per yard.

28. What will 13 coats cost, at £3 16 s. 8 d. each?

WRITTEN WORK.

a	= £13	= cost at £1 each.
4 + a = b	= £52	= " " £4 each.
$\frac{1}{6}$ of a = c	= £ 8 13 s. 4 d.	= cost at 3 s. 4 d. each.
b — c	= £43 6 s. 8 d.	

29. What will 24 tons of iron cost, at £8 2 s. 6 d. per ton?

30. What will 65 pieces of broadcloth cost, at £29 19 s. 8 d. per piece?

31. What will 45 pieces of Irish linen cost, at £3 1 s. 4 d. per piece?

32. What will 96 tons of iron cost, at £7 11 s. 4 d. per ton?

33. If 14 yd. 1 qr. 2 na. 1 in. of cloth cost £12 13 s. 8 d., what will 28 yd. 3 qr. 2 in. of cloth cost?

Suggestion. 28 yd. 3 qr. 2 in. = twice 14 yd. 1 qr. 2 na. 1 in.

34. If 7 lb. 3 oz. 4 dr. of sugar cost 2 s. 9 d. 1 qr., what will 21 lb. 9 oz. 12 dr. cost?

35. If 17 bu. 1 pk. 2 qt. of corn cost $9.39, how much will 51 bu. 3 pk. 6 qt. cost?

36. If 13 gal. 3 qt. 1 pt. 1 gi. of molasses cost $3.87, how much will 41 gal. 2 qt. 1 pt. 3 gi. cost?

37. If 17 A. 3 R. 7 rd. of land cost $849.29, what will 88 A. 3 R. 35 rd. cost?

38. If 98 yd. 3 qr. 2 na. of cloth cost \$44.38, what will 49 yd. 1 qr. 3 na. cost?

39. If 68 bu. 1 pk. 6 qt. of wheat cost \$114.48, what will 22 bu. 3 pk. 2 qt. cost?

40. If 23 lb. 13 oz. 8 dr. of potash can be bought for \$2.12, how much can be bought for \$16.96?

41. If 9 T. 14 cwt. 2 qr. 17 lb. of hay can be bought for \$171.32, how much can be bought for \$513.96?

SECTION XII.

RATIO AND PROPORTION.

NOTE. — If the teacher prefers it, the pupil may omit this section and the two following it, till after he has mastered the sections on interest and the subjects pertaining to business life.

156. *Definitions and Illustrations of Ratio.*

(*a.*) RATIO is the part which one number is of another, or the quotient of one number divided by another: —

Thus, the ratio of 5 to 6 is $\frac{6}{5}$, or $1\frac{1}{5}$, because 6 equals $\frac{6}{5}$ of 5, equals $1\frac{1}{5}$ times 5; or because $6 \div 5 = \frac{6}{5} = 1\frac{1}{5}$.

The ratio of 3 to 12 is $\frac{12}{3}$, or 4, because 12 equals $\frac{12}{3}$ of $3 = 4$ times 3; or because $12 \div 3 = \frac{12}{3} = 4$

NOTE. — Some writers consider that the ratio of 5 to 6, or of 3 to 12, is the part which 5 is of 6, or 3 is of 12, instead of the part which 6 is of 5, or 12 is of 3, as given above.

The difference is not practically of so much consequence as would at first appear to be the case, for the term *ratio* is almost invariably used in some such connection as the following: "The ratio of 4 to 6 equals the ratio of 10 to 15," where, by the first interpretation, we have $\frac{6}{4} = \frac{15}{10}$, or $\frac{3}{2}, = \frac{3}{2}$; and by the second, $\frac{4}{6} = \frac{10}{15}$, or $\frac{2}{3}, = \frac{2}{3}$, both of which are manifestly true. It should, then, be borne in mind, that while the first interpretation is the one usually adopted, the second may be substituted for it, in any case where the change may seem desirable.

(*b.*) A ratio can always be established between abstract numbers, but it can only exist between concrete numbers when they are of the same denomination; for one quantity can never be a fractional part of another quantity of a different kind.

Thus, 8 apples is no part of 4 pears, and hence has no ratio to it.

(*c.*) By the above illustrations, it appears that every ratio is a true fraction, and may be written and dealt with as such.

(*d.*) Ratios are, however, usually expressed by writing one number after the other, and placing two dots between them, thus: —

The ratio of 4 to 6 = 4 : 6, or $\frac{6}{4}$.
The ratio of 9 to 7 = 9 : 7, or $\frac{7}{9}$.

(*e.*) The numbers which form any ratio are called terms of the ratio; the first number, or term, is called the *antecedent* of the ratio; and the second number, or term, is called the *consequent* of the ratio.

Thus, in the ratio 9 : 7, 9 is the consequent, and 7 the antecedent.

(*f.*) Ratios, like fractions, may be SIMPLE, COMPLEX, or COMPOUND.

(*g.*) A SIMPLE RATIO is the ratio of two entire numbers; as, 5 : 7, 4 : 9, or $\frac{7}{5}$, $\frac{9}{4}$.

(*h.*) A COMPLEX RATIO is the ratio of two fractional numbers; as, $\frac{3}{4}$ to $2\frac{1}{2}$, $4\frac{2}{3}$: $3\frac{1}{5}$, or $\frac{2\frac{1}{2}}{\frac{3}{4}}$, $\frac{3\frac{1}{5}}{4\frac{2}{3}}$.

(*i.*) A COMPOUND RATIO is the indicated product of two or more ratios; as (5 : 7) × (8 : 3), or $\frac{7}{5}$ of $\frac{3}{8}$, or $\frac{7}{5} \times \frac{3}{8}$.

(*j.*) A compound ratio is usually expressed by writing the ratios which compose it under each other: —

Thus, $\left.\begin{matrix}4 : 7\\ 9 : 8\end{matrix}\right\}$ expresses that the product of the two ratios, 4 : 7 and 9 : 8, is to be obtained; or, which is the same thing, it means $\frac{7}{4}$ of $\frac{8}{9}$, or $\frac{7}{4} \times \frac{8}{9}$.

157. *Reduction of Ratios.*

(*a.*) Since ratios are really fractions, the principles involved in all operations upon them are precisely the same as

those involved in the corresponding operations on other fractions.

(*b*.) Hence, multiplying or dividing both terms of a ratio by the same number will not alter its value.

1. Reduce $4 : 6$ to its lowest terms.

Solution. — Dividing both terms by 2 gives $4 : 6 = 2 : 3$. This may be proved thus: $4 : 6 = \frac{6}{4} = \frac{3}{2} = 2 : 3$.

Reduce each of the following ratios to its lowest terms: —

2. $6 : 9$.	5. $24 : 48$.	8. $18 : 15$.
3. $12 : 15$.	6. $16 : 12$.	9. $36 : 54$.
4. $21 : 35$.	7. $8 : 6$.	

10. $16 \times 9 : 8 \times 12$.
11. $15 \times 3 \times 7 : 35 \times 9 \times 4$.
12. $14 \times 8 \times 5 : 49 \times 10 \times 12$.
13. $20 \times 7 \times 45 : 9 \times 35 \times 10$.

14. Reduce $4\frac{1}{5} : 6\frac{3}{10}$ to a simple ratio.

First Solution. — Multiplying both terms by 10, the least common multiple of 5 and 10, (see **149**, *e*.) gives $42 : 63 = 2 : 3$.

Second Solution. $4\frac{1}{5} : 6\frac{3}{10} = \frac{21}{5} : \frac{63}{10} = \cancel{21} \times \overset{2}{\cancel{10}} : \cancel{5} \times \overset{3}{\cancel{63}} = 2 : 3$. (See **149**, *c*.)

Reduce the following complex ratios to simple ones: —

15. $3\frac{1}{2} : 4\frac{2}{3}$.	20. $3\frac{5}{6} : 2\frac{7}{9}$.
16. $.03 : 5.7$.	21. $1\frac{4}{5} : 1\frac{2}{15}$.
17. $\frac{2}{3} : \frac{5}{6}$.	22. $2.0007 : 2000.7$
18. $4\frac{3}{7} : 6\frac{1}{7}$.	23. $.0025 : 2500$.
19. $.02 : 005$.	

24. Reduce $\left.\begin{array}{r} 5 : 8 \\ 12 : 14 \\ 21 : 25 \end{array}\right\}$ to a simple ratio.

WRITTEN WORK.

$$\cancel{5} \times \overset{3}{\cancel{12}} \times \overset{3}{\cancel{21}} : \overset{2}{\cancel{8}} \times \overset{2}{\cancel{14}} \times \overset{5}{\cancel{25}} = 9 : 20.$$

Proof.

$$\frac{8}{5} \text{ of } \frac{14}{12} \text{ of } \frac{25}{21} = \frac{\overset{2}{\cancel{8}} \times \overset{2}{\cancel{14}} \times \overset{5}{\cancel{25}}}{\cancel{5} \times \underset{3}{\cancel{12}} \times \underset{3}{\cancel{21}}} = \frac{20}{9} =$$

9 : 20.

Reduce each of the following compound ratios to simple ones: —

25. $\begin{cases} 8 : 12 \\ 9 : 10 \\ 5 : 6 \end{cases}$

26. $\begin{cases} 4 : 7 \\ 6 : 25 \\ 15 : 24 \end{cases}$

27. $\begin{cases} 24 : 49 \\ 25 : 60 \\ 21 : 30 \end{cases}$

28. $\begin{cases} 14 : 11 \\ 81 : 45 \\ 55 : 63 \end{cases}$

29. $\begin{cases} \frac{1}{2} : \frac{2}{3} \\ \frac{4}{7} : \frac{8}{9} \end{cases}$

30. $\begin{cases} 3\frac{1}{2} : 4\frac{1}{3} \\ 8\frac{2}{3} : 4\frac{1}{5} \end{cases}$

NOTE. — From the above, it is obvious that a compound ratio may be reduced to a simple one, by making all its antecedents factors of the new antecedent, and all its consequents factors of the new consequent, and then cancelling all the factors common to both terms.

158. *Definitions and Illustrations of Proportion.*

(*a.*) A PROPORTION is an equality of ratios.

(*b.*) Proportions may be SIMPLE, COMPLEX, or COMPOUND.

(*c.*) A SIMPLE PROPORTION expresses the equality of two simple ratios.

(*d.*) A COMPLEX PROPORTION expresses the equality of two complex ratios, or of a complex and simple ratio.

(*e.*) A COMPOUND PROPORTION expresses the equality of two compound ratios, or of a compound and simple ratio.

(*f.*) A proportion is expressed by writing two ratios one after the other, and placing four dots between them.

Thus, 4 : 6 : : 12 : 18 is a proportion which expresses that the ratio of 4 to 6 equals the ratio of 12 to 18. It would be read, 4 *is to* 6 *as* 12 *is to* 18, *or* 6 *is the same part of* 4 *that* 18 *is of* 12.

$3\frac{1}{2} : 5\frac{1}{3} :: 2\frac{1}{10} : 3\frac{1}{5}$ expresses that the ratio of $3\frac{1}{2}$ to $5\frac{1}{3}$ equals the

ratio of $2\frac{1}{10}$ to $3\frac{1}{5}$. It would be read, $3\frac{1}{2}$ *is to* $5\frac{1}{3}$ *as* $2\frac{1}{10}$ *is to* $3\frac{1}{5}$, *or* $5\frac{1}{3}$ *is the same part of* $3\frac{1}{2}$ *that* $3\frac{1}{5}$ *is of* $2\frac{1}{10}$.

The compound proportion $\left.\begin{array}{r}4 : 7\\ 9 : 16\\ 14 : 3\end{array}\right\} :: 3 : 2$, expresses that the ratio of $4 \times 9 \times 14$ to $7 \times 16 \times 3$ equals the ratio of $3 : 2$. It would be read, 4 *times* 9 *times* 14 *is to* 7 *times* 16 *times* 3 *as* 3 *is to* 2, *or* 7 *times* 16 *times* 3 *is the same part of* 4 *times* 9 *times* 14 *that* 2 *is of* 3.

(*g*.) The sign of equality is sometimes used instead of the four dots.

Thus, instead of $4 : 6 :: 2 : 3$, we may have $4 : 6 = 2 : 3$.

(*h*.) Every proportion may be expressed as the equality of two fractions.

Thus, we may express the first of the above by $\frac{6}{4} = \frac{18}{12}$, the second by $\frac{5\frac{1}{3}}{3\frac{1}{2}} = \frac{3\frac{1}{5}}{2\frac{1}{10}}$, and the third by $\frac{7}{4}$ of $\frac{16}{9}$ of $\frac{3}{14} = \frac{2}{3}$.

(*i*.) The outside terms (i. e., the first and fourth) of a proportion are called the EXTREMES, and the inside terms (i. e., the second and third) the MEANS of the proportion.

Thus, in the first proportion above given, 4 and 18 are the extremes. 6 and 12 are the means; in the second, $3\frac{1}{2}$ and $3\frac{1}{5}$ are the extremes $5\frac{1}{3}$ and $2\frac{1}{10}$ are the means; in the third, $4 \times 9 \times 14$ and 2 are the extremes, and $7 \times 16 \times 3$ and 3 are the means.

159. *Method of finding a missing Term.*

(*a*.) If any term of a proportion is omitted, it may easily be supplied; for from the nature of a proportion, it follows that, —

First. *The missing antecedent of any proportion must be the same part of its consequent that the given antecedent is of its consequent.*

Second. *The missing consequent of any proportion must be the same part of its antecedent that the given consequent is of its antecedent.*

1. Find the missing term of the proportion $5 : 7 :: 25 :$ —

Solution. — The missing term is the same part of 25 that 7 is of 5; i. e., it is $\frac{7}{5}$ of 25, which is 35. Hence, $5 : 7 :: 25 : \frac{7 \times 25}{5} = 35$.

2. Find the missing term of the proportion $9 : 14 :: — : 49$

Solution. — The missing term is the same part of 49 that 9 is of 14; i. e., it is $\frac{9}{14}$ of 49, which is $31\frac{1}{2}$. Hence, $9 : 14 :: \frac{9 \times 49}{14} = 31\frac{1}{2} : 49$.

3. Find the missing term of the proportion $4\frac{2}{3}$ $3\frac{1}{2}$ :: $7\frac{1}{5}$: —.

Solution. — The missing term is the same part of $7\frac{1}{5}$ that $3\frac{1}{2}$ is of $4\frac{2}{3}$; i. e., it is —

$$\frac{3\frac{1}{2}}{4\frac{2}{3}} \text{ of } 7\frac{1}{5} = \frac{\cancel{7} \times 3 \times \overset{9}{\cancel{36}}}{\cancel{2} \times \underset{\cancel{2}}{\cancel{14}} \times 5} = 5\frac{2}{5}.$$

Hence, $4\frac{2}{3} : 3\frac{1}{2} :: 7\frac{1}{5} : \frac{3\frac{1}{2} \times 7\frac{1}{5}}{4\frac{2}{3}} = 5\frac{2}{5}$.

4. Find the missing term of the proportion

$$\left.\begin{matrix} 4 : 9 \\ 7 : 8 \\ 15 : 14 \end{matrix}\right\} :: 25 : —.$$

Solution. — The missing term is the same part of 25 that $9 \times 8 \times 14$ is of $4 \times 7 \times 15$; i. e., it is

$$\frac{9 \times 8 \times 14}{4 \times 7 \times 15} \text{ of } 25 = \frac{\overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times \overset{2}{\cancel{14}} \times \overset{5}{\cancel{25}}}{\cancel{4} \times \cancel{7} \times \underset{\cancel{5}}{\cancel{15}}} = 60.$$

Hence, $\left.\begin{matrix} 4 : 9 \\ 7 : 8 \\ 15 : 14 \end{matrix}\right\} :: 25 \cdot \frac{9 \times 8 \times 14 \times 25}{4 \times 7 \times 15} = 60.$

Find the missing term of each of the following proportions: —

5. $9 : 12 :: 6 : —$.
6. $4\frac{1}{2} : 10\frac{1}{2} :: 26 : —$.
7. $36 : 48 :: — : 60$.
8. $5\frac{3}{4} : 4\frac{2}{3} :: — : 3\frac{3}{7}$.
9. $4 : — :: 20 : 28$.
10. $9\frac{3}{5} : — :: 7 : 28$.
11. $— : 6 :: 13 : 52$.
12. $— : .9 :: 4.9 : .063$

13. $\left\{\begin{array}{rcr} 3 & : & 5 \\ 8 & : & 12 \\ 9 & : & 7 \end{array}\right\} :: 54 : —.$

14. $\left\{\begin{array}{lcl} 3\frac{1}{2} & : & 7\frac{1}{3} \\ 5\frac{1}{2} & : & 2\frac{6}{11} \\ 4 & : & 6 \end{array}\right\} :: 144 : —.$

15. $\left\{\begin{array}{rcr} 21 & : & 17 \\ 42 & : & 32 \\ 51 & : & 54 \end{array}\right\} :: — : 64.$

16. $\left\{\begin{array}{rcr} .03 & : & .007 \\ 3.5 & : & 500 \\ .9 & : & 2.7 \end{array}\right\} :: — : 30.$

160. *Relations of the Terms.*

(*a.*) From the foregoing explanations and illustrations, we may infer that, —

First. *Either extreme is equal to the quotient obtained by dividing the product of the means by the other extreme.*

Second. *Either mean is equal to the quotient obtained by dividing the product of the extremes by the other mean.*

(*b.*) Hence, in a proportion —

The product of the means is equal to the product of the extremes.

161. *Practical Problems.*

(*a.*) The forming of a proportion from the conditions of a problem is called STATING it.

(*b.*) In stating a proportion, it is customary to write the number which is of the same denomination as the answer for the third term.

1. If 8 books cost \$4.32, what will 11 books cost?

Solution. — Since the answer is to be in dollars, we write \$4.32 as the third term, and this being the price of 8 books, 11 books will cost the same part of this that 11 is of 8, and we therefore make 11 the second term, and 8 the first. Hence, the following statement: —

$$8 : 11 :: \$4.32 : \$\frac{4.32 \times 11}{8} = \$5.94 = Ans.$$

NOTE. — The above solution is really equivalent to the following: Since the answer is to be in dollars, we write \$4.32, as the number on which the operation is to be performed. If 8 books cost so much, 11 books will cost $\frac{11}{8}$ of this, which may be found by making 11 a factor

of the numerator, and 8 a factor of the denominator. Hence, the following written work:—

$$\$\frac{4.32 \times 11}{8} = \$5.94.$$

2. If it takes 18 men 7 days to perform a piece of work, how many men will it take to perform it in 9 days?

Solution.—Since the answer is to represent the number of men, we write 18 men as the third term. But since it will take 7 times as many men to do it in 1 day as it will to do it in 7 days, and $\frac{1}{9}$ as many to do it in 9 days as it will to do it in 1 day, it follows that the answer is the same part of 18 that 7 is of 9. Hence,

$$9 : 7 :: 18 : \frac{7 \times 18}{9} \text{ men} = 14 \text{ men} = \textit{Ans.}$$

NOTE.—When the pupil is familiar with the full form, he may abbreviate, thus: Since the answer is to represent a number of men, we write 18 men as the third term. But as this is the number of men which it takes to do it in 7 days, it will take the same part of this number to do it in 9 days that 7 is of 9. This gives the same proportion as before.

3. If it takes a man $7\frac{1}{2}$ days, of $10\frac{1}{2}$ hours each, to earn $\$26\frac{1}{4}$, how many days, of $9\frac{1}{5}$ hours each, will it take him to earn the same sum?

Solution.—Since the answer is to be in days, we write $7\frac{1}{2}$ for the third term. If, when the days are $10\frac{1}{2}$ hours long, it takes so many days, when they are $9\frac{1}{5}$ hours long it will take the same part of this number of days that $10\frac{1}{2}$ is of $9\frac{1}{5}$, which will give $9\frac{1}{5}$ for the first term, and $10\frac{1}{2}$ for the second. Hence, $9\frac{1}{5} : 10\frac{1}{2} :: 7\frac{1}{2} : $ —.

(*c*.) After having written the third term, we can tell in what order to arrange the two remaining numbers, by observing that when the conditions of the question are such as to require an answer greater than the third term, the larger number will be the second term, and the smaller the first; and that when they are such as to require an answer less than the third term, the smaller number will be the second term, and the larger the first.

Thus, in the first example, having written \$4.32 as the third term, we may observe that the cost of 11 books will be greater than the cost of 8 books, and that the ratio is 8 : 11.

In the second example, having written 18 for the third term, we may observe that it will take a less number of men to perform the work in 9 days than to perform it in 7 days, and that the ratio is 9 : 7.

In the third example, having written $7\frac{1}{2}$ days as the third term, we may observe that it will take more days to earn the money when they are $9\frac{4}{5}$ hours long, than when they are $10\frac{1}{2}$ hours long, and that the ratio is $9\frac{1}{5} : 10\frac{1}{2}$.

4. If 24 cows can be bought for $486, for how much can 17 cows be bought?

5. If 17 cows can be bought for $344.25, for how much can 24 cows be bought?

6. If 24 cows can be bought for $486, how many can be bought for $344.25?

7. If 17 cows can be bought for $344.25, how many can be bought for $486?

8. How much would it cost to transport 9 cwt. 2 qrs. 13 lb. of merchandise from Providence to Boston, if it costs $6.43 to transport 12 cwt. 3 qrs. 11 lb. the same distance?

9. How many men will it take to perform a piece of work in $6\frac{2}{3}$ days, which it will take 42 men $12\frac{6}{7}$ days to perform?

10. How many days will it take 81 men to perform a piece of work which 42 men can do in $12\frac{6}{7}$ days?

11. How many men will it take to do a piece of work in $12\frac{6}{7}$ days which 81 men can do in $6\frac{2}{3}$ days?

12. How many days will it take 42 men to do a piece of work which 81 men can do in $6\frac{2}{3}$ days?

13. John walks $3\frac{3}{4}$ miles per hour, and William walks $3\frac{1}{3}$ miles per hour. How many hours will it take John to walk as far as William can walk in 8 hours?

14. If $9\frac{1}{2}$ yards of cloth can be bought for $$44\frac{1}{3}$, how many yards can be bought for $$33\frac{1}{4}$?

162. *Problems in Compound Proportion.*

1. If 6 boxes of soap, each holding 9 pounds, cost $4.59 how much will 11 boxes, each holding 12 pounds, cost?

Solution. — Observing that the answer is to be in dollars, we write \$4.59 as the third term. As this is the cost of 6 boxes of soap, 11 boxes will cost the same part of this that 11 is of 6, which may be expressed by making 11 the second term, and 6 the first. But if boxes each holding 9 pounds cost so much, boxes each holding 12 pounds will cost the same part of this that 12 is of 9, which may be expressed by making 12 a factor of the second term, and 9 a factor of the first. Hence the following compound proportion: —

$$\left.\begin{matrix} 6 : 11 \\ 9 : 12 \end{matrix}\right\} :: 4.59 : \frac{\overset{51}{\cancel{4.59}} \times 11 \times \overset{2}{\cancel{12}}}{\cancel{6} \times \cancel{9}} = \$11.22$$

2. If 6 boxes of soap, each holding 9 pounds, can be bought for \$4.59, how many boxes, each holding 12 pounds, can be bought for \$11.22.

Solution. — We first write 6 boxes for the third term. But as \$11.22, or 1122 cents, will buy the same part of what \$4.59, or 459 cents, will buy, that 1122 is of 459, we make 1122 the second term, and 459 the first. But these boxes hold 9 pounds each, and of boxes holding 12 pounds each, only $\frac{9}{12}$ as many can be bought, which may be expressed by making 9 a factor of the second term, and 12 a factor of the first. Hence the proportion

$$\left.\begin{matrix} 459 : 1122 \\ 12 : 9 \end{matrix}\right\} :: 6 : \frac{\cancel{6} \times \overset{\overset{11}{\cancel{22}}}{\cancel{1122}} \times \cancel{9}}{\underset{\cancel{51}}{\cancel{459}} \times \underset{\cancel{2}}{\cancel{12}}} = 11$$

(*a.*) It will be seen by the above, that, after writing the third term, we consider what ratio an answer depending on any two similar conditions of the question would bear to that third term, and that, after writing this ratio, we consider what ratio an answer depending upon any other two conditions similar to each other would bear to this, and so on till all the conditions are considered. Then, by solving the resulting compound proportion, the answer may be easily obtained.

(*b.*) If in any case the pupil is in doubt how to arrange the terms of a ratio, he may determine it by the method indicated in **161.** *c.*

Examples like those last explained are really equivalent to two or more examples in simple proportion.

Thus, the first is equivalent to the two following: —

1. If 6 boxes of soap cost $4.59, how much will 11 boxes cost?

2. If a certain number of boxes of soap, each containing 9 pounds, cost the answer to the last question, how much will the same number of boxes, each containing 12 pounds, cost? (See note on 215th page.)

3. If it costs $36 to transport 14 tons of merchandise 54 miles, how much will it cost to transport 18 tons 49 miles?

4. If 14 tons of merchandise can be transported 54 miles for $36, how many tons can be transported 49 miles for $42?

5. If 14 tons of merchandise can be transported 54 miles for $36, how many miles can 18 tons be transported for $42?

6. If 18 tons of merchandise can be transported 49 miles for $42, how many tons can be transported 54 miles for $36?

7. If 18 tons of merchandise can be transported 49 miles for $42, how many miles can 14 tons be transported for $36?

8. If it costs $42 to transport 18 tons of merchandise 49 miles, how much will it cost to transport 14 tons 54 miles?

9. If 8 men can earn $216 in 3 weeks, how many dollars can 12 men earn in 2 weeks?

10. If it takes 24 pounds of cotton to make 2 pieces of sheeting, each containing 33 yards, $1\frac{1}{8}$ yard wide, how many pounds of cotton will it take to make 11 pieces of sheeting, each containing 27 yards, $1\frac{1}{4}$ yard wide?

11. If it takes 45 men 8 weeks, working $5\frac{1}{2}$ days per week, and 10 hours per day, to build a road $3\frac{1}{2}$ miles long and 4 rods wide, how many weeks will it take 63 men, working $4\frac{1}{2}$ days per week, and 11 hours per day, to build a road $12\frac{2}{3}$ miles long and 3 rods wide?

12. A company of 40 men agree to perform a piece of work in 50 days, but after working 9 hours per day for 30 days, they find that they have done but half the work. How many more men must they employ, that, by working 10 hours per day, they may finish the job according to agreement?

13. If 9 men, by working 8 hours per day, can mow 3[illegible]

acres of grass in $2\frac{1}{2}$ days, how many acres of grass can 5 men mow in $3\frac{2}{3}$ days, by working $7\frac{1}{2}$ hours per day?

14. How many bottles can be filled from 12 casks of wine, each cask containing 126 gallons, if 1344 bottles can be filled from 3 casks, each cask containing 84 gallons?

15. If, when land is worth $\$46\frac{4}{5}$ per acre, a lot of land, 35 rods long and 24 rods wide, is given for 8 piles of wood, each 45 feet long, 5 feet high, and 4 feet wide, how much ought land to be worth an acre, when 3 lots, each 32 rods long, and 25 rods wide, are given for 21 piles of wood, each $37\frac{1}{2}$ feet long, $4\frac{3}{4}$ feet high, and 4 feet wide?

SECTION XIII.

163. DUODECIMAL FRACTIONS.

(*a.*) A DUODECIMAL FRACTION, or simply a DUODECIMAL, is a fraction whose denominator is 12, or some power of 12.

(*b.*) The denominator of a duodecimal is not written, but is indicated by one or more marks, or accents, placed at the right of the numerator, and a little above it.

Thus, $4' = \frac{4}{12}$; $7'' = \frac{7}{144}$; $11''' = \frac{11}{1728}$; &c.

(*c.*) In reading duodecimals, *twelfths* are usually called PRIMES; *one-hundred-and-forty-fourths* are called SECONDS; &c.

Thus, $4'$ is read, *four primes*; $7''$ is read, *seven seconds*; $11'''$ is read, *eleven thirds*; &c.

(*d.*) Duodecimals are employed in measuring lengths, surfaces, and solids; so that the unit of measure is a foot of either long, square, or cubic measure, according to the nature of the thing measured.

(*e.*) Since a linear inch equals $\frac{1}{12}$ of a linear foot, a square inch $\frac{1}{144}$ of a square foot, and a cubic inch $\frac{1}{1728}$ of a cubic

foot, it follows that in long measure the inch is represented by the prime, in square measure by the second, and in cubic measure by the third.

(*f.*) Duodecimals may be added, subtracted, multiplied, and divided as other fractions are. In performing these operations, it is necessary to notice that a unit of any denomination equals 12 units of the next lower, and $\frac{1}{12}$ of a unit of the next higher denomination; also, that $1' \times 1' = \frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$, or $1''$; that $1'' \times 1' = \frac{1}{144} \times \frac{1}{12} = \frac{1}{1728}$, or $1'''$; &c.

163. *Problems.*

1. What are the contents of a board 13 ft. 4′ long, and 2 ft. 5′ wide?

Reasoning Process.— If the board was 13 ft. 4′ long and 1 ft. wide, it would contain 13 sq. ft. 4′; but being 2 ft. 5′ wide, it must contain $2\frac{5}{12}$ times 13 sq. ft. 4′. This gives the following written work:—

ft.	′	
13	4	
2	5	
5	6	8″
26	8	
32	2′	8″ = 32 sq. ft. 32 sq. in.

Explanation.— First, multiplying by 5′ (i. e., by $\frac{5}{12}$) we have 5′ times 4′ = 20″ = 1′ 8″, (i. e., $\frac{5}{12}$ times $\frac{4}{12} = \frac{20}{144} = \frac{1}{12} + \frac{8}{144}$.) Writing the 8″, we have 5′ times 13 ft. = 65′, and 1′ added, = 66′ = 5 ft. 6″, (i. e., $\frac{5}{12}$ times 13 ft. $= \frac{65}{12}$ ft., &c.,) which we write. Then multiplying by 2, we have 2 times 4′ = 8′; 2 times 13 ft. = 26 ft. Adding these products gives 32 ft. 2′ 8″ for a result, which, being in square measure, = 32 sq. ft. 32 sq. in.

NOTE.— The student should observe that in performing the work, the multiplier is always to be regarded as an abstract number. (See **74**, *g.*) The expression, "multiplying the length by the breadth," means simply that the number representing the length is to be multiplied by the number representing the breadth.

2. What are the contents of a board 14 ft. 5′ long and 1 ft. 11′ wide?

3. What are the contents of a blackboard 17 ft. 9′ long and 5 ft. 3′ wide?

4. What are the contents of a pile of wood 47 ft. 6′ long, 3 ft. 9′ wide, and 5 ft. 4′ high?

NOTE.— These examples can very frequently be performed more easily by reducing the duodecimals to vulgar fractions. Thus, in the last example, 47 ft. 6′ = $47\frac{1}{2}$ ft., 3 ft. 9 = $3\frac{3}{4}$ ft., and 5 ft. 4′ = $5\frac{1}{3}$ ft., which, in conformity with **41,** Note, would give —

$$47\tfrac{1}{2} \times 3\tfrac{3}{4} \times 5\tfrac{1}{3} = \frac{95 \times \overset{5}{\cancel{15}} \times \overset{2}{\cancel{16}}}{\cancel{2} \times \cancel{4} \times \cancel{3}} = 950 \text{ cu. ft.} = \textit{Ans.}$$

5. How many cubic feet of earth would be removed in digging a cellar 15 ft. 9′ long, 13 ft. 4′ wide, and 9 ft. 8′ deep?

6. How many square yards of carpeting will it take to cover a floor 16 ft. 6′ long and 15 ft. 4′ wide?

7. How many square feet and inches in the four walls of a room 23 ft. 5′ long, 18 ft. 9′ wide, and 14 ft. 3 in. high?

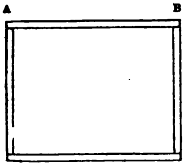
8. How many cords and cord feet of wood in a load 8 ft. long, 3 ft. 11′ wide, and 5 ft. 7′ high?

9. How many yards of plastering are there in the top and walls of a room 16 ft. 4′ long, 13 ft. 2′ wide, and 11 ft. 6′ high, allowing for three windows, each 5 ft. 8′ high and 3 ft. 3′ wide; two doors, each 6 ft. 10′ high and 2 ft. 9′ wide, and for a mop board, 9 inches wide, around the room?

10. Mr. Jackson owned a garden, 137 ft. long and 112 ft. 6′ wide, around which he laid out a gravel walk 4 ft. 8′ wide. How many square feet did the walk contain?

NOTE.— The space used for the walk is supposed to be taken from the contents of the garden.

A B



D C

Suggestion. — To understand this problem, draw a figure like the one annexed. Then suppose the walks along the sides, A B and D C, to be first built. Each of them will be 137 ft. long and 4 ft. 8′ wide, and together they will be equivalent to a walk 2 × 137, or 274 ft. long and 4 ft. 8′ wide. But in building these walks, it is obvious that we have shortened each walk to be built across the ends by just twice the width of the walk, i. e., by twice 4 ft. 8′, which is 9 ft. 4′. This taken from 112 ft. 6 in., the width

of the garden, leaves 103 ft. 2′, which is the length of the walk which remains to be built across each of the two ends. Twice 103 ft. 2′ = 206 ft. 4′, which, added to 274 ft., = 480 ft. 4′, = the whole length of the walk around the garden.

11. A man bought a garden spot, 143 ft. 4′ long and 124 ft. 8′ wide, and after leaving a space for a flower bed, 2 ft. 6′ wide, all around it, he laid out a walk, 3 ft. 10′ wide, within the flower bed, and extending around the garden. How many feet did the walk contain?

12. A man bought a lot of land, 97 ft. 4′ long and 73 ft. 3′ wide, at the rate of 6 cents per sq. ft. He built a tight board fence, 6 ft. 2′ high, around it, for which he paid 3 cents per sq. ft. How much did the land and fence cost him?

13. How many yards of carpeting, $1\frac{1}{4}$ yd. wide, will it take to cover a floor 19 ft. 9′ long and 15 ft. 6′ wide?

14. How many bricks, each 8 in. long, 4 in. wide, and 2 in. thick, will it take to build a wall 47 ft. long, 6 ft. 6′ high, and 1 ft. 8′ thick?

Suggestion. — Since 8 in. = $\frac{2}{3}$ ft., 4 in. = $\frac{1}{3}$ ft., and 2 in. = $\frac{1}{6}$ ft., each brick contains $\frac{2}{3} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{27}$ solid feet, and it will take 27 bricks for each solid foot in the wall.

15. How many bricks, each 8 in. long, 4 in. wide, and 2 in. thick, will it take to build the four walls of a house, 30 ft. 6′ long and 24 ft. 8′ wide, the walls to be 1 ft. thick and 22 ft. 4′ high, allowing for 1 door, 8 ft. high and 3 ft. 10′ wide; for 2 doors, each 8 ft. high and 3 ft. 6′ wide; for 12 windows, each 6 ft. high and 3 ft. 8′ wide; and for 16 windows, each 5 ft. 8′ high and 3 ft. 6′ wide?

NOTE. — The student should be careful to notice how the thickness of the wall will affect his calculations. He must apply a principle similar to that applied in the note to the 10th example.

16. A prison wall is built of Quincy granite, and extends completely round the prison yard, except that a space is left in one of its sides for an iron gate 12 ft. wide and 12 ft. high. The outside length of the wall is 54 ft. 4′, its breadth 49 ft. 6′, and its height 18 ft. Its sides are 4 ft. 3′ thick, and its ends 3 ft. 9′. How many cubic feet in the wall?

SECTION XIV.

ALLIGATION.

164. *Definitions and Explanations.*

(*a.*) MERCHANTS and others often find it convenient to mix articles of different kinds together, so as to obtain a compound differing in value from any of its ingredients. The various problems connected with the subject are called PROBLEMS IN ALLIGATION.

(*b.*) Questions in alligation are usually divided into two classes, viz.: First, ALLIGATION MEDIAL, in which the quantities of the several ingredients and their prices are given, and we are required to find the price of the mixture per pound, per gallon, or per bushel.

Second, ALLIGATION ALTERNATE, in which the prices of the various ingredients are given, and we are required to find what quantities of each must be taken to make a mixture having a given value per pound, per bushel, or per gallon.

165. *Problems.*

1. A trader mixed together 6 lb. of coffee worth 10 cents per pound, 4 lb. worth 8 cents per lb., and 7 lb. worth 16 cents per lb. How much was the mixture worth per lb.?

Solution. 6 lb. at 10 cents are worth 60 cents.
4 " " 8 " " " 32 "
7 " " 16 " " " 112 "

Hence, 17 lb. are worth $2.04, and 1 lb. is worth $\frac{1}{17}$ of $2.04, which is 12 cents = *Ans.*

2. A silversmith melted together 9 oz. of silver, 14 carats fine; 6 oz., 18 carats fine; 12 oz., 22 carats fine; and 23 oz., 24 carats fine. What was the fineness of the mixture?

3. A dry goods dealer sold 25 yd. of sheeting at 9 cents per yd.; 30 yd. of shirting at 10 cents per yd.; 11 yd. of

delaine at 18 cents per yd.; 9 yd. of gingham at 22 cents per yd.; and 25 yd. of linen at 40 cents per yd. What was the average price of the whole per yard?

4. A merchant has sugars at 6, 7, 10, and 13 cents per pound, of which he wishes to make a mixture such that, by selling it for 9 cents per pound, he will neither gain nor lose. How many pounds of each kind must he take?

Solution. — It is obvious that by selling the mixture for 9 cents per pound, he will gain 3 cents on each pound which he puts in of the first kind, and 2 cents on each pound of the second kind; that he will lose 1 cent on each pound of the third kind, and 4 cents on each pound of the fourth; and further, that to make the mixture worth just 9 cents per pound, he must take such proportions of the several kinds as will make his gains equal his losses. Moreover, he may take as many pounds as he chooses of the kinds which cost less than 9 cents, provided he takes enough of the others to counterbalance the gain on them.

Suppose that he takes 8 lb. of the first kind, and 11 lb. of the second. Then, since on 1 lb. of the first he gains 3 cents, on 8 lb. he will gain 8 times 3 cents, or 24 cents; and since on 1 lb. of the second he gains 2 cents, on 11 lb. he will gain 11 times 2 cents, or 22 cents, which, added to the 24 cents, gives 46 cents as the sum of his gains. He must, therefore, take enough of the other kinds to lose 46 cents. Suppose he takes 10 lb. of that at 10 cents. Then, since on 1 lb. he loses 1 cent, on 10 lb. he will lose 10 times 1 cent, or 10 cents, and he must take enough of that at 13 cents to lose 36 cents. Since on 1 lb. he loses 4 cents, he must take as many pounds to lose 36 cents as there are times 4 in 36 which are 9 times Hence, he may take 8 lb. at 6 cents, 11 lb at 7 cents, 10 lb. at 10 cents, and 9 lb. at 13 cents.

The following is a convenient form of writing the work: —

Mean price.	Prices of ingredients	Gain or loss per lb.	Quantities taken.	Sum of gains or losses.
9	6 . . .	+ 3	8 lb. g. 24 cts.	46 cts. gain
	7 . . .	+ 2	11 " g. 22 "	
	10 . . .	— 1	10 " l. 10 "	46 cts. loss.
	13 . . .	— 4	9 " l. 36 "	

Proof.	8 lb. at	6	cents	are	worth	$ .48.
	11 "	" 7	"	"	"	$ .77.
	10 "	" 10	"	"	"	$1.00.
	9 "	" 13	"	"	"	$1.17.
Making	38 lb.,	worth	. . . .			$3.42.

But 38 lb., at 9 cents per lb., would bring just $3.42, which shows that the merchant would get the same sum by selling the mixture at 9 cents per lb. that he would by selling the ingredients separately, at their respective prices.

(*c.*) There is no limit to the number of answers which may be obtained to such questions as the above; for however many or few pounds of any kind we take, we may take enough of other kinds to counterbalance the gain or loss. In the solution, we may as well consider first the number of pounds to be taken of the kinds which cost less than the mean rate, as of those which cost more.

(*d.*) Annexed is a part of the written work of two other solutions to the above example. Let the student complete and explain it, and also prove the correctness of his results.

Quantities taken.	Sum of gains and losses.
lb. g. $.39	$.53 gain.
7 lb. g. $.14	
5 lb. l. $.05	$.53 loss.
12 lb. l. $.48	

Quantities taken.	Sum of gains and losses.
29 lb. g. $ .87	$2.53 gain.
83 lb. g. $1.66	
lb. l.	loss.
26 lb. l. $1.04	

(*e.*) If it should be necessary to make a mixture containing a given number of pounds, we should first get an answer to the question, as though no limit had been specified, and then find how many times as much should be taken to give the required quantity.

Suppose, for instance, that the above question had read, "How many pounds of each kind must he take to make a mixture of 100 lb. worth 9 cents per lb.," we should have the following additional work:—

The first solution gives a mixture containing 38 lb.; and since 100 lb. $= 2\frac{12}{19}$ times 38 lb., we must take $2\frac{12}{19}$ times as much of each of the former ingredients as before, which would give $2\frac{12}{19}$ times 8 lb., or $21\frac{1}{19}$ lb. of the first; $2\frac{12}{19}$ times 11 lb., or $28\frac{18}{19}$ lb. of the second; $2\frac{12}{19}$ times 10 lb., or $26\frac{6}{19}$ lb. of the third; and $2\frac{12}{19}$ times 9 lb., or $23\frac{13}{19}$ lb. of the fourth. The proof is the same as though these quantities had been originally selected.

5. A trader has coffees at 8, 10, 13, and 15 cents per lb. How many pounds of each may he take to make a mixture worth 12 cents per lb.?

6. A trader has molasses at 22, 25, 29, and 33 cents per gallon. How many gallons of each kind may he take to make a mixture worth 26 cents per gallon?

7. A trader has oils at $.95, $1.20, $1.42, and $1.60 per gallon, of which he wishes to make a mixture worth $1.25 per gallon. How many gallons of each kind may he take?

8. A trader wishes to mix 50 lb. of sugar at 7 cents per lb., and 30 lb. at 10 cents, with such quantities at 9 and 6 cents per lb. as will make a mixture worth 8 cents per lb. How many pounds of each may he take?

9. A trader wishes to mix 40 lb. of tea at 40 cents per lb., 30 lb. at 24 cents, and 50 lb. at 45 cents, with enough at 30 cents to make a mixture worth 35 cents per lb. How many pounds of the last must he take?

10. I have salt at 33, 37, and 50 cents per bushel. How many bushels of each kind may I take to make a mixture of 100 bushels worth 40 cents per bushel?

11. A farmer has oats worth 42 cents, barley worth 64 cents, rye worth 87 cents, and wheat worth $1.38 per bushel. How many bushels of each kind may he take to make a mixture of 200 bushels worth 75 cents per bushel?

SECTION XV.

INTEREST.

166. *Introductory.*

When a person hires an article of property of another, it is evident that, at the expiration of the time for which he hires it, he ought to return it, and pay for its use. Moreover, the sum paid for the use of the borrowed article should be proportioned both to its value and the length of time it is kept.

For instance, if I hire two houses, one of which is worth twice as much as the other, I ought to pay twice as much per year for the first as for the second. If the values of the houses are alike, and one is kept one half as long as the other, only one half as much ought to be paid for the first as for the second.

To the Teacher. — It will be well to illustrate the above important principles by questions similar in character to the following: —

If one man hires a horse to go a certain distance, and another hires one to go twice as far, how many times as much ought the second to pay for its use as the first pays? What would have been the answer to the above question, provided the second man had gone 3 times as far as the first? 4 times as far? $3\frac{1}{2}$ times as far? $\frac{1}{2}$ as far? $\frac{5}{6}$ as far? $\frac{2}{3}$ as far? &c. If the horses are hired by the hour, and the first man keeps his horse three times as many hours as the second keeps his, how many times as much ought he to pay for the use of it? What would have been the answer had he kept it 5 times as long as the second? 8 times as long? 6 times as long? $\frac{1}{2}$ as long? $\frac{3}{7}$ as long? $\frac{1}{6}$ as long? &c.

Similar questions should be asked with reference to other objects hired, as houses, money, &c., till the principle is fully understood.

167. *Definitions.*

(*a*.) Money is very frequently hired, and the sum to be paid for its use is determined in accordance with the above principles. (See 177th page, Ex. 24, Note.)

(*b*.) Money thus paid for the use of money is called INTEREST.

(*c*.) The money used is called the PRINCIPAL.

(*d*.) The principal and interest added together form the AMOUNT, or entire sum due at any given time.

(*e*.) The interest of any principal is usually reckoned at a certain *per cent*, i. e., a certain number of *one hundredths* of that principal, for each year it is on interest. This per cent is called the RATE PER CENT, or simply the RATE.

NOTE. — The term *per cent*, from the Latin *per centum*, originally meant *by the hundred;* but as it is now used in arithmetic, it means *one hundredths.* Thus 6 per cent means $\frac{6}{100}$, or .06; 4 per cent means

$\frac{4}{100}$, or .04; $\frac{1}{2}$ per cent means $\frac{\frac{1}{2}}{100}$, or $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$; &c. This term may be applied to any thing else as well as money; and hence the definition (often given in the school room) "so many cents on 100 cents" is not a good one, any more than would be, *so many bushels on* 100 *bushels*, or *so many yards on* 100 *yards*. It is the more objectionable because scholars are sometimes led by it, and by being often called upon to use the term *per cent* in connection with money, to suppose that it has some necessary connection with cents, or with United States money

168. *Legal Rate.*

(*a.*) In most countries, laws have been passed regulating in some way or other the rates of interest.

(*b.*) Such laws are called Usury Laws.

(*c.*) They commonly embrace the following particulars:—

First. They fix the rate which shall be paid when no special rate has been agreed upon by the parties. This is called the Legal Rate.

Second. They forbid persons to receive interest at more than some given rate.

Third. They impose penalties for their violation.*

(*d.*) Any excess over the rates allowed by these laws is called Usury.

(*e.*) In most states of the Union, the legal rate is also the highest rate allowed by law, even on special contracts. The exceptions to this are named in the following statement of the legal rates of interest in the several states.

(*f.*) In New York, South Carolina, Georgia, Michigan, and Wisconsin, the legal rate of interest is 7 per cent per year.

* It may not be amiss to remark that the laws regulating the rate of interest are very often disregarded, while the penalties for their violation are seldom imposed. Very few men continue long in business without paying or receiving interest at more than the legal rate. Money, having, like every thing else, a variable value, will bring what it is worth at the time it is sold or let, and it seems as impossible to regulate by law the price which shall be paid for its use, as to fix by law that which shall be paid for the use of any other article of property

(*g.*) In Alabama and Texas, it is 8 per cent per year.

(*h.*) In Louisiana, it is 5 per cent.

(*i.*) In California, it is 10 per cent.

(*k.*) In all the other states, the legal rate is 6 per cent per year.

(*l.*) By special agreement of the parties, interest may be charged at the rate of 8 per cent per annum in Florida, Mississippi, and Louisiana; at the rate of 10 per cent in Arkansas, Illinois, Michigan, Iowa, and Ohio; at the rate of 12 per cent in Wisconsin and Texas; and at the rate of 18 per cent in California.

(*m.*) On debts in favor of the United States, interest is computed at the rate of 6 per cent.

(*n.*) In each state, interest is reckoned at the legal rate of that state, unless otherwise specified.

(*o.*) In the United States, it is customary, when reckoning interest, to regard a year as 12 months, and a month as 30 days. But in England, the year is regarded as 365 days, and the number of days in the months considered are reckoned as in the calendar.

(*p.*) In England, the legal rate is 5 per cent.

169. *Interest for* 2 *Months,* 200 *Months, &c., at* 6 *per cent.*

(*a.*) If the interest of any sum for one year is 6 per cent of that sum, for $\frac{1}{6}$ of 1 year, or 2 months, it must be $\frac{1}{6}$ of 6 per cent, or 1 per cent of it.

(*b.*) Therefore, at 6 per cent per year, the interest for 2 months is 1 per cent, or .01 of the principal, and may be found by removing the decimal point of the written principal two places towards the left.

Thus, the interest of $75 for 2 months is $.75; of $364.30 is $3.643, &c., &c.

What is the interest of each of the following sums for 2 months?

1. $84	3. $327.41 *	5. $838.75
2. $687	4. $637.86	6. $3.86

7. What is the amount of each of the above sums for 2 months?

(*c.*) If the interest of any sum for 2 months is .01 of that sum, its interest for .1 of 2 months, which is 6 days, must be .1 of .01, or .001 of it.

(*d.*) Therefore, at 6 per cent per year, the interest for 6 days is .001 of the principal, and may be found by removing the decimal point three places to the left.

Thus, the interest of $987 for 6 days is $.987; of $439 is $.439; of $8763.72 is $8.764; † or, carrying the values no farther than cents, the interest of $987 is $.99; of $439 is $.44; of $8763.72 is $8.76; &c.

What is the interest of each of the following sums for 6 days?

8. $586	10. $67	12. $1473.87
9. $930	11. $36.75	13. $142

14. What is the amount of each of the above sums for 6 days?

(*e.*) If the interest of any sum for 2 months is 1 per cent of that sum, its interest for 100 times 2 months, or 200 months, must equal 100 times 1 per cent, or 100 per cent of it, which is the sum itself.

(*f.*) Therefore, at 6 per cent per year, the interest for 200 months, or 16 years 8 months, must equal the principal.

Thus, the interest of $47 for 200 months is $47; of $37.84 is $37.84; of $23 is $23; &c.

What is the interest of each of the following sums for 16 yr. 8 mo., or 200 mo.?

15. $38.73	17. $67.95	19. $.28
16. $57	18. $2763	20. $1.07

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* The denominations below mills need not be given in the answer. Indeed, those below cents need not be given, if, when there are more than 5 mills, 1 be added to the number of cents.

† Since $.00072 = more than ½ of a mill.

21. What is the amount of each of the above sums for 200 months?

(*g.*) If the interest of any sum for 200 months is equal to that sum, its interest for $\frac{1}{10}$ of 200 months, or 20 months, must equal $\frac{1}{10}$ of it.

(*h.*) Therefore, at 6 per cent per year, the interest for 20 months, or 1 year and 8 months, is $\frac{1}{10}$ of the principal, and may be found by removing the decimal point one place to the left.

Thus, the interest of \$63 for 20 months, or 1 yr. 8 mo., is \$6.30; of \$78.60 is \$7.86; of \$8.79 is \$.879, or, carrying the result only to cents, \$.88

What is the interest of each of the following sums for 20 months, or 1 yr. 8 mo.?

22. \$73.86	24. \$673.70	26. \$5.87
23. \$578	25. \$48.63	27. \$63

28. What is the amount of each of the above sums for 20 months?

170. *Recapitulation and Inferences.*

(*a.*) The importance of the above deductions is such as to demand that they should be made perfectly familiar by all who would become expert in casting interest at 6 per cent. We therefore repeat them.

When interest is 6 per cent per year, —

First. *The interest of any sum for* 200 *months, or* 16 *yr.* 8 *mo., will equal that sum.*

Second. *The interest of any sum for* 20 *months, or* 1 *yr.* 8 *mo., will equal* $\frac{1}{10}$ *of that sum, or as many dimes as there are dollars in the principal.*

Third. *The interest of any sum for* 2 *months will equal* .01 *of that sum, or as many cents as there are dollars in the principal.*

Fourth. *The interest of any sum for* 6 *days will equal* .001 *of that sum, or as many mills as there are dollars in the principal.*

(*b.*) It is therefore evident that for $\frac{1}{2}$ of 200 months, the interest will equal $\frac{1}{2}$ of the principal; for $\frac{1}{3}$ of 200 months, $\frac{1}{3}$ of the principal; &c.

(*c.*) For $\frac{1}{2}$ of 20 months, the interest will equal $\frac{1}{2}$ of $\frac{1}{10}$, or $\frac{1}{20}$ of the principal; for $\frac{1}{6}$ of 20 mo., $\frac{1}{6}$ of $\frac{1}{10}$, or $\frac{1}{60}$ of the principal; for 3 times 20 mo., 3 times .1, or .3 of the principal; &c.

(*d.*) For $\frac{1}{3}$ of 2 months, the interest will equal $\frac{1}{3}$ of .01, or $\frac{1}{300}$ of the principal; for $\frac{1}{4}$ of 2 months, $\frac{1}{4}$ of .01, or $\frac{1}{400}$ of the principal; for 7 times 2 months, 7 times .01, or .07 of the principal; &c.

(*e.*) For $\frac{1}{2}$ of 6 days, the interest will be $\frac{1}{2}$ of .001, or $\frac{1}{2000}$ of the principal; for 3 times 6 days, 3 times .001, or .003 of the principal; &c.

171. *Table showing Interest for convenient Fractional Parts of* 200 *months,* 20 *months, &c.*

At 6 per cent per year, the interest for —

200 mo., or 16 yr. 8 mo., =	prin.
$\frac{1}{2}$ of 200 mo., or 100 mo., or 8 yr. 4 mo., =	$\frac{1}{2}$ of prin.
$\frac{1}{3}$ of 200 mo., or $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da., =	$\frac{1}{3}$ of prin.
$\frac{1}{4}$ of 200 mo., or 50 mo., or 4 yr. 2 mo., =	$\frac{1}{4}$ of prin.
$\frac{1}{5}$ of 200 mo., or 40 mo., or 3 yr. 4 mo., =	$\frac{1}{5}$ of prin.
$\frac{1}{6}$ of 200 mo., or $33\frac{1}{3}$ mo., or 2 yr. 9 mo., 10 da., =	$\frac{1}{6}$ of prin.
$\frac{1}{8}$ of 200 mo., or 25 mo., or 2 yr. 1 mo., =	$\frac{1}{8}$ of prin.
$\frac{1}{10}$ of 200 mo., or 20 mo., or 1 yr. 8 mo., =	$\frac{1}{10}$ of prin.
$\frac{1}{12}$ of 200 mo., or $16\frac{2}{3}$ mo., or 1 yr. 4 mo. 20 da. =	$\frac{1}{12}$ of prin.
$\frac{1}{15}$ of 200 mo., or $13\frac{1}{3}$ mo., or 1 yr. 1 mo. 10 da. =	$\frac{1}{15}$ of prin.
$\frac{1}{16}$ of 200 mo., or $12\frac{1}{2}$ mo., or 1 yr. 15 da., =	$\frac{1}{16}$ of prin.
$\frac{1}{2}$ of 20 mo., or 10 mo., =	$\frac{1}{2}$ of $\frac{1}{10}$ of prin.
$\frac{1}{3}$ of 20 mo., or $6\frac{2}{3}$ mo, or 6 mo. 20 da., =	$\frac{1}{3}$ of $\frac{1}{10}$ of prin.
$\frac{1}{4}$ of 20 mo., or 5 mo., =	$\frac{1}{4}$ of $\frac{1}{10}$ of prin.
$\frac{1}{5}$ of 20 mo., or 4 mo., =	$\frac{1}{5}$ of $\frac{1}{10}$ of prin.
$\frac{1}{6}$ of 20 mo., or $3\frac{1}{3}$ mo., or 3 mo. 10 da., =	$\frac{1}{6}$ of $\frac{1}{10}$ of prin.
$\frac{1}{8}$ of 20 mo., or $2\frac{1}{2}$ mo., or 2 mo. 15 da., =	$\frac{1}{8}$ of $\frac{1}{10}$ of prin.

$\frac{1}{10}$ of 20 mo., or 2 mo., =	$\frac{1}{10}$ of $\frac{1}{10}$ of prin.
$\frac{1}{12}$ of 20 mo., or $1\frac{2}{3}$ mo., or 1 mo. 20 da., =	$\frac{1}{12}$ of $\frac{1}{10}$ of prin.
$\frac{1}{15}$ of 20 mo., or $1\frac{1}{3}$ mo., or 1 mo. 10 da., =	$\frac{1}{15}$ of $\frac{1}{10}$ of prin.
$\frac{1}{2}$ of 2 mo., or 1 mo., =	$\frac{1}{2}$ of $\frac{1}{100}$ of prin.
$\frac{1}{3}$ of 2 mo., or $\frac{2}{3}$ of 1 mo., or 20 da., =	$\frac{1}{3}$ of $\frac{1}{100}$ of prin.
$\frac{1}{4}$ of 2 mo., or $\frac{1}{2}$ of 1 mo., or 15 da., =	$\frac{1}{4}$ of $\frac{1}{100}$ of prin.
$\frac{1}{5}$ of 2 mo., or $\frac{2}{5}$ of 1 mo., or 12 da., =	$\frac{1}{5}$ of $\frac{1}{100}$ of prin.
$\frac{1}{6}$ of 2 mo., or $\frac{1}{3}$ of 1 mo., or 10 da., =	$\frac{1}{6}$ of $\frac{1}{100}$ of prin.
$\frac{1}{10}$ of 2 mo., or $\frac{1}{5}$ of 1 mo., or 6 da., =	$\frac{1}{10}$ of $\frac{1}{100}$ of prin
$\frac{1}{15}$ of 2 mo., or $\frac{2}{15}$ of 1 mo., or 4 da., =	$\frac{1}{15}$ of $\frac{1}{100}$ of prin.
$\frac{1}{2}$ of 6 da., or 3 da., =	$\frac{1}{2}$ of $\frac{1}{1000}$ of prin.
$\frac{1}{3}$ of 6 da., or 2 da., =	$\frac{1}{3}$ of $\frac{1}{1000}$ of prin.
$\frac{1}{6}$ of 6 da., or 1 da., =	$\frac{1}{6}$ of $\frac{1}{1000}$ of prin.

172. *Questions on the preceding Table.*

1. How long must a principal be on interest that the interest may equal $\frac{1}{2}$ of it?

2. $\frac{1}{10}$?
3. $\frac{1}{16}$?
4. $\frac{1}{2}$ of $\frac{1}{10}$?
5. $\frac{1}{5}$ of $\frac{1}{10}$?
6. $\frac{1}{20}$?
7. $\frac{1}{40}$?
8. $\frac{1}{15}$ of $\frac{1}{10}$?
9. .01?
10. $\frac{1}{5}$ of .01?
11. $\frac{1}{600}$?
12. .001?
13. $\frac{1}{3000}$?
14. $\frac{1}{4}$?
15. $\frac{1}{5}$?
16. $\frac{1}{6}$ of $\frac{1}{10}$?
17. $\frac{1}{8}$ of $\frac{1}{10}$?
18. $\frac{1}{50}$?
19. $\frac{1}{120}$?
20. $\frac{1}{80}$?
21. $\frac{1}{200}$?
22. $\frac{1}{6}$ of .01?
23. $\frac{1}{1200}$?
24. $\frac{1}{2000}$?
25. $\frac{1}{6000}$?
26. $\frac{1}{1500}$?
27. $\frac{1}{15}$?
28. $\frac{1}{12}$?
29. $\frac{1}{3}$ of $\frac{1}{10}$?
30. $\frac{1}{4}$ of $\frac{1}{10}$?
31. $\frac{1}{30}$?
32. $\frac{1}{12}$ of $\frac{1}{10}$?
33. $\frac{1}{150}$?
34. $\frac{1}{2}$ of .01?
35. $\frac{1}{500}$?
36. $\frac{1}{300}$?
37. $\frac{1}{2}$ of $\frac{1}{1000}$?
38. $\frac{1}{6}$ of .001?

173. *Applications of the foregoing Table.*

1. What is the interest of \$156.96 for each time mentioned in the table?

Answer. — The interest of \$156.96 for 200 mo. = \$156.96; for 100 mo., or 8 yr. 4 mo., = $\frac{1}{2}$ of \$156.96; for $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da., = $\frac{1}{3}$ of \$156.96; &c.

The interest of \$156.96 for 20 mo. = \$15.696; for 10 mo. = $\frac{1}{2}$ of \$15.696; for $6\frac{2}{3}$ mo., or 6 mo. 20 da., = $\frac{1}{3}$ of \$15.696; &c.

The interest of \$156.96 for 2 mo. = \$1.57; for 1 mo. = $\frac{1}{2}$ of \$1.57; for 20 da. = $\frac{1}{3}$ of \$1.57; &c.

NOTE. — The object in giving the answer in the above form is to direct the pupil's attention exclusively to the method of computing the interest; but as soon as he has had practice enough to enable him to tell at once what part of the principal, or of some convenient part of the principal, the interest for any of the above-mentioned times is, he should compute the interest in each case. Thus, the interest of \$156.96 for 100 mo., or 8 yr. 4 mo., = $\frac{1}{2}$ of \$156.96, which is \$78.48; for $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da., = $\frac{1}{3}$ of \$156.96, which is \$52.32; &c.

After a little practice, the final answer may be read at once. Thus, —

The interest of \$156.96 for 100 mo., or 8 yr. 4 mo., is \$78.48; for $66\frac{2}{3}$ mo., or 5 yr. 6 mo. 20 da., is \$52.32; &c.

What is the interest for each of the above-named times of,

2. \$48?	4. \$72?	6. \$24.96?
3. \$42.24?	5. \$144?	7. \$35.42?

8. What is the interest of \$36 for 3 yr. 4 mo.?
9. What is the interest of \$24.72 for $16\frac{2}{3}$ mo.?
10. What is the interest of \$75 for 2 yr. 9 mo. 10 da.?
11. What is the interest of \$54 for 5 yr. 6 mo. 20 da.?
12. What is the interest of \$324 for $33\frac{1}{3}$ mo.?
13. What is the interest of \$231.12 for 1 yr. 15 da.?
14. What is the interest of \$42.24 for 2 yr. 1 mo.?
15. What is the interest of \$500 for $66\frac{2}{3}$ mo.?
16. What is the interest of \$42.24 for 1 yr. 8 mo.?
17. What is the interest of \$150 for 6 mo. 20 da.?
18. What is the interest of \$4.80 for 5 mo.?
19. What is the interest of \$17.70 for 2 mo. 15 da.?
20. What is the interest of \$87.18 for 1 mo. 20 da.?
21. What is the interest of \$537 for 2 mo.?
22. What is the interest of \$288 for 10 da.?

23. What is the interest of $96.34 for 20 da.?
24. What is the interest of $675 for 2 da.?
25. What is the interest of $438.74 for 3 yr. 4 mo.?
26. What is the interest of $73.87 for 1 yr. 15 da.?
27. What is the interest of $144 for 1 yr. 1 mo. 10 da.?
28. What is the interest of $56 for 4 yr. 2 mo.?
29. What is the interest of $1728 for 1 yr. 4 mo. 20 da.?
30. What is the interest of $327.29 for 16 yr. 8 mo.?
31. What is the interest of $44.20 for 10 mo.?
32. What is the interest of $57.60 for 3 mo. 10 da.?
33. What is the interest of $43.65 for 4 mo.?
34. What is the interest of $7.65 for 1 mo. 10 da.?
35. What is the interest of $97.20 for 6 mo. 20 da.?
36. What is the interest of $237.50 for 1 mo.?
37. What is the interest of $541 for 5 da.?
38. What is the interest of $9741 for 6 da.?
39. What is the interest of $82.47 for 3 da.?
40. What is the interest of $32.76 for 2 yr. 9 mo. 10 da.?
41. What is the interest of $93.27 for 6 mo. 20 da.?

174. *Interest for various Times.*

(*a.*) In computing interest for other times than those already mentioned, it is usually most convenient to divide the time into parts, as illustrated below.

(*b.*) The student should bear in mind that in the forms of written work here, as elsewhere, the letter a, b, c, &c., are used merely to indicate how the numbers standing opposite them have been obtained. In practical work they should be omitted.

1. What is the interest of $196.72 for 8 mo. 20 da.?

First Solution.

$$\begin{array}{rrl} a = & \$196.72 & = \text{Principal.} \\ \hline \frac{1}{30} \text{ of } a = b = & 6.557 & = \text{Int. for 6 mo. 20 da} \\ .01 \text{ of } a = c = & 1.967 & = \text{Int. for 2 mo.} \\ \hline b + c = \$ & 8.524 & = \text{Int. for 8 mo. 20 da.} \end{array}$$

Second Solution.

$$\begin{array}{rrl} a = & \$196.72 & = \text{Principal} \\ \hline .04 \text{ of } a = b = & 7.869 & = \text{Int. for 8 mo.} \\ \frac{1}{300} \text{ of } a = c = & .655 & = \text{Int. for 20 da.} \\ \hline b + c = \$ & 8.524 & = \text{Int. for 8 mo. 20 da.} \end{array}$$

Third Solution.

$$\begin{array}{rrl} a = & \$196.72 & = \text{Principal.} \\ \hline \frac{1}{20} \text{ of } a = b = & 9.836 & = \text{Int. for 10 mo.} \\ \frac{1}{150} \text{ of } a = c = & 1.311 & = \text{Int. for 1 mo. 10 da.} \\ \hline b - c = \$ & 8.525 & = \text{Int. for 8 mo. 20 da.} \end{array}$$

Fourth Solution.

Since at 6 per cent the interest for 1 month equals $\frac{1}{2}$ of 1 per cent of the principal, the interest for 8 months must equal 4 per cent of the principal; and since the interest for 1 day equals $\frac{1}{6}$ of .001 of the principal, the interest for 20 days must equal $\frac{20}{6}$ of .001, or $.003\frac{1}{3}$ of the principal. Hence, the interest of \$169.72 for 8 mo. 20 da. $= .04 + .003\frac{1}{3}$ $= .043\frac{1}{3}$ of \$169.72, which, found by the usual method, gives \$8.525 = interest for 8 mo. 20 da.

2. What is the amount of \$649.37 for 17 mo. 15 da.?

First Solution.

$$\begin{array}{rrl} a = & \$649.37 & = \text{Principal.} \\ \hline \frac{1}{12} \text{ of } a = b = & 54.114 & = \text{Int. for 16 mo. 20 da.} \\ \frac{1}{20}{*} \text{ of } b = c = & 2.705 & = \text{Int. for 25 da.} \\ \hline a + b + c = & \$706.189 & = \text{Amt. 17 mo. 15 da.} \end{array}$$

Second Solution.

$$\begin{array}{rrl} a = & \$649.37 & = \text{Principal.} \\ \hline \frac{1}{16} \text{ of } a = b = & 40.585 & = \text{Int. for 12 mo. 15 da.} \\ \frac{1}{40} \text{ of } a = c = & 16.234 & = \text{Int. for 5 mo.} \\ \hline a + b + c = & \$706.189 & = \text{Amt. for 17 mo. 15 da.} \end{array}$$

* Since 16 mo. 20 da. = 500 da.

Third Solution.

$$\begin{aligned} a &= \$649.37 = \text{Principal.} \\ \tfrac{1}{10} \text{ of } a = b &= \quad 64.937 = \text{Int. for 20 mo.} \\ \tfrac{1}{8} \text{ of } b = c &= \quad 8.117 = \text{Int. for 2 mo. 15 da.} \\ \hline a + b - c &= \$706.190 = \text{Amt. for 17 mo. 15 da.} \end{aligned}$$

Fourth Solution.

$$\begin{aligned} a &= \$649.37 = \text{Principal.} \\ \tfrac{1}{20} \text{ of } a = b &= \quad 32.468 = \text{Int. for 10 mo.} \\ \tfrac{1}{40} \text{ of } a, \text{ or } \tfrac{1}{2} \text{ of } b, = c &= \quad 16.234 = \text{Int. for 5 mo.} \\ \tfrac{1}{80} \text{ of } a, \text{ or } \tfrac{1}{2} \text{ of } c, = d &= \quad 8.117 = \text{Int. for 2 mo. 15 da} \\ \hline a + b + c + d &= \$706.189 = \text{Amt. for 17 mo. 15 da.} \end{aligned}$$

Fifth Solution.

Since at 6 per cent the interest for 1 month is $\frac{1}{2}$ of 1 per cent of the principal, the interest for 17 months must equal $\frac{17}{2}$ of 1 per cent $=$ $.08\frac{1}{2} = .085$ of the principal; and since for 1 day the interest equals $\frac{1}{6}$ of .001 of the principal, for 16 days it must equal $\frac{16}{6}$ of .001, or $.002\frac{2}{3}$ of the principal. Hence, the interest of \$649.37 for 17 mo. 15 da. $=$ $.085 + .002\frac{2}{3} = .087\frac{2}{3}$ of \$649.37, which, found by the usual method, gives \$56.819, the interest, which, added to the principal, gives the amount, \$706.189.

NOTE. — Many other solutions might have been given to the above questions, but as they would all involve similar principles, it is unnecessary to add them. Every question in interest admits a great variety of solutions, and the pupil should examine it carefully to determine which he will adopt. Practice will enable him to select a good method at once. One process may be applied to test the correctness of a result obtained by some other. We may remark, that, as a general thing, it is better to divide than to multiply, for in division we have to consider no denomination below the lowest we wish to have in the answer.

3. What is the interest of \$857.63 for 3 mo. 16 da.?
4. What is the interest of \$875.37 for 1 mo. 26 da.?
5. What is the interest of \$93.75 for 9 mo. 29 da.?
6. What is the interest of \$178.43 for 16 mo. 14 da.?
7. What is the interest of \$343.65 for 13 mo. 16 da.?
8. What is the interest of \$237.64 for 19 mo. 24 da.?
9. What is the interest of \$478.96 for 17 mo. 26 da.
10. What is the interest of \$375.81 for 22 mo. 15 da.?
11. What is the interest of \$58.27 for 96 mo. 20 da.?

12. What is the interest of $5789 for 29 mo. 29 da.?
13. What is the interest of $80.32 for 7 mo. 19 da.?
14. What is the interest of $175 for 1 mo. 17 da.?
15. What is the interest of $326 for 8 mo. 23 da.?
16. What is the interest of $27.96 for 1 yr. 3 mc 13 da.?
17. What is the amount of $578.31 for 3 yr. 7 mo. 28 da.?
18. What is the amount of $724.16 for 7 yr. 2 mo. 11 da.?
19. What is the amount of $4369.87 for 3 mo. 26 da.?
20. What is the amount of $25.50 for 9 mo. 27 da.?
21. What is the amount of $117.58 for 3 yr. 1 mo. 18 da.?
22. What is the amount of $313.27 for 6 mo. 9 da.?
23. What is the amount of $57.75 for 9 mo. 1 da.?
24. What is the amount of $35.86 for 11 mo. 25 da.?
25. What is the amount of $17.64 for 1 yr. 1 mo. 13 da.?
26. What is the amount of $378.51 for 1 yr. 5 mo. 17 da.?

175. *Computation of Time, and Application to Problems.*

(*a.*) In business transactions, it is usually necessary to compute the time during which money has been on interest; that is, the time between the dates on which interest began and ended.

(*b.*) The usual method of doing this is to reckon the number of entire years, then the number of entire calendar months remaining, and then the remaining days.

(*c.*) The year is (as before) regarded as 360 days, or 12 months of 30 days each, and each entire calendar month as a month of 30 days; but the days which are left after reckoning the years and months are determined by counting them according to the number in the months in which they occur.

1. What is the time from Jan. 17, 1845, to June 28, 1849?

Solution.—From Jan. 17, 1845, to Jan. 17, 1849, is 4 years; from Jan. 17 to June 17 is 5 months; from June 17 to June 28 is 11 days. Therefore the required time is 4 yr. 5 mo. 11 da.

2. What is the time from April 27, 1846, to Feb. 13, 1851?

Solution. — From April 27, 1846, to April 27, 1850, is 4 years; from April 27, 1850, to Jan. 27, 1851, is 9 months; January having 31 days, there are 4 days left in it, which, added to the 13 in February, give 17 days. Therefore the required time is 4 yr. 9 mo. 17 da.

3. What is the time from Sept. 24, 1849, to March 20, 1852?

Solution. — From Sept. 24, 1849, to Sept. 24, 1851, is 2 years; from Sept. 24, 1851, to Feb. 24, 1852, is 5 months; 1852 being leap year, February has 29 days; hence, there are 5 days left in it, which, added to the 20 in March, give 25 days. Therefore the required time is 2 yr. 5 mo. 25 da.

NOTE. — This method of computing the time, though the one usually adopted by business men when interest is computed for months and days, is unequal in its operation; for the calendar months, though varying in length from 28 to 31 days, are all reckoned as months of 30 days each. Hence, the interest of a sum during the month of February will be as much as during either March or April, though February contains 3 days less than March, and 2 less than April.

By this method, the interest on four notes dated respectively on the 28th, 29th, 30th, and 31st of any one month, and paid on any one day between the 1st and 28th of March, of any year except leap year, would be computed for the same time. Suppose, for instance, that they are dated in October, 1850, and paid March 15, 1851. Then for the first note dated Oct. 28, the time will be found without difficulty to be 4 mo. 15 da. In calculating the time on the others, we proceed thus: Since there are not as many as 29 days in February, 1851, we reckon from the 29th, 30th, or 31st of October to the last day of February as 4 months, to which adding the 15 days in March gives 4 months and 15 days as the time, in each case. The restriction with reference to notes paid in leap year is necessary simply because February has then 29 days. The proposition will *always* be true of notes dated on the 29th, 30th, and 31st, of any month, and paid at any time between the 1st and 28th of March.

Again. Four notes dated respectively on the 28th, 29th, 30th, and 31st of August of any year except the one immediately preceding leap year, and payable in 6 months, would all become due on the same day.

The only strictly accurate method of reckoning time is to actually count the days in each month we consider. Thus, to reckon the time from October 28, 1850, to March 15, 1851, we proceed as follows: From October 28th to 31st, is 3 days; to which adding the 30 days in November, the 31 in December, the 31 in January, the 28 in February, and the 15 in March, gives 138 days as the true time between the two dates.

In England the time is always computed in this way, as it is in this country when notes are payable at the end of a certain number of *days*.

4. What is the time from June 23, 1850, to June 3, 1852?

Answer. 1 yr. 11 mo. 11 da.

5. What is the time from May 13, 1847 to Oct. 8, 1851?

6. What is the time from Jan. 31, 1851, to March 23, 1852?

7. What is the time from Nov. 17, 1849, to Dec. 12, 1851?

8. What is the time from Dec. 31, 1848, to July 6, 1850?

What is the interest —

9. Of $787.36 from May 3, 1843, to Dec. 17, 1845?

10. Of $54.76 from Feb. 14, 1840, to June 2, 1844?

11. Of $476.35 from June 30, 1847, to Dec. 28, 1850?

12. Of $638.29 from May 31, 1851, to Oct. 7, 1852?

13. Of $4937.56 from Dec. 19, 1843, to Feb. 16, 1847?

14. Of $481.74 from Jan. 29, 1847, to March 25, 1851?

15. Of $587.60 from Jan. 31, 1850, to July 18, 1852?

What is the amount —

16. Of $947.84 from May 15, 1850, to June 13, 1851?

17. Of $748.67 from Dec. 14, 1849, to May 4, 1851?

18. Of $1546.61 from April 9, 1847, to June 1, 1851?

19. Of $917.68 from June 5, 1842, to Jan. 1, 1850?

20. Of $8396.58 from April 30, 1847, to March 22, 1850?

21. Of $1449.13 from Dec. 31, 1850, to March 5, 1852?

22. Jan. 15, 1852, George W. Pratt borrowed $237.50 of A. N. Johnson, and Feb. 13, 1852, he borrowed $438.75 more, agreeing to pay interest at 6 per cent per year. He paid the debts March 8, 1852. What was their amount?

23. I have three notes against Arthur Sumner, viz., one for $548.17, dated Jan. 1, 1851, another for $679.18, dated Jan. 27, 1852, and another for $376.89, dated May 31, 1852. What amount will be due on all of them July 8, 1852?

24. Jan. 1, 1851, I borrowed $3468, with which I purchased flour at $6 per barrel. I sold the flour March 17, 1851, for $6.50 per barrel, cash. Interest being reckoned at

6 per cent, did I gain or lose by the transaction, and how much?

176. *Interest by Days.*

(*a.*) Many business men always reduce the time to days, and compute the interest by the method illustrated below. As a general thing, however, this method is not so convenient as the preceding.

(*b.*) Since, at 6 per cent per year, the interest for 1 day is $\frac{1}{6}$ of $\frac{1}{1000}$ of the principal, it follows that the interest for any number of days must be $\frac{1}{6}$ as many thousandths of the principal as there are days.

(*c.*) We may, therefore, find the interest for any number of days, by multiplying the principal by $\frac{1}{6}$ of the number of days, and removing the point three places farther to the left.

It will make no difference with the result, whether we multiply by $\frac{1}{6}$ of the number of days, or by the number of days and divide by 6; but it will usually be better to divide before multiplying.

1. What is the interest of $437.62 for 4 mo. 3 da.?

Solution. — Since 4 mo. 3 da. = 123 da., the required interest must be $\frac{1}{6}$ of .123 of the principal, which is .020$\frac{1}{2}$ of the principal. The work carried to mills would be written thus: —

$$\begin{array}{r} \$437.62 \\ .020\frac{1}{2} \\ \hline 8.752 \\ .218 \\ \hline \$8.970 \end{array} = \textit{Ans.}$$

2. What is the interest of $54.57 for 4 mo. 20 da.?
3. What is the interest of $397.42 for 8 mo. 15 da.?
4. What is the interest of $231.48 for 7 mo. 12 da.?
5. What is the interest of $438.64 for 5 mo. 24 da.?
6. What is the interest of $281.87 for 5 mo. 9 da.?
7. What is the interest of $581.21 for 6 mo. 24 da.?
8. What is the interest of $83.25 for 2 mo. 21 da.?
9. What is the interest of $98.37 for 6 mo 18 da.?

177. *Interest by Dollars, for Months and convenient Parts of a Month.*

(*a.*) We can frequently compute interest with great ease by first finding the interest for 1 day, 1 month, or 1 year, and getting the required interest from this.

(*b.*) When the interest for any of the above times is any convenient sum, as 1 dollar, 1 dime, 1 cent, 1 mill, $\frac{1}{2}$ dollar, $\frac{1}{3}$ dollar, &c., the proposed course will be particularly advantageous.

(*c.*) Since, at 6 per cent per year, the interest of any sum for 1 month is $\frac{1}{200}$ of that sum, —

1. *The interest of* $200 *is* $1 *per month.*
2. *The interest of* $20 *is* $.10, *or* 1 *dime per month.*
3. *The interest of* $2 *is* $.01 *per month.*

1. What is the interest of $200 for 7 mo. 15 da.?

Solution. — The interest of $200 is $1 per month; therefore for 7 mo. 15 da., or $7\frac{1}{2}$ mo., it must be $7\frac{1}{2}$ dollars, or $7.50.

What is the interest of $200 for —

2. 5 mo.?	7. 8 mo. 12 da.?
3. 8 mo.?	8. 9 mo. 15 da.?
4. 6 mo. 10 da.?	9. 1 yr. 7 mo.?
5. 1 yr., or 12 mo.?	10. 8 yr. 8 da.?
6. 2 yr. 1 mo., or 25 mo.?	

What is the interest for each of the above-mentioned times —

11. Of $2? | 12. Of $20? | 13. Of $2000?

14. What is the interest of $100 for 7 mo.?

Solution. — Since the interest of $200 is 1 dollar per month, the interest of $100 must be a half dollar per month, and 7 half-dollars, or $3.50, for 7 months.

What is the interest of $100 for —

15. 9 mo.?	20. 3 mo. 15 da.?
16. 1 yr.?	21. 2 yr. 9 mo.?
17. 3 yr. 4 mo.?	22. 8 mo. 10 da.?
18. 15 da.?	23. 1 yr. 2 mo. 20 da.?
19. 20 da.?	

What is the interest for each of the above-mentioned times —

24. Of $10?	27. Of $50?	30. Of $25?
25. Of $1?	28. Of $5?	31. Of $250?
26. Of $1000?	29. Of $500?	32. Of $2500?

178. *Interest by Dollars, when the Time is in Days, or Months and Days.*

(*a.*) Since the interest of any sum for 6 days is .001 of that sum, the interest of 1 dollar for 6 days must be .001 of 1 dollar, which is 1 mill. If the interest of 1 dollar for 6 days is 1 mill, its interest for 1 day must be $\frac{1}{6}$ of 1 mill. Therefore, the interest of $1 is $\frac{1}{6}$ of 1 mill per day.

(*b.*) From this we have the following statements: —

At 6 per cent, —

1. *The interest of* $6 *is* 1 *mill per day.*
2. *The interest of* $60 *is* 1 *cent per day.*
3. *The interest of* $600 *is* 1 *dime per day.*
4. *The interest of* $6000 *is* 1 *dollar per day.*

NOTE. — When the interest is required for months and days, we may reduce the months to days, and then apply the above; or we may find the interest for the months, as in the last article, and then find it for the days, as above. The pupil should remember that the interest of $6 is 3 cents per month, of $60 is 3 dimes per month, and of $600 is 3 dollars per month.

What is the interest of $6 for —

1. 15 da.?	6. 7 mo. 15 da.?
2. 12 da.?	7. 2 yr. 3 mo. 5 da.?
3. 20 da.?	8. 1 yr. 4 mo. 11 da.?
4. 1 mo. 3 da.?	9. 4 yr. 7 mo. 20 da.?
5. 3 mo. 10 da.?	

What is the interest for each of the above-mentioned times —

10. Of $60?	11. Of $600?	12. Of $6000?

13. What is the interest per day of $750?

Solution. — The interest of \$6000 being 1 dollar per day, the interest of \$750, which is $\frac{1}{8}$ of \$6000, must be $\frac{1}{8}$ of 1 dollar per day.

What is the interest per day —

14. Of \$1000?	18. Of \$10?	22. Of \$30?
15. Of \$2000?	19. Of \$150?	23. Of \$180?
16. Of \$4000?	20. Of \$15?	24. Of \$800?
17. Of \$1200?	21. Of \$120?	25. Of \$500?

26. What is the interest per month of each of the above sums?

What is the interest of \$3000 for —

27. 8 da.?	32. 27 da.?
28. 15 da.?	33. 9 da.?
29. 1 mo. 10 da.?	34. 25 da.?
30. 20 da.?	35. 11 da.?
31. 2 mo. 10 da.?	

36. What is the interest of \$1200 for each of the above-mentioned times?

37. Of \$100?	40. Of \$120?	43. Of \$600?
38. Of \$12?	41. Of \$500?	44. Of \$250?
39. Of \$.12?	42. Of \$1.20?	45. Of \$.60?

179. *When to disregard Cents.*

If the time is not very long, the interest of any sum less than a dollar can be computed with sufficient accuracy by referring the principal to the nearest convenient aliquot part of a dollar.

Thus, the interest of 24 cents, for any ordinary time of calculating interest, will differ but a trifle from that of 25 cents, or $\frac{1}{4}$ of a dollar, for the same time; the interest of 35 cents will differ but a trifle from that of $33\frac{1}{3}$ cents, or $\frac{1}{3}$ of a dollar, &c.

1. What is the interest of \$599.77 for 9 mo. 17 da.?

Solution. — \$599.77 = \$600 — 23 cents. The interest of \$600 for 9 mo. 17 da. (being \$3 per month, and 1 dime per day) is \$28.70. As 23 cents is very near 25 cents, its interest must be very near $\frac{1}{8}$ of a cent per month, which will be about 1 cent for 9 mo. 17 da. This taken from \$28.70 leaves \$28.69 as the interest required.

2. What is the interest of $60.49 for 5 mo. 11 da.?
3. What is the interest of $59.67 for 8 mo. 13 da.?
4. What is the interest of $299.51 for 11 mo. 23 da.?
5. What is the interest of $150.32 for 7 mo. 19 da.?
6. What is the interest of $6.24 for 9 mo. 5 da.?
7. What is the interest of $200.42 for 8 mo. 15 da.?
8. What is the interest of $599.66 for 5 mo. 13 da.?
9. What is the interest of $119.94 for 13 mo. 7 da.?
10. What is the interest of $50.31 for 3 mo. 23 da.?

180. *Interest at various Rates, obtained from that at 6 per cent.*

When the interest is other than 6 per cent per year, we may first find the interest at 6 per cent, and then take such part of this as the given rate is of 6 per cent.

Thus, the interest of any sum at 8 per cent is $\frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$ times its interest at 6 per cent; at $4\frac{1}{2}$ per cent the interest is $\frac{4\frac{1}{2}}{6} = \frac{3}{4}$ of the interest at 6 per cent, &c.

1. What is the interest of $367.32 for 1 yr. 9 mo. 20 da., at $7\frac{1}{2}$ per cent?

Solution.

$a =$	$367.32	= principal.
$\frac{1}{10}$ of $a = b =$	36.732	= int. 20 mo. at 6 per cent.
$\frac{1}{12}$ of $b = c =$	3.061	= int. 1 mo. 20 da. at 6 per cent.
$b + c = d =$	39.793	= int. 21 mo. 20 da. at 6 per cent.
$\frac{1}{4}$ of $d = e =$	9.948	= int. 21 mo. 20 da. at $1\frac{1}{2}$ per cent.
$d + e =$	$ 49.741	= int. 1 yr. 9 mo 20 da. at $7\frac{1}{2}$ per cent. = *Answer.*

NOTE. — The work may frequently be facilitated by observing that the interest of any sum at other than 6 per cent is equal to the interest at 6 per cent of the same part of that sum that the required rate is of 6 per cent. Thus, the interest of any sum for a given time at 3 per cent is equal to the interest of $\frac{3}{6}$, or $\frac{1}{2}$, of that sum for the same time at 6 per cent. The interest of any sum at $4\frac{1}{2}$ per cent is equal to the interest of $\frac{4\frac{1}{2}}{6}$, or $\frac{3}{4}$ of that sum, at 6 per cent, &c.

Again. **The interest of any sum at other than 6 per cent is equal to its interest at 6 per cent for the same part of the given time that the required rate is of 6 per cent. Thus, the interest of any sum for a given time at 2 per cent is equal to its interest for $\frac{2}{6}$, or $\frac{1}{3}$, of the given time at 6 per cent.**

What is the interest of —

2. \$847.38 for 2 yr. 4 mo. 10 da., at 5 per cent?
3. \$483.94 for 3 yr. 5 mo. 26 da., at 7 per cent?
4. \$150 for 8 mo. 27 da., at 9 per cent?
5. \$46.38 for 11 mo. 19 da., at 3 per cent?
6. \$512.59 for 4 yr. 7 mo. 17 da., at $6\frac{1}{2}$ per cent?
7. \$437.95 from June 17, 1848, to May 19, 1850, at $6\frac{3}{4}$ per cent?
8. \$978.31 from Jan. 27, 1850, to Sept. 5, 1852, at 5 per cent?
9. \$87.63 from April 21, 1848, to Jan. 7, 1852, at $5\frac{1}{2}$ per cent?
10. \$450 from Aug. 12, 1852, to Oct. 7, 1852, at $7\frac{1}{2}$ per cent?
11. \$75 from Nov. 5, 1850, to Jan. 4, 1852, at 3 per cent?
12. \$240 from Sept. 30, 1848, to May 23, 1851, at $1\frac{1}{2}$ per cent?

181. *Interest at various Rates, obtained directly.*

Methods similar in character to those illustrated in the following examples and explanations will usually be more brief than the preceding.

1. What is the interest of \$549.84 for 1 yr. 4 mo. 15 da., at 8 per cent?

Solution.

$$
\begin{array}{rrl}
a = & \$549.84 & = \text{principal.} \\
.08 \text{ of } a = b = & 43.987 & = \text{int. for 1 yr. at 8 per cent.} \\
\frac{1}{3} \text{ of } b = c = & 14.662 & = \text{int. for 4 mo. at 8 per cent.} \\
\frac{1}{8} \text{ of } c = d = & 1.833 & = \text{int. for 15 da. at 8 per cent.} \\
b + c + d = & \$\ 60.482 & = \text{int. for 1 yr. 4 mo. 15 da. at 8 per cent} =
\end{array}
$$

Answer.

Second Solution. — Since the rate is 8 per cent of the principal, the interest for $1\frac{1}{4}$ years, or 15 months, must be $1\frac{1}{4}$ times 8 per cent = 10 per cent = $\frac{1}{10}$ of the principal. Hence,

$$
\begin{array}{rrl}
 & a = \$549.84 & = \text{principal.} \\
\frac{1}{10} \text{ of } a = b = & 54.984 & = \text{int. for 15 mo. at 8 per cent.} \\
\frac{1}{10} \text{ of } b = c = & 5.498 & = \text{int. for 1 mo. 15 da. at 8 per cent.} \\
b + c = & \$\ 60.482 & = \text{int. for 16 mo. 15 da. at 8 per cent} = \textit{Ans.}
\end{array}
$$

2. What is the interest of \$537.48 for 3 yr. 8 mo. 15 da. at $7\frac{1}{2}$ per cent?

Solution. — The interest for 2 yr. at $7\frac{1}{2}$ per cent must be 2 times $7\frac{1}{2}$, or 15 per cent of the principal. Hence we have the following written work: —

$$
\begin{array}{rrl}
 & a = \$537.48 & = \text{principal.} \\
.15 \text{ of } a = b = & 80.622 & = \text{interest 2 yr.} \\
\frac{1}{2} \text{ of } b = c = & 40.311 & = \text{interest 1 yr.} \\
\frac{1}{3} \text{ of } a = d = & 26.874 & = \text{interest 8 mo.} \\
\frac{1}{16} \text{ of } d = e = & 1.679 & = \text{interest 15 da.} \\
b + c + d + e = & \$149.486 & = \text{interest 3 yr. 8 mo. 15 da. at } 7\frac{1}{2} \text{ per cent.}
\end{array}
$$

Second Solution. — The interest for 4 yr. at $7\frac{1}{2}$ per cent must be 4 times $7\frac{1}{2}$, or 30 per cent, = $\frac{3}{10}$ of the principal. Hence,

$$
\begin{array}{rrl}
 & a = \$537.48 & = \text{principal.} \\
.3 \text{ of } a = b = & 161.244 & = \text{interest for 4 yr.} \\
\frac{1}{16} \text{ of } b = c = & 10.077 & = \text{interest for 3 mo.} \\
\frac{1}{6} \text{ of } c = d = & 1.679 & = \text{interest for 15 da.} \\
b - c - d = & \$149.488 & = \text{interest for 3 yr. 8 mo. 15 da.}
\end{array}
$$

3. What is the interest of \$537.47 for 1 yr. 10 mo. 15 da. at $3\frac{1}{3}$ per cent?

Solution. — The interest of any sum for 3 years at $3\frac{1}{3}$ per cent per year will be 10 per cent, or $\frac{1}{10}$ of that sum. Hence,

$$
\begin{array}{rrl}
 & a = \$537.47 & = \text{principal.} \\
\frac{1}{20} \text{ of } a = b = & 26.873 & = \text{int. for 1 yr. 6 mo.} \\
\frac{1}{4} \text{ of } b = c = & 6.718 & = \text{int. for 4 mo. 15 da.} \\
b + c = & \$33.591 & = \text{int. for 1 yr. 10 mo. 15 da.}
\end{array}
$$

Second Solution.

$$\begin{aligned}
a &= \$537.47 = \text{principal.}\\
\tfrac{1}{20} \text{ of } a = b &= 26.873 = \text{int. for 1 yr. 6 mo.}\\
\tfrac{1}{6} \text{ of } b = c &= 4.479 = \text{int. for 3 mo.}\\
\tfrac{1}{2} \text{ of } c = d &= 2.239 = \text{int. for 1 mo. 15 da.}\\
b + c + d &= \$33.591 = \text{int. for 1 yr. 10 mo. 15 da.}
\end{aligned}$$

NOTE. — It will be seen that, by this method, we first get the interest for any convenient time, and then take such part or parts of this as will give the interest for the required time.

4. What is the interest of \$279.64 for 6 yr. 5 mo. 22 da. at $\frac{1}{4}$ per cent?

Solution. — The interest for 4 years at $\frac{1}{4}$ per cent must be 4 times $\frac{1}{4}$, or 1 per cent of the principal. Hence,

$$\begin{aligned}
a &= \$279.64 = \text{principal.}\\
.01 \text{ of } a = b &= 2.796 = \text{int. for 4 yr.}\\
\tfrac{1}{2} \text{ of } b = c &= 1.398 = \text{int. for 2 yr.}\\
\tfrac{1}{9}{}^{*} \text{ of } b = d &= .310 = \text{int. for } 5\tfrac{1}{3} \text{ mo., or 5 mo. 10 da.}\\
\tfrac{1}{60}{}^{\dagger} \text{ of } c = e &= .023 = \text{int. for 12 da.}\\
&\ \$4.527 = \text{int. for 6 yr. 5 mo. 22 da.}
\end{aligned}$$

What is the interest of —

5. \$483.79 for 3 yr. 3 mo. 18 da. at $4\frac{1}{2}$ per cent?
6. \$538.71 for 1 yr. 7 mo. 24 da. at 9 per cent?
7. \$875.37 for 2 yr. 7 mo. 13 da. at $8\frac{1}{2}$ per cent?
8. \$63.29 for 5 yr. 11 mo. 10 da. at $8\frac{1}{3}$ per cent?

Suggestion. — The interest for 6 years, at $8\frac{1}{3}$ per cent, is 50 per cent, or $\frac{1}{2}$ of the principal, and since 20 days $= \frac{1}{3}$ of 2 months $= \frac{1}{18}$ of 1 year $= \frac{1}{108}$ of 6 years, the interest for 20 days must equal $\frac{1}{108}$ of the interest for 6 years.

9. What is the interest of \$56.84 from March 5, 1850, to May 25, 1851, at 5 per cent?

---

* Since $5\frac{1}{3}$ mo. $= \frac{1}{9}$ of 4 yr., or 48 mo.

† Since 12 da. $= \frac{1}{5}$ of 2 mo., and 2 mo. $= \frac{1}{12}$ of 2 yr., 12 da. must equal $\frac{1}{5}$ of $\frac{1}{12}$, or $\frac{1}{60}$ of 2 yr.

10. What is the interest of $138.46 from July 1, 1848, to Aug. 26, 1852, at $2\frac{1}{2}$ per cent?

11. What is the interest of $273.81 from Sept. 28, 1847, to Oct. 31, 1852, at 7 per cent?

12. What is the amount of $783.25 from Aug. 28, 1846, to Feb. 29, 1852, at $4\frac{1}{2}$ per cent?

13. What is the amount of $57.84 from Jan. 19, 1849, to Dec. 29, 1852, at $6\frac{2}{3}$ per cent?

14. What is the amount of $278.49, from Sept. 13, 1841, to Oct. 10, 1846, at $7\frac{1}{2}$ per cent?

15. Jan. 17, 1850, I borrowed 837 dollars, agreeing to pay interest at the rate of 6 per cent per year, and immediately put it on interest at the rate of $7\frac{1}{2}$ per cent. Aug. 27, 1852, I collected the amount due to me, and paid that which I owed. How much did I gain by the transaction?

Suggestion. — Since I paid 6 per cent and received $7\frac{1}{2}$ per cent interest on the sum I borrowed, my gain must have been $1\frac{1}{2}$ per cent per year.

16. A merchant, wishing to purchase 9 acres of land at $378.43 per acre, borrowed money for the purpose at the rate of 5 per cent. At the end of 3 yr. 9 mo. 15 da. he sold the land, receiving $400 per acre for $\frac{1}{3}$ of it, and $475.28 per acre for the remainder. Did he gain or lose, and how much?

17. Bought 397 yards of cloth at $3.75 per yard, payable in 6 months, with interest at $7\frac{1}{2}$ per cent per year, and immediately sold it for $4 per yard, payable in 6 months, without interest. When the 6 months had elapsed, I collected the money due me, and paid my debt. Did I gain or lose, and how much?

18. Bought 397 yards of cloth at $4 per yard, payable in 6 months, and immediately sold it at $3.75, cash, and put the money on interest at the rate of $7\frac{1}{2}$ per cent. At the end of 6 months I called in the money I had lent, and paid that which I owed. Did I gain or lose by the transaction, and how much?

182. *Promissory Notes.*

(*a.*) A PROMISSORY NOTE, a NOTE OF HAND, or, as it is more commonly called, A NOTE, is a written promise to pay a specified sum of money.

(*b.*) Annexed is a form of a note, which, with the subsequent explanations, will illustrate the principal points connected with this subject.

$100. Providence, May 1, 1855.

For value received, I promise to pay George Smith, or order, one hundred dollars, on demand, with interest.

John Brown.

(*c.*) The above note is a promise made by John Brown to pay George Smith, or order, one hundred dollars, and is equivalent to the following: —

Providence, May 1, 1855.

Because of an equivalent value received from George Smith, I promise to pay to him, or to whomsoever he may order me to pay it, one hundred dollars, whenever the payment may be demanded of me by presenting this note; and also to pay the interest of one hundred dollars from this date till the time of payment.

John Brown.

(*d.*) In considering this note, we may observe, —

First. The "$100" placed at the left hand upper corner. These figures do not form an essential part of the note, but are written in order to enable a person to tell at a glance the amount for which it was given, and also to guard against any changes which might be made in the body of the note.

Second. The date, which shows when and where it was written.

Third. The words "value received," which are designed as an acknowledgment that the signer of the note has received an equivalent from the person to whom it is to be paid.

Fourth. "I promise to pay George Smith, or order," which means the same as, "I promise to pay to George Smith, or to whomsoever he may order me to pay it."

Fifth. The sum, "one hundred dollars." This should always be written out in words.

Sixth. The phrase "on demand," which means whenever he shall demand payment.

Seventh. The phrase "with interest," which means that interest is to be paid from the time the note is dated.

Eighth. The signature, "John Brown," which gives validity to the note, and must be written by Mr. Brown himself, or by some one specially authorized by him.

(*e*.) The words "value received" are regarded as essential to a note, it being a principle in law, that no person shall be compelled to pay money for which he has not received an equivalent in some form or other.

(*f*) The words "or order" may be omitted, or the words "or bearer" may be substituted for them. If they are omitted, the note can only be collected by the person named in it. Such a note is not negotiable. (See **183**, *a*.)

(*g*.) Some specified time, as "in sixty days," "in three months," &c., might be substituted for the phrase "on demand." The meaning then would be in so many days or months from the date of the note. Such a note would be called A NOTE ON TIME.

(*h*.) Notes on time are not regarded as due till three days after the time specified in the note. Thus, a note payable in sixty days is not due till the end of sixty-three days. The three days thus added are called DAYS OF GRACE.

(*i*.) If grace is not to be allowed, the form of the note should be, "in so many days or months without grace."

(*j*.) If the last day of grace comes on Sunday, or upon a legal holiday, as the Fourth of July, Thanksgiving, &c., the note is payable on the preceding day, and if that be Sunday, or a legal holiday, it is payable on the first day of grace.

(*k*.) If the phrase "with interest" should be omitted, the note would not be on interest. If, however, a note on demand is not paid when the demand is made, or if a note on time is not paid when due, interest may be afterwards charged, though no mention of it is made in the note; so that two notes, one payable in three months, and the other payable in three months, with interest afterwards, would both be on interest after the expiration of three months.

(*l*.) The form of notes may be varied somewhat without affecting their meaning or value. Thus, it makes no difference where the phrases "value received" and "on demand" are placed, provided they are above the signature. The phrase "to the order of George Smith" may be substituted for "George Smith or order." These and other changes will be illustrated in the forms of notes given in subsequent parts of the book.

(*m.*) The person whose name appears on a note as promisor is called the MAKER of the note.

(*n.*) The person to whom a note is to be paid is called the PAYEE or the PROMISEE, or the HOLDER of the note.

(*o.*) Every note should be written, or at least signed, by the maker of it, or by some one specially authorized by him. It is then taken by the payee, and is his property until it is paid, or until he transfers it to another. When paid, it should be given up to the signer, who should immediately tear off or erase his signature.

(*p.*) If the maker of a note refuses or neglects to pay it, when the demand is legally made, at the proper place and time, the note is said to be DISHONORED.

(*q.*) A demand for payment, to be legal, must be made by actually presenting the note; or, if it is payable at a certain place on a given day, by having the note deposited at that place ready to be presented to the maker, should he call to pay it.

NOTE. — The maker of a note is not obliged to regard a request for its payment as a legal demand, unless the note be exhibited and tendered to him at the time the request is made; but if he waives his right to see the note, the demand is legal.

183. *Negotiable Notes, Indorsements, and Protests.*

(*a.*) A NEGOTIABLE NOTE is one that may be transferred or sold by one person to another.

(*b.*) Negotiable notes are of two kinds, viz., those payable to *a person, or order*, (as "*George Smith, or order,*") and those payable to *a person, or bearer*, (as "*George Smith, or bearer,*") or simply to "*the bearer.*"

(*c.*) Those of the second kind are negotiable by mere delivery; but those of the first require a written order of the payee to authorize any one else to collect the money due on them.

(*d.*) Such an order is commonly written on the back of the note, and is called *an indorsement.*

Thus, if George Smith wishes to transfer the above note to Charles Woods, he might write on the back of it, "Pay to Charles Woods, or order. — George Smith."

(*e.*) This would be an indorsement in full, and would give Charles Woods the same title to the note, and the same claim on John Brown on account of it that George Smith originally had.

(*f.*) If the indorsement had been, "Pay to Charles Woods, or

bearer," it would give to Charles Woods, or whosoever might obtain legal possession of the note, the full title to it.

The note would then be negotiable by mere delivery.

(*g.*) If it had been, "Pay to Charles Woods only," it would give to Charles Woods the full title to the note, but would prevent his transferring it to any one else.

(*h.*) If George Smith had simply written his name on the back of the note, it would have been an indorsement equivalent to, "Pay to the bearer," and would make the note negotiable by mere delivery. Such an indorsement is called an INDORSEMENT IN BLANK, and is the form most frequently used.

(*i.*) By either of the foregoing forms of indorsement, George Smith would not only authorize some one else to collect the note, but he would make himself responsible for several important points. He would guaranty, —

First. That the note is genuine, and just what it purports to be.

Second. That he has legal possession of it, and a right to transfer it.

Third. That it shall be paid if payment is demanded by presenting the note to the signer at the proper time.

Fourth. That if not so paid, he will pay it himself, if properly notified.

If he does not wish to guaranty the last two points, he writes the words "without recourse" before his name. The indorsement in this form is a guaranty of the first two points, but not of the last two.

(*j.*) Any person may write his name as a special indorser on a note which he does not own. Such an indorsement would not affect the negotiability of a note, but it would make the indorser responsible for its payment, in case the maker should not pay it. Indeed, it might impose upon him all the obligations with regard to the note, which rest upon the original maker.

(*k.*) A person who indorses a note under any circumstances incurs all the obligations of such an indorsement, even though he may be ignorant of them at the time.*

(*l.*) If a note is indorsed by several persons, each of them makes himself responsible for these points to whoever may afterwards get legal possession of it.

(*m.*) A person to whom an indorsed note is transferred, by the mere act of receiving it, agrees with all the indorsers, except the special ones, and under some circumstances with them, —

---

* Many a man has been reduced from affluence to poverty, by merely writing his name on the back of a note "just to accommodate a friend."

First. That payment shall be demanded of the signer *within a reasonable time*, if the note is payable on demand ; and *on the very day it becomes due*, if it be on time.

Second. That he will not consent to any delay in the time of payment.

Third. That if the note is dishonored he will at once get it protested, (see (*u.*) (*v.*) p. 269,) or, on that day or the next, inform the indorser that it is dishonored, and notify him that he will be held responsible for its payment. This information should be given by telling him personally, or by leaving a letter at his place of business or dwelling, or by mailing one to his address.

(*n.*) If the holder of a note fails to comply with any one of these conditions, he releases every indorser except the special ones, and sometimes even them, from all obligation to pay it; but he does not in any way affect the obligation of the original promisor.

(*o.*) The question, What is a reasonable time in the case of a note payable on demand ? must depend for its answer somewhat on circumstances.

In Massachusetts, the indorser of a note payable on demand is by statute excused, if payment be not demanded within sixty days from the date of the note.

(*p.*) If no particular place for payment is specified in the note, it may be presented at the signer's counting house or place of business, in business hours, or at his dwelling house, at a time when he may reasonably be supposed to be at home, and not to have gone to bed.

(*q.*) If it is payable at some specified place, as at a bank or at some counting room, the demand must be made at that place. In case the promisor does not appear there to pay the note on the day it falls due, it is dishonored, and notice should be sent to the indorsers, as though payment had been demanded and refused.

(*r.*) When there are several indorsers to the note, notice that it is dishonored should be sent to each whom the possessor of the note wishes to hold responsible for its payment. He may collect the money of either, or, if he resorts to legal measures, may commence suits against any of them, or against all at once. When, however, he collects the money of any one, his demands against the others are void.

(*s.*) Each indorser may require any one whose name precedes his own to make good to him the loss he may sustain on account of the note, provided he gives notice of his intention to do so the day that he receives his own notice, or the day after.

For instance, suppose that the foregoing note comes to me indorsed by George Smith, Charles Woods, Silas Bacon, and Edward Jones, and that it is dishonored. I shall immediately inform Smith, Woods, Bacon

and Jones of the fact, and tell each that I shall look to him for payment. Jones will on the same day, or day after receiving the notice, inform Smith, Woods, and Bacon of it, and also that he expects them to make good to him any money he may have to pay on account of it. Bacon would write a similar letter to Smith and Woods, and Woods would write one to Smith; and if Smith should be obliged to pay it, he could hold only the signer of the note responsible.

(*t.*) A PROTEST is a formal declaration, made by an officer called a Notary Public, that a note is dishonored, and that the indorser will be held responsible for it.

(*u.*) The common method of protesting a note is for the holder to take it to a notary public, and get him to demand payment of the promisor. If the demand be refused, the notary makes out a protest, i. e., he writes (usually on a copy of the note) a formal declaration that it is dishonored, and sends a copy of it to each indorser. The protest should be made out on the very day the note is due; otherwise it has no value.

(*v.*) Although a notice sent by the holder of the note is usually regarded as sufficient to make the indorsers liable for its payment, it is a safer course to have a formal protest made out.

184. *Joint and Several Notes.*

(*a.*) Notes are sometimes signed by more than one person. If the note is so worded as to show that the signers are together responsible for its payment, it is a "joint note;" but if so worded as to show that the signers together and separately are responsible for its payment, it is a "joint and several note."

(*b.*) There seems to be but little difference between a joint note and a joint and several note, as far as regards the liabilities of the promisors. Any signer to either *may* be compelled to pay its whole amount. The chief difference between them is, that an action commenced to enforce payment of a joint note must be commenced against all the promisors jointly, while an action commenced to enforce payment of a joint and several note may be commenced against them jointly or individually.

(*c.*) If judgment is given in favor of the holder of a joint note, he may compel either promisor to pay its full amount, as much as if it had been a joint and several note. One who is thus compelled to pay the amount of the note may recover from each of the other promisors the share which each ought justly to pay. In a joint and several note, or a several note, he may or may not recover from the other promisors, according to the circumstances under which the note was given.

$50. Providence, May 1, 1855.

For value received, we jointly promise to pay o James Clark, or order, fifty dollars, on demand, with interest.

Freeman Sprague,
Walter Hale.

(*d.*) The above is a joint note, because it is the promise of Sprague and Hale to pay jointly, or together, a specified sum of money.

(*e.*) The holder of the note may demand payment of either Sprague or Hale; but if he wishes to hold any indorser or indorsers responsible, he must demand payment of both. If he commences an action on account of it, he must commence it against both jointly.

(*f.*) When a note is signed by one person as principal, and another as surety, the holder of it may demand payment of, and commence action against, the surety instead of the principal, if for any reason he chooses so to do.

185. *Renewal of Notes.*

(*a.*) If the holder of a note allows it to run for six years without collecting any thing on it, or otherwise getting it renewed, it becomes outlawed, and he cannot afterwards compel the promisor to pay it.

(*b.*) A note under seal, and in some states (as Massachusetts and Connecticut) an attested note, is not outlawed under twenty years.

(*c.*) Any act by which the promisor acknowledges the validity of a note renews it. Care should be taken to have the renewal made in the presence of witnesses, or to have evidence of it in the handwriting of the promisor. For instance, —

(*d.*) When a part of the money due on a note is paid, a receipt for it should be written on the note. Such a receipt is called an indorsement. Thus, if twelve dollars be received on some note, the following would be written: "Jan. 1, 1855, received twelve dollars." This indorsement should be written by the one who pays the money, as it is equivalent to a renewal of the note, and it may afterwards be important for the holder to prove that the money was really paid at that time on account of the note.

186. *Exercises.*

$500.   Boston, May 1, 1855.

Ninety days after date, I promise to pay James Drew, or order, five hundred dollars. Value received.

Samuel Johnson.

Who is the maker of the above note? To whom does he promise to pay it? Who shall take possession of the note? Can he transfer it? If so, how? What form of indorsement would be used if he wished to transfer it to Francis Baker, or his order? To Francis Baker only? To Francis Baker, or whoever should have possession of it? To whoever should have possession of it without naming any person? How should it be indorsed in each of the above cases, if the indorser does not wish to make himself responsible for its payment? Is it on interest? On what month, and what day of the month, will the note become due? If this note should be indorsed first by James Drew, then by Francis Baker, and then by William Davis, and then Charles Morton should come into legal possession of it, of whom and when ought he to demand payment? If payment is refused, what ought he to do? What ought each indorser to do on receiving notice that the note is dishonored? Who should take possession of it when it is finally paid? What disposition should be made of it? What effect would it have on the negotiability of the above note, if the words "or order" were omitted? What if the words "or bearer" were substituted?

(1.)

$500.   Providence, Feb. 1, 1855.

On demand, with interest after three months, I promise to pay to the order of Henry Gray five hundred dollars. Value received.

Alfred Baker.

Attest, John Clapp.

(2.)

Salem, April 7, 1855.

I promise to pay George Barton, or bearer, one thousand dollars, in 40 days, without grace, for value received.

$1000.

Jabez Fisher.

(3.)

$800.

Bath, August 4, 1855.

For value received, we, John Brown, as principal, and James Jackson, as surety, promise to pay Edward Reed eight hundred dollars, on demand, with interest.

John Brown
James Jackson

Let the scholar write each of the above notes and explain their meaning, and the meaning of all their points; let him also change their form, and indorse them in various ways.

187. *Banks and Banking.*

(*a.*) A bank is an institution or corporation for the purpose of trafficking in money.

(*b.*) Banks usually receive money on deposit, loan money on interest, and issue bank notes, or bank bills, i. e., notes payable in specie to the bearer on demand at the bank, and intended to circulate as money.

(*c.*) When money is loaned by a bank, it is commonly made payable at the end of a given number of days, and the interest for that time and the three days of grace is deducted at the time it is borrowed.

Thus, on a note of $500, payable in 30 days, I shall receive at a bank $500, minus the interest of $500 for 33 days, i. e., $500 — $2.75 = $497.25.

(*d.*) By this arrangement the banks receive interest on a larger sum of money than they lend.

Thus, in the above example, the bank receives interest on $500, while the sum actually lent is only $497.25.

(*e.*) Bank interest is called DISCOUNT, because it is thus deducted from the face of the note, i. e., from the sum for which the note is given.

(*f.*) The note on which the money is received is said to be DISCOUNTED.

(*g.*) To present a practical illustration of this subject, we will suppose the following case: —

On May 9, 1855, George Guild, being in want of money, wrote a note promising to pay at the Merchants' Bank, to the order of Alfred Hall, $600, in 60 days, and got Mr. Hall to indorse it. He then applied to the officers of the bank to discount it, and they decided to do so. He forthwith presented the note to the cashier, who deducted the interest of $600 for 63 days, and paid him the balance, $593.70. Guild took the money, and had the use of it till July 11, when the note became due. He then paid to the cashier of the bank the $600 due on the note, and the transaction was settled.

(*h.*) By considering the above, it will be seen that Guild paid to the bank the $593.70 which he had borrowed, together with the interest of $600; so that he paid the interest of $6.30 (i. e., of the bank discount) more than he had the use of.

(*i.*) If he had wished to keep the money as much longer, he would on the last day of grace have written a new note, differing from the former only in the date, and have got it indorsed as before. As this new note would be worth $593.70 at the bank, he could by giving it, and $6.30 besides, to the cashier, pay the amount due at the bank. At the end of 63 days he would again owe the bank $600.

(*j.*) Now, it is obvious that during all this time he has been paying the interest of $600, while he has had the use of but $593.70, and that therefore he has paid the interest of $6.30 more than he has used. Besides this he loses the use for 63 days of the $6.30 he paid on renewing the note. Hence, as the use of a sum is worth its interest, he virtually pays the interest of $6.30 more than he receives for 126 days + 63 days, or 189 days.

(*k.*) If Mr. Guild should fail to appear at the bank to pay the note before the close of bank hours on the last day of grace, the note would be protested, and notice sent by a notary public to Mr. Hall, who would then be held responsible for its payment.

1. A note of $1200, payable in 60 days, was discounted at a bank at 6 per cent. How much was received on it?

Solution. — The interest of $1200 for 63 days, being 2 dimes per day, is $12.60, which, deducted from $1200, leaves $1187.40 as the sum received.

2. How much would be received at a bank on a note of $200, payable in 90 days?

3. How much would be received at a bank on a note of $360, payable in 30 days?

4. I got my note for $1000, payable in 90 days, discounted at a bank, and immediately put the money received on it at interest. When the note became due, I collected the amount of what I had put on interest, and paid my note at the bank. How much did I lose by the transaction? How does the sum lost compare with the *interest* of the *bank discount* for the given time?

5. My note for $1000, payable in 6 months, was discounted at a bank, and I immediately put the money received on it at interest. When the note became due, I collected the sum due me, and paid that which I owed at the bank. How much did I lose by the transaction?

6. I had my note for $500, payable in 2 months, discounted at a bank, and immediately put the money on interest. When the note became due, I renewed it for the same time as before; and when the new note became due, I collected the amount due me, and paid my note at the bank. How much did I lose?

Suggestions. — From a consideration of the methods of reckoning interest at banks, it is evident that from the time the first note was discounted to the time the second was paid, I paid interest on the bank discount more than I received, and that at the end of two months three days I paid a sum equal to the bank discount. Hence, I lost the interest of the bank discount for 4 mo. 6 da., plus 2 mo. 3 da., = 6 mo. 9 da.

Or, since I paid nothing at the bank, except the bank discount at the time of renewing the note, and the second note when it became due, the actual value, at the time of settlement, of the sums paid, will be the amount of the bank discount for 2 mo. 3 da., plus the face of the note

The sum received will be the amount for 4 mo. 6 da. of the money obtained at the bank on the first note. The difference between the values paid and received is the loss.

7. I had my note for \$600, payable in 4 months, discounted at a bank, and immediately lent the money received on it for just one year. When my note at the bank became due, I renewed it for the same time as before, and when this new note became due, I renewed it for such time that it became due at the end of the year, when I collected the amount of the sum I had lent, and paid my note at the bank. How much did I lose by the transactions?

188. *English Method of computing Interest.*

(*a.*) In England, time is reckoned in years and days, but never in months. The year is regarded as 365 days. Interest is usually computed by first finding it for the years, and then for the days.

(*b.*) In computing it for the days, it is well to notice that 73 days $= \frac{1}{5}$ of 1 year, that 5 days $= \frac{1}{73}$ of 1 year, and that 1 day $= \frac{1}{365}$ of 1 year.

(*c.*) When any part of the principal is expressed in shillings, pence, and farthings, it should be reduced to the decimal of a pound.*

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* Probably the simplest method of doing this is to regard each shilling as $\frac{1}{20}$, or .05 of £1, and each farthing as $\frac{1}{960}$, or $.001\frac{1}{24}$ of £1. We shall then have as many times .05 of £1 as there are shillings, plus as many times $.001\frac{1}{24}$ of £1 as there are farthings in the pence and farthings. But as all values less than $\frac{1}{2}$ of .001 of £1 are so small that they may be disregarded, the result will be sufficiently accurate for ordinary purposes, if we regard each farthing as .001 of £1, observing to add .001 if there are more than 12 and less than 36 farthings, and .002 if there are more than 36. By adding this result to the value of the shillings, we shall have the decimal expression required. For example: To find what part of £1 is equal to 9 s. 8 d. 1 qr., we have 9 s. = 9 times £.05 = £.45; 8 d. 1 qr. = 33 qr. = £.033 + £.001 = £.034 Therefore, 9 s. 8 d. 1 qr. = £.45 + £.034 = £.484 The reverse operation will get the value of the decimal expression, in terms of shillings, pence, and farthings.

1. What is the interest of £327 17 s. 7 d. from May 7, 1851, to Sept. 4, 1852, at 5 per cent?

Solution. — From May 7, 1851, to May 7, 1852, is 1 year. There are 24 days left in May, to which adding the 30 in June, the 31 in July, the 31 in August, and the 4 in September, gives 120 days. The time, then, is 1 year 120 days. The principal equals £327.879 Hence we have the following written work: —

$a =$	£327.879	= principal.
.05, or $\frac{1}{20}$ of $a = b =$	16.39395	= int. for 1 yr.
$\frac{1}{365}$ of $b = c =$	.044914	= int. for 1 da.
120 times $c = d =$	5.389680	= int. for 120 da.
$b + d = e =$	£21.78363	= int. for 1 yr. 120 da.

Or we may have the following: —

$a =$	£327.879	= principal.
.05, or $\frac{1}{20}$ of $a = b =$	16.39395	= int. for 1 yr.
$\frac{1}{73}$ of $b = c =$	.224574	= int. for 5 da.
23 times $c = d =$	5.165202	= int. for 115 da.
$b + c + d = e =$	£21.783726	= int. for 1 yr. 120 da.

Calling this £21.784, we have £21 15 s. 8 d. 1 qr. as the answer. The multiplications required in solving these examples render it necessary to carry out the work to places below thousandths, though we do not care to have them appear in the answer.

2. What is the interest of £47 9 s. 4 d. 1 qr. from May 17, 1849, to Aug. 23, 1852, at 5 per cent?

3. What is the interest of £148 19 s. 9 d. 3 qr. from Oct. 23, 1850, to Nov. 11, 1852, at 5 per cent?

4. What is the amount of £361 13 s. 2 d. 1 qr. from July 18, 1847, to April 12, 1850, at 5 per cent?

5. What is the amount of £248 18 s. 10 d. 3 qr. from Dec. 5, 1849, to March 3, 1852, at $4\frac{1}{2}$ per cent?

6. What is the interest of £548 15 s. 7 d. 3 qr. from July 29, 1847, to March 12, 1850, at $2\frac{1}{2}$ per cent?

7. What is the amount of £258 19 s. 5 d. 2 qr. from Jan. 1, 1849, to Sept. 29, 1852, at 4 per cent?

8. What is the amount of £329 7 s. 1 d. 3 qr. from Nov. 13, 1850, to Dec. 1, 1852, at 3 per cent?

9. What is the interest of £481 13 s. 5 d. 1 qr. from April 19, 1842, to May 3, 1847, at 5 per cent?

10. What is the amount of £222 2 s. 2 d. 2 qr. from Feb. 29, 1848, to Jan. 1, 1852, at 4 per cent?

189. *Partial Payments.*

(*a.*) The principle adopted by the Supreme Court of the United States, and by that of Massachusetts and most of the other states, as the one to be applied in determining the sum due on a promissory note or bond on which payments have been made at different times, is, that as much of the payment as is necessary to pay the interest due at the time the payment is made should be appropriated to that purpose, and the surplus to the payment of the principal. The balance then due will form a new principal on interest as was the original principal. If, however, any payment is less than the interest at the time due, the principal remains unaltered, and on interest, till some payment is made, which, with the preceding neglected payments, is more than sufficient to pay the interest; when we proceed as if a single payment, equal to the sum of the last payment and the preceding neglected ones had been made.*

EXAMPLE.

$750.00 Boston, April 7, 1848.

For value received, I promise to pay James Sullivan, or order, seven hundred and fifty dollars, on demand, with interest. Edward Delano.

* The method adopted by the court of Connecticut differs from the above only in this respect — that if a payment greater than the interest at the time due be made before the principal has been on interest one year, the person making it is allowed interest on it to the end of the year; that is, its amount from the time it was made to the end of the year, is deducted from the amount of the principal to the same time. If settlement be made before the principal has been on interest one year, interest is allowed on the payments from the time they were made to the time of settlement.

On this note are the following indorsements: —

Jan. 17, 1849. Received one hundred dollars.
March 13, 1850. Received twenty-five dollars.
Feb. 19, 1851. Received thirty dollars.
Aug. 3, 1851. Received two hundred dollars.
Jan. 1, 1852. Received one hundred and fifty dollars.

What was due on the note at the time of settlement, Aug. 14, 1852?

(*b.*) The following exhibits a good form of writing the work in such examples, and, in connection with the explanations following, will be a sufficient illustration of the process: —

\$750.00	= 1st principal, April 7, 1848.	
35.00	= int. 280 days at $1\frac{1}{4}$ dimes per day.	
\$785.00	= amt. due Jan. 17, 1849.	
\$100.00	= 1st payment.	
\$685.00	= balance due Jan. 17, 1849.	
68.50	= int. 20 mo.	2 yr. 6 mo. 17 da.
34.25	= int. 10 mo.	
1.712	= int. 15 da.	
.228	= int. 2 da.	
\$789.69	= amt. due Aug. 3, 1851.	
255.00	= 2d, 3d, and 4th payments.	
\$534.69	= bal. due Aug. 3, 1851.	
13.367	= int. 5 mo.	5 mo. — 1 da., or 4 mo. 29 da.
.089	= int. 1 da.	
\$547.968	= amt. due Jan. 1, 1852.	
150.00	= 5th payment.	
\$397.968	= bal. due Jan. 1, 1852.	
13.265	= int. 6 mo. 20 da.	7 mo. 13 da.
1.326	= int. 20 da.	
.198	= int. 3 da.	
\$412.757	= amt. due Aug. 14, 1852, = *Ans.*	

Explanation. — As it is obvious that the first payment was greater than the interest at the time due, we get the amount of the note to that time, and deduct from it the payment. The remainder is a new princi-

pal The second payment, $25, is evidently less than the interest then due; for the time is over one year, while the principal, between $600 and $700, gives more than $36 interest per year. Similar considerations will at once show that the interest to the time of the third payment must be greater than the second and third payments together. But the fourth payment, together with the second and third, is very evidently more than sufficient to pay the interest then due; therefore we get the amount of the new principal to the time of the fourth payment, and subtract from it the sum of the second, third, and fourth payments, thus getting our third principal. As it is evident that the interest of this principal to the time of the fifth payment is less than that payment, we find its amount, and from it subtract the payment. This gives us the fourth principal, which is on interest till the time of settlement; and hence its amount is the sum due.

(c.) The above is really equal to the following simple problems, each of which is very easy of solution: —

Is the interest of $750 from April 7, 1848, to Jan. 17, 1849, more or less than $100, the first payment? What, then, is the amount of $750 for that time? How much will be due after paying the $100? Is the interest of this to March 13, 1850, more or less than the $25 at that time paid? Is the interest to Feb. 19, 1851, greater or less than $25 + $30, or $55, the sum of the second and third payments? Is the interest from Jan. 17, 1849, to Aug. 3, 1851, greater or less than $265, the sum of the second, third, and fourth payments? What, then, is the amount of $685 from Jan. 17, 1849, to Aug. 3, 1851? How much will be due after deducting the $255? Is the interest of this from Aug. 3, 1851, to Jan. 1, 1852, greater or less than $150? What, then, is the amount of $534.69 for that time? How much will be due after deducting $150, the fifth payment? What is the amount of this from Jan. 1, 1852, to Aug. 14, 1852? What, then, was due Aug. 14, 1852?

Every problem in partial payments can be resolved into simple ones; and if the pupil will use a little care in determining what these are, and be sure that he performs each correctly, he may obtain a true result the first time of performing the work. Nothing short of this should satisfy him.

2. $850.00 Boston, April 7, 1847.

For value received, I promise to pay Albert Simmons, or order, eight hundred and fifty dollars, on demand, with interest. Isaac Goodrich.

On this note were the following indorsements: —

June 19, 1848. Received one hundred and twenty-five dollars

Jan. 7, 1849. Received eighty-three dollars.
Sept. 27, 1849. Received one hundred dollars.
May 1, 1850. Received twenty dollars.
Aug. 28, 1850. Received two hundred dollars.
Jan. 1, 1851. Received one hundred dollars.

How much was due on the note Oct. 13, 1851?

3. $1000.00 Providence, Nov. 28, 1848.

I promise to pay Bradford Allen, or order, one thousand dollars, on demand, with interest. Value received.

Henry Williams.

On this note were the following indorsements: —

July 23, 1849. Received eighty dollars.
Feb. 28, 1850. Received fifteen dollars.
June 27, 1850. Received twenty dollars.
April 2, 1851. Received twenty-five dollars.
Dec. 20, 1851. Received five hundred dollars.
May 17, 1852. Received three hundred dollars.

How much was due Aug. 14, 1852?

4. 645\frac{75}{100}$ Worcester, Dec. 20, 1846.

For value received, we promise to pay Alfred Lincoln, or order, six hundred and forty-five dollars and seventy-five cents, in three months, with interest after.

Thompson & French.

Indorsements: —

Nov. 8, 1848. Received forty dollars.
April 16, 1849. Received three hundred dollars.
March 10, 1851. Received two hundred and fifty dollars.
Sept. 8, 1851. Received sixty dollars.

How much was due Jan. 1, 1852?

5. $1275.00 Bridgewater, Sept. 29, 1845.

For value received, we promise to pay Lincoln and Wood twelve hundred and seventy-five dollars, on demand, with interest. Paine, Root, & Co.

Indorsements : —

Aug. 5, 1846. Received three hundred dollars.
Sept. 22, 1847. Received four hundred dollars.
May 25, 1848. Received two hundred dollars.
June 17, 1849. Received one hundred and fifty dollars.
Nov. 13, 1850. Received one hundred and fifty dollars.

What was the balance due March 1, 1851?

6. $3000.00 Lowell, April 3, 1849.

For value received, I promise to pay the order of James Wyman three thousand dollars, on demand, with interest after four months.

Edward Robinson.

Attest, George Stone.

Indorsements : —

Nov. 1, 1849. Received five hundred dollars.
Dec. 27, 1850. Received ninety dollars.
March 25, 1851. Received fifty dollars.
July 18, 1851. Received six hundred dollars.
Sept. 13, 1851. Received one thousand dollars.
Jan. 1, 1852. Received one thousand dollars.

The note was settled Nov. 8, 1852. How much was due?

190. *Merchants' Method when Debts are paid within a Year.*

(*a.*) When notes or debts of any kind, on which partial payments have been made, are paid in full within one year from the time interest commences, merchants often determine the sum to be paid on settlement, as they would if nothing is due on a note till it is paid in full; that is, they find the amount of the note to the time of settlement, and the amount of each payment from the time it was made till the time of settlement, and then consider the excess of the amount of the note over the sum of the amounts of the several payments to be the sum due on settlement.

1. $500.00 Worcester, July 8, 1851.

For value received, I promise to pay John F. Barnard, or order, five hundred dollars, on demand, with interest.

William H. West.

Indorsements : —

Sept. 23, 1851. Received sixty dollars.
Nov. 20, 1851. Received one hundred dollars.
Jan. 17, 1852. Received two hundred dollars.
Feb. 8, 1852. Received fifty dollars.

How much was due May 11, 1852?

Solution.

$500.00 = principal.
25.25 = int. 10 mo. 3 da.

$525.25 = amt. of note to May 11, 1852.

$ 60.00 = 1st payment, Sept. 23, 1851.
2.28 = int. 7 mo. 18 da.
100.00 = 2d payment, Nov. 20, 1851.
2.85 = int. 5 mo. 21 da.
200.00 = 3d payment, Jan. 17, 1852.
3.80 = int. 3 mo. 24 da.
50.00 = 4th payment, Feb. 8, 1852
.775 = int. 3 mo. 3 da.

$419.705 = amt. of payments, May 11, 1852.

$106.55 = bal. due May 11, 1852, = *Ans.*

2. $728.00 Springfield, Sept. 7, 1849.

For value received, I promise to pay A. Parish, or order, seven hundred and twenty-eight dollars, on demand, with interest.

William Mitchell.

Indorsements : —

Oct. 3, 1849. Received eighty dollars.
Dec. 1, 1849. Received ninety dollars.
Feb. 4, 1850. Received one hundred dollars
March 2, 1850. Received forty dollars.
June 1, 1850. Received eighty dollars.

How much was due Aug. 2, 1850?

3. $\$583\frac{75}{100}$ Lowell, Jan. 18, 1850.

For value received, I promise to pay C. C. Chase, or order, five hundred and eighty-three dollars and seventy-five cents, on demand, with interest. A. H. Fiske.

Indorsements: —

March 5, 1850. Received fifty dollars.
April 1, 1850. Received seventy-five dollars.
June 17, 1850. Received twenty-eight dollars.
July 3, 1850. Received one hundred dollars.
Oct. 2, 1850. Received ninety dollars.
Dec. 27, 1850. Received seventy dollars.

How much was due Jan. 7, 1851?

191. *To find the Time.*

The methods illustrated in the following solutions will enable us to find the time, when we know the principal, interest, and rate.

1. How long must \$420 be on interest at 6 per cent to gain \$32.27?

Solution. — The interest of \$420 for 6 days is \$.42 If it takes 6 days to gain \$.42, it will take $\frac{1}{42}$ of 6 days to gain \$.01, and 3227 times the last result to gain \$32.27

$$\frac{3227}{42} \text{ of } 6 \text{ days} = \frac{\overset{461}{\cancel{3227}} \times \cancel{6}}{\underset{\cancel{7}}{\cancel{42}}} \text{ days} = 461 \text{ days} = 15 \text{ mo. } 11 \text{ days.}$$

2. How long must \$357 be on interest at 6 per cent to gain \$29.869?

Solution. — The interest of \$357 for 6 days is \$.357 If it takes 6 days to gain \$.357, it will take $\frac{29869}{357}$ of 6 days to gain \$29.869

$$\frac{29869}{357} \text{ of } 6 \text{ days} = \frac{\overset{251}{\cancel{29869}} \times \overset{2}{\cancel{6}}}{\underset{\cancel{119}}{\cancel{357}}} \text{ days} = 502 \text{ days} = 16 \text{ mo. } 22 \text{ days.}$$

3. How long must $136.80 be on interest at 7 per cent to gain $2.793?

Solution. — The interest of $136.80 for 1 year, or 360 days, at 7 per cent, is $9.576. If it takes 360 days to gain $9.576, it will take $\frac{2793}{9576}$ of 360 days to gain $2.793

$$\frac{2793}{9576} \text{ of } 360 \text{ da.} = \frac{\overset{21}{\cancel{2793}} \times \overset{5}{\cancel{360}}}{\underset{133}{\cancel{9576}}} \text{ da.} = 105 \text{ da.} = 3 \text{ mo. } 15 \text{ da.}$$

NOTE. — By this method of solution, we first select some convenient time for which to compute the interest. Then the required time will be the same part of the time selected, that the given interest is of the interest for the selected time. The selected time should be one for which the interest can be easily computed, as, when the rate is 6 per cent per year, 6 da., 60 da. or 2 mo., 600 da. or 20 mo., 6000 da. or 200 mo.; and when the rate is other than 6 per cent, 1 year = 360 da., or such part of 1 year as shall give for the interest 1 per cent, or some other equally convenient part of the principal.

When there is a fractional part of a day in the result, the fraction may be omitted if it be less than $\frac{1}{2}$; but if it be more than $\frac{1}{2}$, 1 may be added to the number of days.

In how long time at interest will —

4. $427.32 gain $19.68 at 6 per cent?
5. $186.75 gain $12.45 at 6 per cent?
6. $378.50 gain $4.542 at 7 per cent?
7. $56.34 gain $18.78 at 8 per cent?
8. $873.70 gain $17.474 at 5 per cent?
9. $594.00 gain $60.654 at 4 per cent?

192. *Equation of Payments.*

(*a.*) When one man owes another sums of money payable at different times, it may be desirable to determine when the whole can be paid without gain or loss to either party. The process of doing this is called EQUATION OF PAYMENTS, and the time sought is called the EQUATED TIME.

(*b.*) It is obvious that if a debt be not paid till after it has become due, the debtor gains the use of it from the time it

became due to the time of payment; while if it be paid before it becomes due, the debtor loses the use of the sum paid from the time of payment to the time when it would justly have been due. The use of any sum of money is regarded as worth its interest for the time it is used.

(*c.*) The application of the foregoing principles is illustrated in the following problems and solutions: —

1. Mr. Lincoln owes Mr. Wood \$400, due in 5 months, \$600, due in 9 months, and \$200, due in 12 months. When can the whole be paid without gain or loss to either party?

Solution. — By the conditions of the question, Mr. Lincoln is entitled to the use or

Interest of \$400 for 5 mo. = \$10.00
Interest of \$600 for 9 mo. = \$27.00
Interest of \$200 for 12 mo. = \$12.00

Or to use \$1200 till its int. = \$49.00

The interest of \$1200 being \$6 per month, he is entitled to keep it as many months as there are times \$6 in \$49, which are $8\frac{1}{6}$ times. Therefore he is entitled to keep it $8\frac{1}{6}$ months, or 8 months and 5 days.

First Proof. — By paying the whole at the equated time, Mr. Lincoln gains the use of the first debt from the time it was due to the equated time, and loses that of the second and third from the equated time to the time when they would otherwise have been due. That is, he

gains interest of \$400.00 for 3 mo. 5 da. = $\$6.33\frac{1}{3}$
and loses interest of \$600 00 for 25 da. = \$2.50
and loses interest of \$200.00 for 3 mo. 25 da. = $\$3.83\frac{1}{3}$

Sum of losses = $\$6.33\frac{1}{3}$ = the gain,

which shows the work to be correct.

Second Proof. — If each debt should be paid when it becomes due, Mr. Wood will, when the last debt is paid, have had the use of \$400 for 7 mo. and of \$600 for 3 mo., which at 6 per cent is equivalent to \$14 + \$9 = \$23 interest. If, however, the sum of the debts should be paid at the equated time, Mr. Wood would, at the end of 12 months, when the last debt would otherwise have been paid, have had the use of \$1200 for 3 mo. 25 da., which, at 6 per cent, is worth \$23 interest. This shows that he would have the same interest in one case as in the other, and thus proves the first result correct.

Second Solution. — Another solution similar in character to the last can be obtained by ascertaining how much would be gained or lost by

paying the entire debt at any assumed time, and from that getting the equated time.

For instance, suppose that the entire debt had been paid at the end of 9 months. Then Mr. Lincoln would have

gained interest on \$400 for 4 months = \$8.00
and lost interest on \$200 for 3 months = \$3.00

equivalent to a gain of \$5.00

which shows that 9 months is as many days longer than the true time as it will take for \$1200 to gain \$5 at interest. We find (by **191**) that it will take \$1200, at 6 per cent interest, 25 days to gain \$5. Therefore the equated time = 9 mo. — 25 da. = 8 mo. 5 da.

Third Solution. — When the numbers are convenient, as in this example, a method like the following can be used to advantage: —

The sum of the debts is \$1200, of which the first debt is $\frac{1}{3}$, the second $\frac{1}{2}$, and the third $\frac{1}{6}$. But the use of $\frac{1}{3}$ of a sum 5 mo. is worth as much as the use of the whole of it for $\frac{1}{3}$ of 5 mo., or $1\frac{2}{3}$ mo.; the use of $\frac{1}{2}$ of a sum for 9 mo. is worth as much as the use of the whole of it for $\frac{1}{2}$ of 9 mo., or $4\frac{1}{2}$ mo.; and the use of $\frac{1}{6}$ of a sum for 12 mo. is worth as much as the use of the whole of it for $\frac{1}{6}$ of 12 mo., or 2 mo. Therefore Mr. Lincoln is entitled to the use of the sum of the debts for $1\frac{2}{3}$ mo. + $4\frac{1}{2}$ mo. + 2 mo. = $8\frac{1}{6}$ mo. = 8 mo. 5 da.

Fourth Solution. — The following method is much used, but we think the method by interest will ordinarily be found preferable: —

The use of \$400 for 5 mo. is worth as much as the use of \$1 for 400 times 5 mo., or 2000 mo. The use of \$600 for 9 mo. is worth as much as the use of \$1 for 600 times 9 mo., or 5400 mo. The use of \$200 for 12 mo. is worth as much as the use of \$1 for 200 times 12 mo., or 2400 mo. Therefore Mr. Lincoln is entitled to the use of the entire debt for such time as will be equivalent to the use of \$1 for 2000 mo. + 5400 mo. + 2400 mo., or 9800 mo. But as the use of \$1 for 9800 mo. is equivalent to the use of \$1200 for $\frac{1}{1200}$ of 9800 mo., or $8\frac{1}{6}$ mo., he can keep the entire debt $8\frac{1}{6}$ mo., or 8 mo. 5 da.

NOTES. — First. As the equated time will be the same, whatever be the rate of interest, the rate may be considered to be that which is most easily calculated.

Second. The equated time will frequently contain a fraction of a day; but if the fraction be less than $\frac{1}{2}$, it may be disregarded, or if it be more than $\frac{1}{2}$, 1 may be added to the number of days.

2. A owes B \$250, due in 3 mo., \$400, due in 6 mo., and \$350, due in 8 mo. What is the equated time of payment?

3. I owe \$700, payable as follows: \$150 in 3 mo., \$184

in 7 mo., and the rest in 11 mo. When can I pay the whole without gain or loss?

4. I owe $960, payable as follows: $180 in 4 mo. 20 da., $348 in 6 mo. 15 da., $234 in 8 mo. 5 da., and the rest in 10 mo. 13 da. Required the equated time of payment.

5. A trader bought $1800 worth of goods, agreeing to pay $\frac{1}{3}$ of the money down, $\frac{1}{6}$ of it in 5 mo., $\frac{1}{9}$ of it in 6 mo., $\frac{1}{3}$ of it in 9 mo., and the rest in 12 mo. At what time may the whole be paid?

6. Bought a lot of goods, for which I agreed to pay $437.75 in 3 mo., $394.25 in 6 mo., and $628.19 in 8 mo When may the whole be paid without gain or loss?

7. A owes B $800, payable in 10 mo.; but to accommodate B, he pays $250 down. When ought the remainder to be paid?

Solution.— After paying $250, he will owe $800 — $250 = $550, which he ought to keep till its interest shall equal the interest of $800 for 10 months. But the interest of $800 for 10 mo., equals the interest of one dollar for 800 times 10 mo., or 8000 mo. equals the interest of $550 for $\frac{1}{550}$ of 8000 mo., which is $14\frac{6}{11}$ mo. Hence it ought to be paid in $14\frac{6}{11}$ mo.

8. I owe $1000, payable in 9 mo.; but to accommodate my creditor, I pay $300 down, and agree to pay $300 more in 2 mo. How long ought I, in justice, to keep the remainder?

9. I owe $600, payable in 8 mo. 15 da., and $400, payable in 12 mo.; but afterwards agree to pay $400 down, and $300 in 2 mo. 20 da., on condition that I may keep the remainder enough longer to compensate for my loss. When will the remainder become due?

10. A owes B $480, due in 1 yr., and B owes A $720, due in 1 yr. 6 mo. If A should pay his debt at once, when ought B to pay his?

193. *To find Date of Equated Time.*

(*a.*) The best method of solving such examples as the following is to see how much interest will be gained or lost by paying the sum of the debts at any assumed time.

(*b.*) It will be well as a general thing, to select for the assumed time a date on which one of the debts becomes due, as by that means we shall avoid the necessity of reckoning interest on that debt. Reference should also be had to the probable equated time.

(*c.*) The time is reckoned by counting the days between the dates considered, as in the English method of computing interest.

1. James Brown owes William Greene the following debts, viz.: \$534.83, due Jan. 7, 1855; \$285, due April 4, 1855; \$327.38, due July 3, 1855; and \$438,75, due Aug. 17, 1855. When may the whole be paid without gain or loss?

Solution. — Suppose that April 4, 1855, be selected as the assumed time. Then Mr. Brown would gain interest on

\$534.83 from Jan. 7 to April 4, 88 da. =		\$7.84
\$285.00 due at assumed time,		\$0.00
and lose int. on		
\$327.38 from April 4 to July 3, 90 da. =	\$4.91	
\$438.75 from April 4 to Aug. 17, 135 da. =	\$9.87	
Sum of debts = \$1585.96 Sum of losses =		\$14.78
Excess of losses over gains		\$6.94

Showing that Mr. Brown is entitled to keep \$1585.96, the entire debt due, as many days after April 4 as it will take it to gain \$6.94 interest. This, found by **191**, is 26 days, plus a fraction less than $\frac{1}{2}$.

Therefore the equated time is 26 days after April 4, which is April 30.

NOTE. — The above shows that on April 4th Mr. Brown could justly have settled the account by paying \$1585.96 — \$6.94 = \$1579.02.

Again. Suppose that July 3 be selected as the assumed time. Then Mr. Brown would gain interest on

\$534.83 from Jan. 7 to July 3, 178 da., =	\$15.86	
\$285.00 from April 4 to July 3, 90 da., =	4.27	
\$327.38 due at assumed time,	00.00	
Giving for sum of gains,		\$20.13
and lose int. on		
\$438.75 from July 3 to Aug. 17, 45 da., =		3.29
Sum of debts. = \$1585.96 Excess of gain over loss,		\$16.84

Showing that Mr. Brown ought to pay $1585.96, the entire debt due, as many days before July 3 as it will take it to gain $16.84 interest. This, found as before, is 64 days nearly. Therefore the equated time is 64 days before July 3, which is April 30, as before.

NOTE. — The above shows that if the account should not be settled till July 3, Mr. Brown ought justly to pay $1585.96 + $16.84 = $1602.80.

Proof. — By paying the debt on April 30, Mr. Brown will gain interest on

$534.83 from Jan. 7 to April 30, 114 da. = $10.16
$285.00 from April 4 to April 30, 26 da. = 1.23

Making sum of gains = ... $11.39
He will lose interest on

$327.38 from April 30 to July 3, 64 da. = $3.49
$438.75 from April 30 to Aug. 17, 109 da. = $7.97

Making sum of losses, = ... $11.46
Excess of loss over gain, = ... $00.07

which, being less than the interest of $1585.96 for a half day, shows that April 30 is the correct equated time.

2. I owe $387.53, due Nov. 7, 1851; $467.81, due Dec. 21, 1851; $256.19, due Feb. 11, 1852; $136.43, due March 1, 1852; and $387.59, due May 3, 1852. What is the equated time of payment?

3. I owe $2867, due April 15, 1850; $1642, due July 27, 1850; $4371, due Oct. 8, 1850; and $5940, due Jan. 1, 1851. What is the equated time of payment?

4. I owe $628.13, due Dec. 17, 1852; $427.19, due Dec. 23, 1852; $371.16, due Dec. 30, 1852; $587.83, due Jan. 3, 1853; $987.62, due Jan. 7, 1853; and $843.28, due Jan. 14, 1853. What is the equated time of payment?

How much is due on the above Jan. 1, 1853?

5. I owe $543.28, due April 24, 1855; $723.13, due May 11, 1855; $484, due Sept. 3, 1855; $426.18, due Oct. 10, 1855; $236, due Nov. 10, 1855. What is due on the above Sept. 1, 1855, interest being reckoned at 5 per cent?

6. What is the equated time for paying the following debts: $600, due March 7, 1850; $400, due June 11, 1850;

$800, due Aug. 17, 1850; $500, due Oct. 3, 1850; and $1000, due Nov. 27, 1850?

194. *Equation of Accounts.*

(*a.*) The method of finding the equated time when each party owes the other, that is, when there are entries on both the *debit* and *credit* side of an account, does not differ in principle from that in which there are entries only on one side. The following example and solution will illustrate it: —

1. The account books of A and B show that

A owes B	And that B owes A
$426.70, due Jan. 6, 1855.	$148.37, due Dec. 22, 1854.
$413.65, due Feb. 2, 1855.	$173.19, due Jan. 29, 1855.
$169.28, due April 13, 1855.	$587,23, due May 7, 1855.
$328.57, due Aug. 29, 1855.	$658.45, due Sept. 30, 1855.

When ought the balance to be paid?

Solution. — Suppose that April 13, 1855, be the assumed time of payment. Then A will gain interest on each of his debts which becomes due to B before that time, and on each of B's debts which become due to him after that time; for he will have the use of each for a longer time than he is justly entitled to. He will lose interest on each of his debts which becomes due to B after that time, and on each of B's debts which becomes due to him before that time; for he will not have the use of them for so long a time as he is justly entitled to. Hence A will gain the interest of

$426.70 from Jan. 6 to April 13, 97 da.,	= $ 6.90	
$413.65 from Feb. 2 to April 13, 70 da.,	= $ 4.83	
$169.28 from Feb. 13, ...	$ 0.00	
$587.23 from April 13 to May 7, 24 da.,	= $ 2.35	
$658.45 from April 13 to Sept. 30, 170 da.,	= $18.65	
Sum of gains = . . .		$32.73

A will lose the interest of

$328.57 from April 13 to Aug. 29, 138 da., =	$7.56	
$148.37 from Dec. 22, 1854, to April 13, 1855, 112 da., =	$2.76	
$173.19 from Jan. 29, to April 13, 74 da., =	$2.14	
Sum of losses, ...		$12.46
Excess of A's gain over his loss, or of B's loss over his gain,		$20.27

But the sum of A's debts is $1338.20, and of B's is $1567.24.
$1567.24 — $1338.20 = $229.04, the balance which B owes A.

The question now resolves itself into this: If by B's paying A $229.04 April 13, 1855, A gains and B loses $20.27 interest, when can he pay it without any gain or loss of interest? The answer evidently is, As many days after April 13, 1852, as it will take $229.04, or, disregarding the cents, $229, to gain $20.27 interest. This, found by methods before explained, is 531 days = 1 yr.* 166 da., and shows the equated time to be Sept. 26, 1853, which may be proved as were the former examples.

NOTE. — Although accounts like the above are sometimes settled by notes payable at the equated time, they are more frequently settled by notes payable at some more convenient time, or by cash. In all such cases, allowance is made for the interest gained or lost. Thus, if the above account should be settled by cash April 13, 1852, $20.27 would be deducted from the balance due from B to A, in order to compensate B for the interest he would lose; that is, B would pay A $229.04 — $20.27 = $208.77. If it should be paid May 1, 1852, B would have to pay A $.69 (the interest of the balance due A from April 13 to May 1) more than if he had paid it April 13; or, which is the same thing, he would have to pay the balance $229.04, minus its interest $19.58, from May 1, 1852, to the equated time. If the balance due at any given time had been originally required, it should have been found directly by making the given time the "assumed time."

2. By the respective accounts of Henry Lane and William Pond, it appears that

Pond owes Lane	And that Lane owes Pond
$876.37, due April 5, 1852.	$228.13, due April 28, 1852.
579.48, due May 3, 1852.	347.16, due June 3, 1852.
487.83, due June 11, 1852.	313.27, due July 28, 1852.
145.38, due Aug. 8, 1852.	839.42, due Sept. 1, 1852.
$2089.06 = amt. due Lane.	$1727.98 = amt. due Pond.

$2089,06 — $1727.98 = $361.08 = balance due Lane.

When can this balance be paid without gain or loss to either party?

Solution. — Suppose it to be paid June 11, 1852. Then will Mr Pond gain the interest of —

* Reckoning the year as 365 days, as is always done in such cases, unless it includes February of leap year, when it is reckoned as 366 days.

$876.37 from April 5 to June 11, 67 da., = $ 9.79
579.48 from May 3 to June 11, 39 da., = 3.77
487.83, due June 11, ... = 0.00
313.37 from June 11 to July 28, 47 da., = 2.45
839.42 from June 11 to Sept. 1, 82 da., = 11.47

Sum of gains = ... $27.48
He will lose the interest of
$145,38 from June 11 to Aug. 8, 58 da., = $1.41
228,13 from April 28 to June 11, 44 da., = 1.67
347,16 from June 3 to June 11, 8 da., = .46

Sum of losses, ... $ 3.54
Excess of gain over loss, ... 23.94

As Mr. Pond gains this interest on money which he owes, he ought to pay the debt ($361.08, the balance of the account) as many days before June 11, 1852, as it will take for it to gain $23.94 interest. This gives for the equated time 398 days before June 11, 1852, which is May 10, 1851. The sum necessary to settle the account after the equated time will be the amount of the balance, $361.08, from the equated time to the time of settlement.

3. When was the balance of the following account due?

Dr. George Ide, in account with James Snow. Cr.

1849.				1849.			
Jan. 17.	To Mdse. .	$336	18	Feb. 1.	By Mdse. .	$421	30
Jan. 31.	To Mdse. .	443	17	Feb. 27.	By Mdse. .	620	00
March 7.	To Cash, .	218	63	Mar. 13.	By Mdse. .	283	17
April 17.	To Mdse. .	500	00	April 29.	By Mdse. .	482	29
May 28.	To Mdse. .	84	36	June 1.	By Mdse. .	825	13

4. When was the balance of the following account due?

Dr. George Black in account with John Brown. Cr.

1850.				1850.			
May 13.	To Mdse. 4 mo.	$431	17	June 1.	By Mdse. 3 mo.	$223	62
July 25.	To Mdse. 3 mo.	256	38	July 7.	By Mdse. 6 mo.	150	00
Aug. 8.	To Mdse. 6 mo.	431	72	July 22.	By Mdse. 3 mo.	250	00
Sept. 23.	To Mdse. 3 mo.	585	41	Sept. 1.	By Cash, .	300	00
Nov. 7.	To Mdse. 3 mo.	738	16	Nov. 23.	By Mdse. 2 mo.	138	16
				Dec. 1.	By Mdse. 3 mo.	122	31

NOTE.—4 mo., 3 mo., &c., means that goods were sold at so many months' credit.

5. What was due on the following account Jan. 1, 1853?

Dr. George Mann in account with Henry Guild. Cr.

1852.				1852.			
May 5.	To Bal. 3 mo.	$513	43	April 20.	By Mdse. 6 mo.	$328	13
June 27.	To Mdse. 4 mo.	624	27	May 10.	By Mdse. 3 mo.	143	27
July 3.	To Mdse. 6 mo.	831	13	June 13.	By Mdse. 4 mo.	837	19
Sept. 5.	To Mdse. 2 mo.	47	62	July 7.	By Mdse. 6 mo.	56	18
Sept. 17.	To Mdse. 3 mo.	125	53	Aug. 20.	By Mdse. 4 mo.	123	42
Oct. 19	To Cash, .	387	00	Oct. 1.	By Mdse. 3 mo.	78	36
Dec. 1.	To Cash, .	629	28	Nov. 23.	By Mdse. 2 mo.	127	14

6. What was due on the following account Jan. 1, 1853, interest being 7 per cent, and 4 months' credit being allowed on each entry?

Dr. David H. Daniels in account with George W. Dean. Cr.

1852.				1852.			
July 8.	To Mdse. .	$ 236	17	July 3.	By Mdse. .	$439	27
Aug. 1.	To Sundries,	819	63	July 25.	By Mdse. .	213	16
Sept. 4.	To Mdse. .	142	13	Sept. 13.	By Mdse. .	100	00
Nov. 13.	To Mdse. .	947	22	Oct. 24.	By Mdse. .	262	18
Dec. 8.	To Sundries,	1050	00	Nov. 30.	By Mdse. .	327	48
				Dec. 21.	By Mdse. .	520	75

195. *To find the Principal, or Interest, from the Amount, Rate, and Time.*

(*a.*) When the amount, time, and rate are given to find the principal or interest, we find what part any principal, or (if the interest be required) its interest for the given time, at the given rate, is of its amount, and then take this part of the given amount.

(*b.*) The first step towards this is to find the fraction expressing what part any interest for the given time, at the

given rate, is of its principal. This fraction will *always* be the same part of the given annual rate that the given time is of 1 year, or 360 days; or, *if the rate is* 6 *per cent,* it will equal the fraction expressing the part which the given time is of 200 months, or 6000 days. The amount, of course, will equal the principal, plus the fractional part of it which the interest equals.

(*c.*) Thus, interest for 1 yr. 7 mo., or 19 months, at 6 per cent per year, $= \frac{19}{200}$ of the principal, and the amount $= \frac{200}{200} + \frac{19}{200} = \frac{219}{200}$ of the principal. Hence, $\frac{1}{200}$ of the principal $= \frac{1}{219}$ of the amount, and the entire principal $= \frac{200}{219}$, and the interest $\frac{19}{219}$, of the amount for 19 mo. at 6 per cent. (The same fractions would have been obtained by considering the interest to be $\frac{19}{12}$ of $\frac{6}{100}$ of the principal.)

(*d.*) Again. Interest for 19 months at $4\frac{1}{2}$ per cent per year $= \frac{19}{12}$ of $\frac{4\frac{1}{2}}{100} = \frac{19}{12}$ of $\frac{9}{200} = \frac{57}{800}$ of the principal, and the amount $= \frac{800}{800} + \frac{57}{800} = \frac{857}{800}$ of the principal. Hence, the principal $= \frac{800}{857}$, and the interest $= \frac{57}{857}$, of the amount for 19 mo. at $4\frac{1}{2}$ per cent.

(*e.*) Again. Interest for 2 yr. 3 mo. 2 da., or 812 days, at 6 per cent per year, $= \frac{812}{6000} = \frac{203}{1500}$ of the principal, and the amount $= \frac{1500}{1500} + \frac{203}{1500} = \frac{1703}{1500}$ of the principal. Hence, the principal $= \frac{1500}{1703}$, and the interest $= \frac{203}{1703}$, of the amount for 2 yr. 3 mo. 2 da. at 6 per cent. (The same fractions would have been obtained by considering the interest to be $\frac{812}{360}$ of $\frac{6}{100}$ of the principal.)

(*f.*) Again. Interest for 5 mo. 14 da., or 164 days, at 7 per cent per year, $= \frac{164}{360}$ of $\frac{7}{100} = \frac{41}{90}$ of $\frac{7}{100} = \frac{287}{9000}$ of the principal, and the amount $= \frac{9000}{9000} + \frac{287}{9000} = \frac{9287}{9000}$ of the principal. Hence, the principal $= \frac{9000}{9287}$, and the interest $= \frac{287}{9287}$, of the amount for 5 mo. 14 da. at 7 per cent.

1. What principal on interest at 6 per cent per year will amount to $884.125 in 1 yr. 2 mo. 10 da.?

Solution. — Since, at 6 per cent per year, interest for 1 day $= \frac{1}{6000}$ of the principal, interest for 1 yr. 2 mo. 10 da., or 430 days, must equal $\frac{430}{6000}$, or $\frac{43}{600}$, of the principal, and the amount must equal $\frac{600}{600} + \frac{43}{600}$, or $\frac{643}{600}$, of the principal. Hence, $\frac{1}{600}$ of the principal must equal $\frac{1}{643}$, and the principal itself must equal $\frac{600}{643}$ of the amount. $\frac{600}{643}$ of

$884.125 = $825 = principal required. The mere numerical work may be indicated thus : —

1 yr. 2 mo. 10 da. = 430 da.

$\frac{430}{6000}$, or $\frac{43}{600}$, + $\frac{600}{600} = \frac{643}{600}$

$\frac{600}{643}$ of $884.125 = $825 = principal required.

Second Solution. — The amount of $1 for 1 yr. 2 mo. 10 da. = 1.07\frac{1}{6}$, and if one dollar is required to gain 1.07\frac{1}{6}$, as many dollars will be required to gain $884.125 as there are times 1.07\frac{1}{6}$ in $884.125 But $884.125 ÷ 1.07\frac{1}{6}$ = $884.125 ÷ $\frac{643}{600} = \frac{600}{643}$ of $884.125 = $825, as before.

Proof. — The amount of $825 for 1 yr. 2 mo. 10 da. is $884.125

2. What principal on interest at 7 per cent per year will amount to $703.551 in 4 mo. 27 da.?

Solution. — At 7 per cent per year, interest for 4 mo. 27 da., or 147 days, = $\frac{147}{360}$ of $\frac{7}{100} = \frac{343}{12000}$ of the principal, and the amount = $\frac{12000}{12000} + \frac{343}{12000} = \frac{12343}{12000}$ of the principal. Hence the principal must equal $\frac{12000}{12343}$ of the amount, which in this case is $\frac{12000}{12343}$ of $703.551 = $684. The mere numerical work may be indicated thus : —

4 mo. 27 da. = 147 da. $\frac{147}{360}$ of $\frac{7}{100} = \frac{343}{12000}$.

$\frac{343}{12000} + \frac{12000}{12000} = \frac{12343}{12000}$.

$\frac{12000}{12343}$ of $703.551 = $684 = principal required.

Second Solution. — The amount of $1 at 7 per cent for 4 mo. 27 da. = 1.02\frac{103}{120}$; and if one dollar is required to gain 1.02\frac{103}{120}$, as many dollars would be required to gain $703.551 as there are times 1.02\frac{103}{120}$ in $703.551. But $703.551 ÷ 1.02\frac{103}{120}$ = $703.551 ÷ $\frac{12343}{12000} = \frac{12000}{12343}$ of $703.551 = $684, as before.

Proof. — The amount of $684 for 4 mo. 27 da. at 7 per cent equals $703.551.

What principal will amount to —

3. $569.296 in 8 mo. 16 da. at 6 per cent?
4. $573.16 in 2 yr. 9 mo. 10 da. at 6 per cent?
5. $922.13 in 1 yr. 4 mo. at 8 per cent?
6. $378.82 in 1 yr. 4 mo. 20 da. at 6 per cent?

7. $57.72 in 8 mo. 20 da. at 10 per cent?
8. $899.944 in 5 mo. 14 da. at 6 per cent?

196. *Discount and Present Worth.*

(*a.*) DISCOUNT, as it is technically called, furnishes the most common application of the processes of the preceding article to the problems of business life.

(*b.*) It is obvious that the true present value of a debt due at a future time is that sum of money which, put on interest at the present time, will amount to the given debt at the time it becomes due.

Thus, when the rate of interest is 6 per cent per year, a debt of $106 due in one year is worth the same as a debt of $100 due now; for if the money received on the second debt be put on interest, it will amount to $106 in one year; that is, when the first debt becomes due.

(*c.*) A debt due at a future time may be regarded, then, as the amount of a principal on interest from the present time to the time when the debt will become due. This principal is usually called the PRESENT WORTH of the debt, and its interest is called the DISCOUNT, because, if discounted or deducted from the debt, it leaves the present worth.

(*d.*) From this, it follows that to ask what is the present worth of $651, due in 6 mo. 20 da., money being worth 6 per cent per year, is equivalent to asking what principal on interest at 6 per cent will amount to $651 in 6 mo. 20 da.; and that the solution of the first question is the same as that of the second. To test the correctness of any result, see if the amount of the present worth equals the given debt.

1. What is the present worth of $438.18 due in 1 yr. 6 mo. at 6 per cent per year?

Partial Solution. — The present worth required is that sum of money which, put on interest at 6 per cent, will amount to $438.18 in 1 yr. 6 mo., or 18 mo. The interest for 18 mo. $= \frac{18}{200} = \frac{9}{100}$ of the principal, and the amount =, &c., as in the last article.

What is the present worth of —

2. $83.45 due in 4 yr. 2 mo. at 6 per cent?
3. $89.88 due in 1 yr. at 7 per cent?

4. $142.56 due 2 yr. hence at 4 per cent?
5. $122.94 due 4 mo. 27 da. hence at 6 per cent?
6. $475.64 due 1 yr. 8 mo. hence at 6 per cent?
7. $578.50 due 3 yr. $1\frac{1}{2}$ mo. hence at 8 per cent?
8. $731.52 due 3 yr. 4 mo. hence at 6 per cent?
9. $1323.70 due 7 mo. 15 da. hence at $5\frac{1}{3}$ per cent?
10. What is the discount of $195.87 due 1 yr. 5 mo. 19 da. hence, at 6 per cent per year?

Direction. — Find what part of the debt the discount is, and get that part of $195.87. For proof, subtract the discount thus found from $195.87, and see if the interest of the remainder for the given time equals the discount. We may also get the discount by finding the present worth, and subtracting it from the debt.

What is the discount of —

11. $3946.11 due 2 yr. 5 mo. 15 da. hence at 6 per cent?
12. $6392.43 due 15 mo. 7 da. hence at 6 per cent?
13. $1241.27 due 1 yr. 5 mo. 23 da. hence at 6 per cent?
14. $6255 due 3 yr. 2 mo. hence at 5 per cent?
15. $179.96 due 2 yr. 3 mo. 6 da. hence at 4 per cent?
16. I own a note for $976, payable on demand with interest, and another for $1034.56, payable in just 1 year, with interest afterward. Allowing money to be worth 6 per cent per year, which debt is justly worth the most at the present time, and how much the most? Which will be worth the most at the end of the year? Which will be worth the most at the end of 6 months? Which will be worth the most at the end of 2 years?

17. If two notes are given on the same day, one for a certain sum due at a future time, with interest afterwards, and the other for the true present worth of the first note, payable on demand with interest, their true values will be the same at the time they are given, and also at the time the first becomes due; but at all other times they will differ. At any time between the day of their date and the day when the first note becomes due, the true value of the second note will be greater than that of the first; but at any time after the first

note becomes due, the true value of the first note will be greater than that of the second. Show why this is so.

197. *Business Method of Discount.*

(*a.*) Business men are usually willing to allow on money paid for goods before it is due, a discount equal to, or greater than, its interest from the time of payment to the time when, by the conditions of the sale, it would have become due.

Thus, when only the interest of the debt is discounted, \$824 due in 6 months, interest being 6 per cent per year, is regarded as worth \$824 — .03 of \$824 = \$824 — \$24.72 = \$799.28, whereas it ought to be worth \$800.

(*b.*) This is always an advantage to the person owing the money, as it enables him to pay his debt for less than the sum which, put at interest, would amount to the debt at the time it would become due.

(*c.*) It is common to deduct as much as five per cent from the face of a bill due in four or six months, and even more is sometimes deducted.

(*d.*) The present worth thus obtained may be called the ESTIMATED PRESENT WORTH, to distinguish it from the TRUE PRESENT WORTH, obtained by the method explained in **196.**

1. I owed a debt of \$8692, payable June 1, 1852; but my creditor, offering to allow me discount estimated by the business method at the rate of 6 per cent per year, if I would pay the debt Jan. 1, 1852, I borrowed money for the purpose at 6 per cent interest. June 1, 1852, I paid the amount of the borrowed money. What was my gain by the transaction?

2. Owing a debt of \$1545, due in 6 months, when money is worth 6 per cent per year, what shall I gain by hiring money enough to pay it now, allowing the usual business discount on the debt, and then paying the borrowed money with interest, when the original debt would otherwise have become due?

3. At 6 per cent per year, what is the difference between

the bank discount and the true discount of a note for $2059.40, payable in 60 days? *

4. Received for my note of $600, payable in 6 months, its true present worth. How much more did I receive on it than I should have received at a bank, money being worth 6 per cent? How much interest money shall I have gained, when the note becomes due, over what I should have gained on the present worth, as determined at the bank?

5. For how much must a note, payable in 30 days, be given, that, when discounted at a bank, $900 may be received on it, money being 6 per cent?

Solution. — The money received on a note discounted at a bank equals the sum for which the note is given, minus its interest for the time before it becomes due. Since, at 6 per cent, the interest for 30 days and grace, or 33 days, $= \frac{33}{6000} = \frac{11}{2000}$ of the principal, the sum received must equal $\frac{2000}{2000} - \frac{11}{2000} = \frac{1989}{2000}$ of the face of the note. Therefore the face of the note $= \frac{2000}{1989}$ of the sum received on it, $= \frac{2000}{1989}$ of $900 = $904,977, or, as it would in practice be considered, $904.98.

The numerical work can be expressed thus: —

$$\frac{33}{6000} = \frac{11}{2000}. \quad \frac{2000}{2000} - \frac{11}{2000} = \frac{1989}{2000}. \quad \frac{2000}{1989} \text{ of } \$900 = \frac{200000}{221} =$$

$904.977;

hence face of note = $904.98.

6. How much would be received at a bank on a note for $904.98, payable in 30 days?

NOTE. — The above example suggests the method of proving the 5th.

7. For how much must a note payable in 3 months be given, that, when discounted at a bank, $1000 may be received on it, money being worth 6 per cent per year?

8. For how much must a note payable in 6 mo. be given, that, when discounted at a bank, $1800 may be received on it, money being worth 6 per cent per year?

* Remember that three days' grace are always allowed on a note, unless the contrary is specified.

9. Obtained at a bank, on my note payable in 6 mo., money enough to buy 20 acres of land at \$100 per acre. The day my note at the bank became due, I sold the land for \$2062.32 cash. Did I gain or lose by the transaction, and how much, money being worth 6 per cent per year?

10. Obtained at a bank, on my note payable in 4 months, money enough to buy 20 acres of land at \$100 per acre. The day the note became due, I sold the land for cash, at such rate that the price of 18 acres was just sufficient to pay the note. How much did I gain by the transaction, money being worth 6 per cent?

11. Bought goods to the amount of \$864.27 on a credit of 6 months; but the seller offering to deduct 5 per cent from the face of the bill if I would pay cash, I hired the requisite amount of money, giving my note payable in 6 months, with interest at 6 per cent per year, to be reckoned from date. For how much less than the value of the original bill could I pay the amount of this note?

12. I owed \$800, due in 6 months; but my creditor offering to deduct 5 per cent of the debt for cash, I paid \$380 down. How much did I still owe?

Suggestion. — Since 5 per cent of the debt was to be deducted for cash, the cash payment would be 95 per cent, $= \frac{95}{100} = \frac{19}{20}$, of the part of debt it would cancel; or the part cancelled would be $\frac{20}{19}$ of the cash paid.

13. I owed \$900, payable in 4 months; but my creditor offering to deduct 4 per cent of the debt for ready money, I paid \$696 down. How much did I still owe?

198. *To find the Rate.*

Problems in which, the principal, interest, and time being given, we are required to find the rate, rarely occur in business life. The following solutions illustrate the principles which apply to them.

1 At what rate per cent must $648 be on interest to gain $81.873 in 2 yr. 3 mo. 17 da.?

Solution. — The principal being equal to 648,000 mills, and the interest to 81,873 mills, the interest is $\frac{81873}{648000} = \frac{9097}{72000}$ of the principal. 2 yr. 3 mo. 17 da. = 27 mo. 17 da. = 827 da. If the interest for 827 da. = $\frac{9097}{72000}$ of the principal, the interest for 1 day must equal $\frac{1}{827}$ of $\frac{9097}{72000}$ of the principal, and the interest for 1 yr., or 360 da., must equal 360 times the last result, or $\frac{360}{827}$ of $\frac{9097}{72000} = \frac{11}{200} = .05\frac{1}{2} = 5\frac{1}{2}$ per cent.

Required the rate of interest when —

2. $624 gains $74.88 in 1 yr. 2 mo. 12 da.
3. $57.25 gains $5.038 in 1 yr. 5 mo. 18 da.
4. $855 gains $46.55 in 2 yr. 2 mo. 4 da.
5. $64.80 gains $6.246 in 11 mo. 17 da.

199. *To find the Principal from the Interest.*

Problems in which, the interest, rate, and time being given, we are required to find the principal, are, like those in the last article, of rare occurrence.

1. What principal on interest at 6 per cent will gain $37.47 in 1 yr. 3 mo.?

Solution. — At 6 per cent per year, the interest of any principal for 15 mo. = $\frac{15}{200} = \frac{3}{40}$ of the principal. If $37.47 is $\frac{3}{40}$ of the principal, $\frac{1}{40}$ of the principal must equal $\frac{1}{3}$ of $37.47, and the principal must equal 40 times the last result, or $\frac{40}{3}$ of $37.47, which is $449.60.

2. What principal on interest at 8 per cent will gain $26.18 in 1 yr. 4 mo. 15 da.?

Solution. — Since 1 yr. 4 mo. 15 da. = $16\frac{1}{2}$ mo. = $\frac{16\frac{1}{2}}{12}$ yr. = $\frac{11}{8}$ of 1 yr., the interest must equal $\frac{11}{8}$ of 8 per cent, or 11 per cent of the principal. If $26.18 is 11 per cent of the principal, 1 per cent of the principal must be $\frac{1}{11}$ of $26.18, which is $2.38, and 100 per cent, or the principal, must equal 100 times the last result, which is $238.

What principal on interest —

3. At 6 per cent will gain $8.73 in 5 mo.?
4. At 6 per cent will gain $4.77 in 1 yr. 5 mo. 20 da.?

5. At 5 per cent will gain $4.27 in 2 yr. 6 mo.?

6. At $7\frac{1}{2}$ per cent will gain $116.127 in 4 yr. 4 mo. 4 da.?

200. *Compound Interest.*

When interest is to be paid at regular intervals, or, if unpaid, is to be added to the principal, to form a new principal on which interest is to be computed, it is called Compound Interest. The following example illustrates this: —

1. What is the compound interest of $784 for 2 yr. 8 mo. at 6 per cent, payable annually?

Solution.

a =	$784.	= principal.
.06 of a = b =	47.04	= interest for 1st year.
a + b = c =	831.04	= amount due at end of 1st year.
.06 of c = d =	49.862	= interest for 2d year.
c + d = e =	880.902	= amount due at end of 2d year.
.04 of e = f =	35.236	= interest for 8 mo.
e + f = g =	916.138	= amount due at end of 2 yr. 8 mo.
a =	784.	= principal.
g — a = h =	$132.138	= compound interest for 2 yr. 8 mo.

2. To what sum will $437 amount in 2 yr. 6 mo. at 4 per cent, payable semiannually?

Solution.

a =	$437	= principal.
.02 of a = b =	8.74	= interest 1st 6 mo.
a + b = c =	445.74	= amount at end of 6 mo.
.02 of c = d =	8.915	= interest 2d 6 mo.
c + d = e =	454.655	= amount at end of 12 mo.
.02 of e = f =	9.093	= interest 3d 6 mo.
e + f = g =	463.748	= amount at end of 18 mo.
.02 of g = h =	9.275	= interest 4th 6 mo.
g + h = i =	473.023	= amount at end of 2 yr
.02 of i = j =	9.4[illegible]	= interest 5th 6 mo.
i + j = k =	$482.483	= amount due at end of 2 yr. 6 mo.

3. What is the compound interest of $938.63 for 4 yr. 6 mo. at 6 per cent, payable annually?

4. What is the compound interest of $573.32 for 2 yr. 3 mo. at 8 per cent, payable quarterly?

5. To what sum will $1000 amount in 3 yr. 2 mo. at 6 per cent, payable annually?

6. To what sum will $500 amount in 4 yr. 3 mo. at 5 per cent, payable semiannually?

201. *Table for Compound Interest.*

Table showing what part of any principal is equal to its amount at 3, 4, 5, 6, 7, and 8 per cent annual compound interest, for any number of years not exceeding 25.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1.	1.03	1.04	1.05	1.06	1.07	1.08
2.	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664
3.	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4.	1.125509	1.169858	1.215506	1.262477	1.310796	1.360489
5.	1.159274	1.216653	1.276281	1.338225	1.402552	1.469328
6.	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7.	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8.	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9.	1.304773	1.423312	1.551328	1.689479	1.838459	1.999005
10.	1.343916	1.480244	1.628894	1.790848	1.967151	2.158925
11.	1.384234	1.539454	1.710339	1.898298	2.104852	2.331639
12.	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13.	1.468534	1.665073	1.885649	2.132928	2.409845	2.719624
14.	1.512590	1.731676	1.979931	2.260903	2.578534	2.937194
15.	1.557967	1.800943	2.078928	2.396558	2.759031	3.172169
16.	1.604706	1.872981	2.182874	2.540351	2.952164	3.425943
17.	1.652848	1.947900	2.292018	2.692772	3.158815	3.700018
18.	1.702433	2.025816	2.406619	2.854339	3.379932	3.996019
19.	1.753506	2.106849	2.526950	3.025599	3.616527	4.315701
20.	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21.	1.860294	2.278768	2.785962	3.399563	4.140562	5.033834
22.	1.916103	2.369918	2.925260	3.603537	4.430402	5.436540
23.	1 973586	2.464715	3.071524	3.819749	4.740530	5.871464
24.	2.032794	2.563304	3.225100	4.048934	5.072367	6.341181
25.	2.093778	2.665836	3.386355	4.291870	5.427433	6.848475

USE OF THE TABLE.—The amount of any sum of money, at compound annual interest, for any time and rate mentioned in the table, may be found by multiplying the principal by the appropriate number selected from the table. This will be illustrated in the following examples and solution.

1. What is the amount of \$245.73 for 12 yr. 6 mo. 20 da. annual compound interest at 6 per cent?

Solution.—By the table, it appears that the amount of a sum for 12 years, at 6 per cent, compound interest, is 2.012196 times the principal. Multiplying \$245.73 by 2.012196, we have \$494.45692308, or, omitting the denominations below mills,

$a =$ \$494.457 = amount for 12 years.
$\frac{1}{30}$ of $a = b =$ 16.481 = interest for 6 mo. 20 da.

$a + b = c =$ \$510.938 = amount for 12 yr. 6 mo. 20 da

What is the amount at annual compound interest of—

2. \$578.67 for 11 yr. 4 mo. at 3 per cent?
3. \$147.43 for 22 yr. 4 mo. 24 da. at 5 per cent?
4. \$1467 for 18 yr. at 7 per cent?

What is the compound interest of—

5. \$1137.38 for 13 yr. 6 mo. at 6 per cent?
6. \$328.96 for 9 yr. 3 mo. at 4 per cent?

202. *Commission.*

Money received for services in buying and selling goods for others is called COMMISSION, and is usually reckoned at a certain per cent of the cost of the goods bought, or price of hose sold. A merchant who makes it his business to buy and sell on commission is called a COMMISSION MERCHANT. (See 177th page, Ex. 21, Note.)

1. A commission merchant sold goods for a manufacturer to the amount of \$568.36, for which he charged a commission of $2\frac{1}{2}$ per cent. What was his commission, and how much will be due to the manufacturer?

Answer. — His commission = $.02\frac{1}{2}$ of \$568.36 = \$14.209, and the sum due the manufacturer = \$568.36 — \$14.21 = \$554.15.

2. A commission merchant bought goods for a country trader to the amount of \$738.27, for which he charged a commission of $1\frac{1}{2}$ per cent. What was his commission, and what sum must the trader remit to pay for the goods and commission?

Answer. — His commission = $.01\frac{1}{2}$ of \$738.27 = \$11.07, and the trader must remit \$738.27 + \$11.07 = \$749.34.

3. I sold for Seth Jones 2024 pounds of butter at 19 cents per pound, and 5276 pounds of cheese at $7\frac{1}{4}$ cents per pound. What was the value of my commission at $2\frac{1}{2}$ per cent on the sales? How much money ought I to pay him?

4. I bought for Francis Jackson 50 pairs of boots at $\$3.87\frac{1}{2}$ per pair, 100 pairs at \$2.75 per pair, 75 pairs at $\$2.16\frac{2}{3}$ per pair, 100 pairs of shoes at \$1.56 per pair, and 80 pairs at \$1.08 per pair. What was the value of my commission at 2 per cent on the purchase? How much money must Mr. Jackson remit to me to pay for the goods and my commission?

5. I have sent \$5000 to my agent in New Orleans, directing him to expend it for cotton, first deducting his commission of 2 per cent on the purchase. What will be his commission, and what will he expend for cotton?

Suggestion. — The \$5000 sent includes the sum to be actually invested and my agent's commission of 2 per cent on that sum, and is therefore $\frac{102}{100}$ of the purchase money. Hence, the purchase money, or sum to be expended by my agent, is $\frac{100}{102}$ of \$5000, and his commission is $\frac{2}{102}$ of \$5000. For proof, see if the sum expended, plus .02 of it, equals \$5000.

6. I have received \$5600 from my correspondent in St. Louis, with directions to expend it in merchandise, first deducting my commission of $2\frac{1}{2}$ per cent on the money expended. How much ought I to expend?

7. My agent in Rochester writes that he has purchased a lot of flour for me at \$5 per barrel, and that the entire cost

of the flour, including his commission of $2\frac{1}{2}$ per cent, is $1600 How many barrels of flour were purchased?

8. Briggs & Grant sold for Jenks, Clarke, & Co. 1000 brooms at $.25 apiece, for which they charge a commission of 4 per cent. Pursuant to instructions, they invest the balance in sugar at 8 cents per pound, first deducting their commission of 2 per cent on the purchase. How many pounds of sugar did they buy?

9. My agent at Cincinnati writes that he has purchased on my account a lot of provisions, and that his commission of $1\frac{1}{2}$ per cent on the purchase is $13.50. How many dollars worth of provisions has he bought?

10. My agent at New Orleans has purchased for me a lot of cotton at 6 cents per pound, for which he charges a commission of $1\frac{2}{3}$ per cent. His commission amounts to $38.80. How many pounds of cotton has he bought, and how much money must I remit to him to pay for the cotton and his commission?

11. Haskell & Latham sell for me, at auction, goods to the amount of $8732. Their charges are as follows: Auction tax, 1 per cent; commission, 4 per cent; guarantying the sales, 3 per cent; advertising, $23.25; truckage and storage, $11.25. How much money will be due me?

Suggestion. — The auction tax, commission, and guaranty amount to 8 per cent of the value of the goods, which, together with the other expenses, must be deducted from the value of the goods, to leave the sum due me.

12. Lewis & Johnson sell for Field & Dean 96 cases of cassimere, each case containing 276 yards, at $1.25 per yard, for which they charge a commission of $3\frac{1}{2}$ per cent. They receive instructions from Field & Dean to remit them half of the net proceeds, and to invest the remainder, after deducting a commission of $1\frac{1}{4}$ per cent on the purchase, in wool, at 50 cents per pound. What was the value of the cash remitted to Field & Dean? How many pounds of wool were bought?

203. *Stocks.*

(*a.*) Money invested in any property designed to yield an income is called STOCK.

The money invested by a man in his business is called his STOCK IN TRADE; that invested in government securities, bonds, &c., is called GOVERNMENT STOCK.

(*b.*) The CAPITAL STOCK of any incorporated company is the money paid in by its members for the general purposes for which the company was formed. It is divided into equal parts, called SHARES. Any person owning one or more of these shares is a STOCKHOLDER, or member of the corporation. STOCKS is a general term applied to the shares themselves, which may be bought or sold like any other property.

(*c.*) The value at which the shares of any corporation are rated in estimating its capital stock, that is, their first or original value, is called their NOMINAL VALUE, or their PAR VALUE, and is always the same. The price which they will bring, if exposed for sale, is their TRUE OR REAL VALUE, and is different at different times. If the real value equals the par value, the stocks are at par; if it be greater, they are above par, and sell at a PREMIUM, or ADVANCE; if it be less, they are below par, and sell at a DISCOUNT.

(*d.*) The profits accruing to the corporation, if any, are at intervals distributed among the members in proportion to the number of shares each holds, and are then called *dividends.* The dividends are usually reckoned at a certain per cent of the par value of the shares.

1. How much will 11 shares Fall River Railroad stock cost, at an advance of 6 per cent, the par value being $100 per share?

Solution.

$1100 = par value of 11 shares.
66 = 6 per cent premium.

$1166 = real value, or required cost.

2. How much will 11 shares Providence Railroad stock

cost, at a discount of 6 per cent, the par value being $100 per share?

Solution.

$1100 = par value of 11 shares.
66 = 6 per cent discount.

$1034 = real value, or required cost.

3. How much will 53 shares Suffolk Bank stock cost at an advance of 23 per cent, the per value being $100 per share?

4. How much will 43 shares Vermont Central Railroad stock cost, at a discount of 78 per cent, the par value being $50 per share?

5. How many shares of stock at an advance of 5 per cent on the par value of $100 per share, can be bought for $1995?

6. How many shares of stock at a discount of 5 per cent from the par value of $100 per share, can be bought for $1805?

7. A broker paid $1776.50 for stock at an advance of 4½ per cent. What was the par value of the stock bought?

8. A broker paid $4850 for bank stock, at a discount of 3 per cent. What was the nominal value of the stock bought?

9. A broker sold on consignment 376 shares bank stock, par value $100 per share, at an advance of 7 per cent. He charged 25 cents per share for his services. How much ought he to remit to the person consigning the stock?

10. I bought 40 shares of railroad stock, at 33 per cent below par, and after keeping them 10 months, sold them at 20 per cent below. How much did I gain on them, allowing that I hired money for the investment at 6 per cent interest, and that the par value of the shares was $100 each?

11. Alfred E. Potter bought 10 shares of bank stock at par, which was $50 per share. At the end of 3 months he received a dividend of 4 per cent, and at the end of 9 months he received another of 3 per cent. At the end of one year he sold the stock at an advance of 3 per cent. Money being worth 6 per cent per year, how much did he gain by the transactions?

NOTE — Interest should be reckoned on the dividends from the time they were made.

12. Crocker & Guild sent $972.63 to a stock broker, directing him to invest it in Fall River Railroad stock. He bought the stock at a premium of 7 per cent, the par value being $100 per share, and he charged a commission of 1 per cent on the money invested. How many shares did he buy?

13. I paid $7398 for stock at 10 per cent below par, and some time after sold the stock at 10 per cent above par. How much did I gain on it?

14. Mr. Hamblin bought stock at 10 per cent above par, but he was obliged to sell it at 10 per cent below par. Allowing that he lost $138.40 on it, what did he pay for it?

204. *Insurance.*

(*a.*) INSURANCE is an obligation assumed by one individual or company to pay to another a certain sum of money on the occurrence of some contingent event.

(*b.*) A house is insured against loss by fire, when some individual or company agrees to pay the owner a specified sum if it is burned within a given time.

(*c.*) A ship in like manner may be insured against loss by fire or by any of the perils of the sea.

(*d.*) When the obligation is to pay the person insured a certain sum if he is sick, it is called HEALTH INSURANCE; when it is to pay to his heirs, or to some particular person, a specified sum if the person insured dies, it is called LIFE INSURANCE.

(*e.*) The written certificate of insurance is called a POLICY.

(*f.*) The sum paid for insurance is called the PREMIUM.

(*g.*) If property is insured, the premium is a certain per cent of the sum covered, i. e., of the sum for which it is insured, and is usually paid at the time of effecting the insurance.

(*h.*) When health or life is insured, the premium is usually a given sum paid annually during the time for which the insurance continues; and its amount is calculated from tables prepared by the different insurance companies. It will, therefore, be unnecessary to give examples in life or health insurance.

1. What would be the expense of getting $1000 insured

on a house at a premium of $1\frac{1}{2}$ per cent, the charge for the policy being $1?

2. I had $1000 insured on my house for 7 years, for which I paid a premium of $2\frac{1}{2}$ per cent, and $1 for the policy. At the end of 5 yrs. 7 mo. 15 da. the house was burned, and I received the amount for which it was insured. Money being worth 6 per cent per year compound interest, how much did I really save by having the house insured?

3. Oct. 1, 1854, I bought a lot of flour for $6000, giving my note payable in 6 months, and immediately shipped it for England. Oct. 8, 1854, I got it insured for $6000, paying a cash premium of $1\frac{1}{2}$ per cent, and $1 for the policy. Oct. 10, 1854, I paid a bill of $50 for cartage and other expenses. Nov. 1, 1854, I received information that the vessel was lost at sea; and Dec. 1, the insurance company paid me the insurance. I immediately put it on interest at 6 per cent, till my note for the flour became due, when I made a complete settlement. Had I gained or lost by the transactions, and how much?

205. *Assessment of Taxes.*

(*a.*) TAXES are duties or assessments laid on persons or property, usually for some public purpose.

(*b.*) A tax on persons is called a POLL TAX, and a tax on property is called a PROPERTY TAX.

The poll tax is only assessed on males between certain ages, as between twenty-one and seventy; and in some states is not assessed

(*c.*) In assessing taxes, it is necessary, —

First. To estimate the value of all the property to be taxed, and make a complete inventory of it.

Second. To find the number of polls, i. e., the number of persons liable to pay a poll tax.

Third. To determine what portion of the tax is to be raised upon the polls, and to divide it equally among them.

Fourth. To find how much must be paid on each dollar of the taxable property, to raise the remainder of the tax.

This last may be done by dividing the amount to be raised by the estimated value of the property on which it is to be raised. It will then be easy to find the tax of any individual.

1. A tax of $4800 is to be raised by a certain town. The taxable property is valued at $960,000, and there are 320 polls, each taxed $1.50. What will be the tax on each dollar, and what will be the tax of each of the following persons?

A, who pays a tax on $5700, and 2 polls.
B, who pays a tax on $728, and 1 poll.
C, who pays a tax on $8976, and 3 polls.
D, who pays a tax on $1147, and 1 poll.

Solution. — The tax on 320 polls at $1.50 each is $480, which, subtracted from $4800, leaves $4320 to be assessed on the property.

Since 960,000 is to be taxed $4320, one dollar will be taxed $\frac{1}{960000}$ of $4320, which is $4\frac{1}{2}$ mills.

A's tax on 2 polls would be twice $1.50, or $3, and on $5700 property would be 5700 times $4\frac{1}{2}$ mills, which is $25.65, and added to $3 gives $28.65 as the amount of A's tax.

The tax of the others may be found in the same way.

2. A tax of $4800 is to be raised in a certain town. The taxable property is valued at $1,228,000, and there are 575 polls to be taxed $1.40 each. How much is the tax on $1? How much is the tax on each of the following persons?

J. S., who pays a tax on $1728, and 1 poll.
S. R., who pays a tax on $4235, and 2 polls.
L. M., who pays a tax on $7945, and no poll.
F. G., who pays a tax on $2794, and 1 poll.

206. *Orders.*

(*a.*) If a person should wish to have money which is due him paid to some one else, he might write an order for it, i. e., a paper requesting the one who owes it to pay it to the third person.

(*b.*) Suppose, for instance, that Edward Mellen owes Richard Jackson five hundred dollars, and that Mr. Jackson owes Stephen Baker one hundred dollars.

(*c.*) Suppose that Mr. Baker presents his bill for payment, and tha Mr. Jackson, not having the money by him, gives him the following order on Mr. Mellen:—

$100. *Providence, July 1, 1855.*

Edward Mellen, Esq.

Please pay to the order of Stephen Baker one hundred dollars, and charge the same to me.

Richard Jackson.

(*d.*) In considering this order, we may notice,—

First. The "$100" and the date at its head. See **182,** (*d.*)

Second. "Edward Mellen, Esq.," which is written to show to whom the order is addressed.

Third. The request, "Please pay to the order of Stephen Baker one hundred dollars." The explanations of **182,** (*d.*) will show the use of the various parts of this.

Fourth. "And charge the same to me," which authorizes Mr. Mellen to charge the money to Mr. Jackson, as though it had been paid directly to him. The phrase "on my account" would mean the same thing.

(*e.*) This order, when written, would be taken to Mr. Mellen by Mr Baker. If Mr. Mellen pays it, Mr. Baker indorses it, or writes the words "Received payment," with his name, upon it, and hands it to Mr. Mellen, who keeps it as evidence that he has actually paid out so much money on account of Mr. Jackson.

(*f.*) By this arrangement it will be seen that Mr. Jackson pays the one hundred dollars which he owes to Mr. Baker, and that Mr. Mellen pays one hundred dollars of his indebtedness to Mr. Jackson.

(*g.*) If Mr. Mellen should refuse to pay the above order, Mr. Baker would have no right to commence legal proceedings against *him*, but he would have the same claim against Mr. Jackson that he had before the order was given.

(*h.*) If Mr. Mellen is willing to pay it, but is unable to do so the day that it is presented, he should accept it, by writing his name, or the word "accepted" and his name, upon it. This is called an *acceptance*, and would give Mr. Baker the same legal claims against Mr. Mellen that he would have had upon a note for the same amount.

(*i.*) An order may be given to pay in a given time. as "thirty days

after date." Such an order should at once be presented for acceptance, so that the person on whom it is drawn may be held responsible for its payment.

(*j.*) An order may be payable in some given time *after sight*, i. e., in some given time after it is exhibited to and accepted by the person to whom it is directed. Usage varies with regard to grace on such orders, though it is commonly allowed.

207. *Bills of Exchange.*

(*a*) When an order is drawn on a person living in a distant place, it is called a BILL OF EXCHANGE.

(*b.*) Bills of exchange are of two kinds — foreign and inland; the former being drawn on persons living in foreign countries, and the latter on those living within our own. There are, however, no other essential differences between foreign and inland bills, than such as are connected with the different business customs existing in different countries.

(*c.*) It is customary to write two or three copies of a foreign bill of exchange, and to send them by different vessels, so as to render it reasonably certain that one of them shall reach the person to whom it is sent. These bills constitute a SET OF EXCHANGE, and are called the first, second, and third of exchange. They are so worded as to express that the payment of one renders the others void.

(*d.*) The following illustrates their form: —

Boston, June 1, 1855.

Exchange for £1000.

Twenty days after sight of this first of exchange, (second and third unpaid) pay to the order of Brown and Butler one thousand pounds, and charge the same to our account.

Potter & Hammond.

To French, Harris, & Co.

Liverpool.

(*e.*) The other bills of the set would be dated and directed like this, but the second would read, —

"Twenty days after sight of this second of exchange, (first and third unpaid,) pay," &c., as before.

A corresponding change would be made in the third.

(*f.*) To illustrate the use of such bills, let us suppose that Brown and Butler, of Boston, owe Miller and Jones, of Liverpool, one thousand pounds, and that French, Harris, & Co., of Liverpool, owe Potter and Hammond, of Boston, one thousand pounds.

(*g.*) It is obvious that if the parties should pay their debts in specie, they must incur the expense and risk of transporting two thousand pounds across the Atlantic, and that then as much money would have been brought back to each country as has been sent out from it.

(*h.*) But if Brown and Butler should buy of Potter and Hammond, a set of exchange for one thousand pounds on French, Harris, & Co., and should indorse it to Miller and Jones, and send it to them, and they should collect it, all these debts would be cancelled without any transfer of specie from one country to the other.

(*i.*) For Potter and Hammond would have received from Brown and Butler an equivalent for their claim against French, Harris, & Co., and have thus got the means of paying Miller and Jones. Miller and Jones would have received from French, Harris, & Co. an equivalent for their claim against Brown and Butler; and hence each would have paid out the value of what he owed, and have received the value of what was due to him.

(*j.*) Such transactions as the above are so manifestly to the advantage of the parties concerned in them, that a demand for bills of exchange would naturally spring up in countries having much business intercourse with each other, as between the United States and England, or the United States and France.

(*k.*) If our merchants should owe the merchants of England more than they owe us, there would be more persons here wishing to buy than sell bills on England. The demand would thus be greater than the supply, and bills would bring more than the sum for which they were drawn, or would be at a *premium.*

(*l.*) If, however, English merchants should owe us more than we owe them, more of our merchants would wish to sell than to buy, and bills on England would be at a *discount.*

(*m.*) There would be a limit to the premium on one side, and to the discount on the other; for merchants would not pay more premium for bills of exchange than it would cost them for freight and insurance to transport specie across the ocean.

(*n.*) When exchange on a foreign country is at a premium, it is said to be against us, for it not only makes our merchants pay more than the amount of their debts, but it indicates that a balance of specie is due to

that country. When it is at a discount, it is said to be in our favor, for it not only enables our merchants to pay their debts for less than their amount, but indicates that a balance of specie is due to us.

(*o.*) These variations in value are called the COURSE OF EXCHANGE.

(*p.*) It is an important thing with the merchant to watch the course of exchange between different countries, as he can often save considerable money by making his remittances through an indirect route. Thus, it may happen that bills on England command a much higher premium than bills on France; at the same time it may happen that exchange from France on England can be bought at a low rate. In such circumstances it may happen that by first buying a bill on France, and then sending it to a banking house there, with directions to buy a bill on England, a merchant may pay his debt for less than he could by direct remittance.

(*q.*) The pound sterling varies, as has been said, from $4.83 to $4.86, but in exchange its par value is assumed to be 4.44\frac{4}{9}$; so that when a pound of exchange brings its true value, it should be very nearly 9 per cent above par.

1. Paine & Colman, of Providence, bought of Robert Greene & Co. a set of exchange for £1500 on R. & S. Hubbard, of Liverpool, paying 10 per cent premium. How many dollars did it cost them?

2. What will a set of exchange on London for £2000 cost at 9$\frac{3}{4}$ per cent premium?

3. What will a set of exchange on France for 5000 francs cost at 1 per cent discount, the par value of the franc being $.186?

4. Which will be the most profitable, and how much the most, to buy a set of exchange on London for £1500 at 10$\frac{1}{2}$ per cent premium, or to buy sovereigns enough at $4.86 to pay the debt, allowing that it will cost 2$\frac{1}{4}$ per cent of their value for freight and insurance?

5. How much will it cost to remit £3000 to Liverpool through France, when exchange on France can be bought at the rate of 5.4 francs for $1, and exchange on England can be bought in France at the rate of £1 for 24 francs, allowing that a banking house in Paris will charge $\frac{1}{2}$ per cent for purchasing the exchange in that city?

202. *Profit and Loss.*

Most problems in profit and loss come under one of three classes, viz.: —

First. Those in which it is required to find for what price articles must be sold that their owner may gain or lose a certain per cent of their cost.

Second. Those in which it is required to find what per cent of the given cost will be gained or lost by selling articles at a given price.

Third. Those in which it is required to find the cost, or some per cent of the cost, of articles on which a certain per cent will be gained or lost by selling them at a given price.

They involve similar principles to those involved in other examples in percentage and fractions.

1. A speculator bought a lot of land for \$2473. For how much must he sell it to gain 25 per cent of its cost?

Suggestion. — He must sell it for the cost, plus 25 per cent of the cost; or, since 25 per cent $= \frac{1}{4}$, he must sell it for the cost, plus $\frac{1}{4}$ of the cost.*

2. Mr. Huntington bought a lot of grain at 54 cents per bushel; but it being damaged, he was obliged to sell it at a loss of $16\frac{2}{3}$ per cent. For how much per bushel did he sell it?

Suggestion. — Since $16\frac{2}{3}$ per cent $= \frac{1}{6}$, he must have sold it for the cost, minus $\frac{1}{6}$ of the cost.*

3. Mr. Shelley bought a lot of sugar at 8 cents per pound. For how much per pound must he sell it to gain $12\frac{1}{2}$ per cent?

4. For how much per yard must cloth, costing \$2.50 per yard, be sold, to gain 10 per cent on its cost?

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* When the given per cent can be reduced to a convenient vulgar fraction, as in this example, it is ordinarily best to use the vulgar fraction instead of the given per cent.

5. I bought cloth at $2 per yard, and sold it at $2.33 per yard. What per cent of its cost did I gain?

Solution. — Since I gave $2 per yard for the cloth, and received $2.33, I received $.33 per yard more than I gave; and as $.33 = $\frac{33}{200}$ of $2, my gain must be $\frac{33}{200}$ = $16\frac{1}{2}$ per cent of the cost.

6. What per cent shall I lose by selling molasses, which cost me $.30 per gallon, for $.25 per gallon?

Solution. — Since I bought the molasses for 30 cents, and sold it for 25 cents, per gallon, I lost 5 cents per gallon on it; and, as 5 cents = $\frac{5}{30}$ = $\frac{1}{6}$ of 30 cents, I lost $\frac{1}{6}$, or $16\frac{2}{3}$ per cent of the cost.

7. How much per cent shall I gain by selling flour, which cost me $6.50 per barrel, for $7.215 per barrel?

8. Mr. Winsor bought flour at $7.50, and sold it at $6.75 per barrel. How much per cent did he lose?

9. What per cent shall I gain by selling linen cloth, which cost 45 cents per yard, at 50 cents per yard?

10. What per cent shall I lose by selling linen cloth, costing 50 cents per yard, at 45 cents per yard?

11. A merchant bought some molasses at $.20 per gallon, and some oil at $1.16 per gallon. He sold the molasses at $.25 per gallon, and the oil at such rate that he gained the same per cent on it that he gained on the molasses. For how much per gallon did he sell the oil?

12. I gained $17.28 by selling a lot of sugar for 16 per cent more than it cost. How many dollars did it cost, and for how many did I sell it?

Suggestion. — Since I gained 16 per cent, or $\frac{4}{25}$ of the cost, I must have sold it for $\frac{29}{25}$ of the cost. Hence, the cost = $\frac{25}{4}$, and the price for which I sold it = $\frac{29}{4}$ of the gain.

13. Mr. Kent bought a lot of apples, and sold them for 20 per cent more than they cost, by which he gained $24.80. How much did they cost him, and for how much did he sell them?

14. Mr. Kilburn sold 43 barrels of apples for $6.45 less than they cost him, thereby losing 10 per cent of their cost.

What did they cost him, and what did he get per barrel for them?

15. Mr. Thurbur sold 146 yards of cloth for $71.54 more than it cost him, thereby gaining 14 per cent. How much did he receive per yard for it?

16. Logee & Drown sold a large lot of goods for 6700.43\frac{1}{3}$, thereby gaining 18 per cent on their cost. How much did the goods cost them?

Suggestion. — Since they gained 18 per cent of the cost, 6700.43\frac{1}{3}$ must be 118 per cent, or $\frac{59}{50}$ of the cost.

17. What is the cost of a lot of goods on which 15 per cent will be gained by selling them for $288.65?

18. What is the cost of a lot of goods on which 8 per cent will be gained by selling them for $622.215?

19. What is the cost of a lot of goods on which 7 per cent will be lost by selling them for $442.68?

Suggestion. — Since 7 per cent will be lost by the sale, $442.68 must equal 93 per cent of the cost.

20. What is the cost of a lot of goods on which 30 per cent will be lost by selling them for $874.846?

21. What is the cost of a lot of goods on which 9 per cent will be lost by selling them for $9009?

22. A man bought corn at 50 cents per bushel, for which he asked 25 per cent more than it cost him; but it falling in price, he was obliged to sell it for 25 per cent less than his asking price. Did he gain or lose, and how much per cent? How much on each bushel?

Solution. — Since his asking price was 125 per cent, or $\frac{5}{4}$ of the cost, and his selling price was 75 per cent, or $\frac{3}{4}$ of his asking price, his selling price must have been $\frac{3}{4}$ of $\frac{5}{4}$, or $\frac{15}{16}$ of the cost. Therefore he lost $\frac{1}{16}$, or 6$\frac{1}{4}$ per cent of the cost; and since each bushel cost $.50, his loss per bushel must have been $\frac{1}{16}$ of $.50, which is $.03$\frac{1}{8}$.

23. A merchant asked for a lot of goods 12$\frac{1}{2}$ per cent more than they cost him, but was obliged to deduct 12$\frac{1}{2}$ per cent from his asking price. What part of the cost did he lose?

24. I asked for a lot of cloth 10 per cent more than it cost

me, but was obliged to deduct 10 per cent from my asking price. What per cent of its cost did I lose?

25. A watch dealer bought a watch for \$75, and asked for it $33\frac{1}{3}$ per cent more than it cost. He was obliged to sell it for 10 per cent less than his asking price. What per cent did he gain on the investment? How many dollars?

26. A merchant asked for a quantity of goods 25 per cent more than they cost him, but was obliged to sell them for $12\frac{1}{2}$ per cent less than his asking price. He gained \$98.70 by the transaction. How much did the goods cost? For how much did he sell them? What was his asking price?

Suggestion.—His asking price was $\frac{5}{4}$ of the cost, and his selling price was $\frac{7}{8}$ of his asking price, or $\frac{7}{8}$ of $\frac{5}{4} = \frac{35}{32}$ of the cost. Hence, the gain, \$98.70, $= \frac{3}{32}$ of the cost.

27. A merchant asked for a lot of cloth 20 per cent more than it cost him; but being in want of money, he sold the cloth for 25 per cent less than his asking price. Allowing that he lost \$.275 per yard, for how much per yard did he sell it?

28. I sold a lot of goods for \$4268, which was $8\frac{1}{3}$ per cent less than my asking price. My asking price was 44 per cent more than the cost of the goods. How much did they cost?

29. By selling flour at \$6.65 per barrel, I shall lose 5 per cent of its cost. For how much per barrel must I sell it to gain 5 per cent?

Suggestion.—\$6.65 = 95 per cent of the cost. Hence, 105 per cent of the cost must equal $\frac{105}{95}$ of \$6.65.

30. By selling land at \$88.14 per acre I shall gain 13 per cent. For how much must I sell it to gain 20 per cent?

31. By selling a lot of goods for \$113.75 I lost 9 per cent. For how much ought I to have sold it to gain 9 per cent?

32. I bought 400 bushels of grain, and sold half of it at 75 cents per bushel, which was 20 per cent more than it cost. I sold the remainder at an advance of 25 per cent on the cost. For how much per bushel did I sell the last lot? How many dollars did I gain?

33. What must be the asking price of cloth costing $2.55 per yard, that I may fall 10 per cent on it, and still gain 14 per cent on the cost?

Suggestion. — As the selling price is to be 90 per cent, or $\frac{90}{100}$ of the asking price, and 114 per cent of the cost, the asking price must be $\frac{100}{90}$ of the selling price, and $\frac{100}{90}$ of $\frac{114}{100} = \frac{114}{90}$ of the cost.

34. What must be the asking price of cloth costing $3.29 per yard, that I may deduct $12\frac{1}{2}$ per cent from it, and still gain $12\frac{1}{2}$ per cent on the cost?

35. What must be the asking price of boots costing $2.75 per pair, that I may fall $16\frac{2}{3}$ per cent on it, and still gain 20 per cent on their cost?

36. If, by selling cloth at $2.34 per yard, I gain 4 per cent of its cost, what per cent shall I gain by selling it at $2.79 per yard?

Suggestion and Answer. — Since $2.34 = 4 per cent more than the cost, it must equal 104 per cent of the cost. Hence, $2.79 = $\frac{279}{234}$ of 104 per cent, = 124 per cent of the cost, and the gain equals 24 per cent.

37. If, by selling cloth at $4.37 per yard, I gain 15 per cent of its cost, what per cent shall I gain by selling it at $4.94 per yard?

38. If, by selling flour at $6.75 per barrel, I gain 8 per cent of its cost, what per cent shall I lose by selling it at $6 per barrel?

39. If, by selling oil at $1.254 per gallon, I lose 5 per cent of its cost, what per cent shall I gain by selling it at $1.584 per gallon?

40. I bought 96 acres of land at $84 per acre, and sold $\frac{3}{4}$ of it for the cost of the whole. What per cent did I gain on the part sold?

Suggestion. — The part sold cost $\frac{3}{4}$, and was sold for $\frac{4}{4}$ of the cost of the entire lot. Hence, it was sold for $\frac{4}{3}$ of its cost.

41. I bought 28 tons of iron at $48 per ton, and sold $\frac{6}{7}$ of it for the cost of the whole. What per cent did I gain on the part sold?

42. I bought 83 barrels of beef at $12.50 per barrel, and was obliged to sell it for what $\frac{4}{5}$ of it cost. What per cent did I lose?

43. A. & M. J. Miles bought of Cragin & Cleveland, for cash, goods to the amount of $423.75, and the same day sold them at an advance of 16 per cent, receiving in payment a note on 3 months. This note they got discounted at a bank at the rate of 6 per cent per year. How much did they gain on the goods?

44. Armington, Horswell, & Kilburn bought of Godding, Briggs, & Co. goods to the amount of $1000, payable in 6 months, without grace. One month afterwards they sold the goods for cash, at an advance of 10 per cent, and immediately put the money at interest at 6 per cent. When the 6 months had expired, they collected the amount of the money they had lent, and paid the bill due Godding, Briggs, & Co. Did they gain or lose, and how many dollars?

45. I bought a lot of coffee at 12 cents per pound. Allowing that 5 per cent of the coffee will waste in weighing it out, and that 10 per cent of the sales will be bad debts, for how much per pound must I sell it to make a clear gain of 14 per cent on the cost?

Answer. 16 cents per pound.

46. What must be the asking price of raisins costing $7.29 per cask, that I may fall 10 per cent from it and still gain 10 per cent on the cost, allowing that 10 per cent of the sales will be bad debts?

47. I bought 8 casks of oil, each containing 133 gallons, at $1.20 per gallon, and paid $5.32 for having it brought to my store. Allowing that there will be a waste of 5 per cent in measuring, that 3 per cent of my sales will be bad debts, and that it will cost 1 per cent of the remainder to collect it, for how much per gallon must I sell it to make a net gain of 33 per cent on its cost at my store, nothing being allowed for interest?

209. *Partnership.*

Two or more persons, uniting for the purpose of carrying on business together, form what is called a PARTNERSHIP, FIRM, or COMPANY. The capital invested by them is called their STOCK IN TRADE.

It is evident that the profit or loss made by the company should be shared among its members in proportion to what the use, or interest, of each man's stock for the time it was invested is worth.

When the stocks of the several partners are invested for the same length of time, their use, or interest, will be proportioned to the stocks themselves, and hence each partner's gain or loss will be the same part of his stock that the entire gain or loss is of the entire stock; or it will be the same part of the entire gain or loss that his stock is of the entire stock.

1. A, B, and C trade in company. A puts in $250, B puts in $750, and C puts in $500. At the end of 6 months they find that they have gained $472.50. What is each man's share of the gain?

First Solution. — Since A's stock = $250, B's = $750, and C's = $500, the entire stock = $250 + $750 + $500 = $1500; and as the gain = $472.50, it must equal $\frac{47250}{150000}$, or $\frac{63}{200}$ of the stock. Therefore, each man's gain will be $\frac{63}{200}$ of his stock, which gives for A's gain $78.75, for B's gain $236.25, and for C's gain $157.50.

Second Solution. — Since A's stock = $250, B's = $750, and C's = $500, the entire stock must equal $1500, of which A's stock = $\frac{250}{1500} = \frac{1}{6}$, B's = $\frac{750}{1500} = \frac{1}{2}$, C's = $\frac{500}{1500} = \frac{1}{3}$. Therefore, A should have $\frac{1}{6}$, B should have $\frac{1}{2}$, and C should have $\frac{1}{3}$ of the gain. $\frac{1}{6}$ of $472.50 = $78.75 = A's share; $\frac{1}{2}$ of $472.50 = $236.25 = B's share; $\frac{1}{3}$ of $472.50 = $157.50 = C's share.

2. X, Y, and Z traded in company for 1 year. X put in $1000, Y put in $1500, and Z put in $2000. At the end of the year they found that they had gained $1800. What was each man's share of the gain?

3. A man, failing in business, finds that he owes A $424, B $638, C $197, D $338, and E $574, and that his whole available property amounts only to $1173. How much ought he to pay to each creditor?

Suggestion. — Since he owes $2171, and has but $1173, he can pay but $\frac{1173}{2171}$ of his debts. Therefore, he ought to pay A $\frac{1173}{2171}$ of $424, B $\frac{1173}{2171}$ of $638, &c.

4. The stock of a bankrupt is valued at $1200, and he owes $4200. How many dollars ought he to pay the person to whom he owes $546? to whom he owes $338.73?

5. A, B, C, and D agree to cut 500 cords of wood for $300. When the job is finished, they find that A has cut 125 cords, B 100 cords, C 150 cords, and D the rest. How many dollars ought each to receive?

6. A and B traded in company. A put in $200, and B put in $300. A's share of the gain was $84.56. What was B's share?

7. A and B traded in company, and gained $348, of which B's share was $261. If A's stock was $175, what was B's stock, and A's share of the gain?

8. Samuel Greene and Joseph Irons traded in company. Greene paid in 3 times as much of the stock as Irons, and they gained $1176. What was each one's share of the gain?

Suggestion. — Since Greene paid in 3 times as much as Irons, both together must have paid in 4 times as much as Irons. Therefore, Irons paid in $\frac{1}{4}$, and Greene $\frac{3}{4}$ of the stock.

9. William Balch and Joseph Adams bought a ship together, Balch paying in twice as much money as Adams. At the end of one year they sold her, and found that they had realized a profit of $15,000 from her. What was each partner's share?

10. Anderson and Parker, after trading in company for 2 years, found that their profits had been $2400. Allowing that Anderson's stock was $\frac{2}{3}$ of Parker's, how many dollars of the profit ought each to have?

11. A, B, and C traded in company. A put in $\frac{1}{3}$ of the stock, B put in $\frac{1}{2}$ of it, and C put in the rest. On dividing the gain, they found that C's share of it was $321. What was the gain of each of the other partners?

12. William Hall, Edward Johnson, and Henry Whiting

traded in company, and gained $6534, of which Johnson's share was $1089. If Johnson's and Whiting's stock was together equal to twice Hall's, what was Hall's share of the gain? What was Johnson's share?

13. A small estate belonged to a large number of heirs: 2 members of the family of A each owned $\frac{1}{150}$ of the estate; 4 of the family of B each owned $\frac{1}{300}$ of it; 4 of the family of C each owned $\frac{1}{15}$ of it; 2 of the family of D each owned $\frac{1}{30}$ of it; 4 of the family of E each owned $\frac{2}{75}$ of it; 3 of the family of F each owned $\frac{2}{525}$ of it; 4 of the family of G each owned $\frac{2}{375}$ of it; 6 of the family of H each owned $\frac{2}{105}$ of it; 3 of the family of I each owned $\frac{2}{45}$ of it. Mr. Byram, as agent for the above-named individuals, sold *their interest* in the estate for $350. How many dollars ought he to give to each?

Answer.

$ 3.123 to each member of A's family.
$ 1.562 to each member of B's family.
$31.234 to each member of C's family.
$15.617 to each member of D's family.
$12.493 to each member of E's family.
$ 1.785 to each member of F's family.
$ 2.499 to each member of G's family.
$ 8.924 to each member of H's family.
$20.823 to each member of I's family.

NOTE. — The above example is a statement of transactions which actually occurred. It was brought to the author for solution, by the agent of the parties.

210. *Partnership on Time.*

In dividing the gain or loss among the partners, when their shares of the stock are invested for unequal times, it becomes necessary to consider both the stock and the time, or to consider the interest of each man's stock for the time it was in trade. The following examples and solutions will illustrate this: —

1 A, B, and C traded in company. A put in $750 for 10

months, B put in \$375 for 12 months, and C put in \$1125 for 16 months. They gained \$860. What was each man's share of the gain?

First Solution.

$$
\begin{aligned}
\$37.50 &= \text{interest of A's stock for 10 mo.}\\
22.50 &= \text{interest of B's stock for 12 mo.}\\
90.00 &= \text{interest of C's stock for 16 mo.}\\
\hline
\$150.00 &= \text{interest of whole.}
\end{aligned}
$$

Therefore, A should have $\frac{3750}{15000}$, or $\frac{1}{4}$, of the gain, = \$215.
B should have $\frac{2250}{15000}$, or $\frac{3}{20}$, of the gain, = \$129.
C should have $\frac{9000}{15000}$, or $\frac{3}{5}$, of the gain, = \$516.

Second Solution.

The use of \$ 750 for 10 mo. is worth the use of \$ 7500 for 1 mo.
The use of \$ 375 for 12 mo. is worth the use of \$ 4500 for 1 mo.
The use of \$1125 for 16 mo. is worth the use of \$18000 for 1 mo.

Use of whole stock is worth the use of \$30000 for 1 mo.

Therefore, A should have $\frac{7500}{30000}$, or $\frac{1}{4}$, of the gain, = \$215.
B should have $\frac{4500}{30000}$, or $\frac{3}{20}$, of the gain, = \$129.
C should have $\frac{18000}{30000}$, or $\frac{3}{5}$, of the gain, = \$516.

When the stocks of the several partners are convenient multiples or fractional parts of each other, a very neat solution can be given. Thus, in the above example, by noticing that B's stock equals $\frac{1}{2}$ of A's, and that C's stock equals $\frac{3}{2}$ of A's, we may have the following: —

Third Solution. — The use of A's stock 10 mo. = use of 10 times A's stock for 1 mo.

The use of B's, or $\frac{1}{2}$ of A's stock, 12 mo. = use of $\frac{12}{2}$, or 6 times A's stock for 1 mo.

The use of C's, or $\frac{3}{2}$ of A's stock, 16 mo. = use of $\frac{48}{2}$, or 24 times A's stock for 1 mo.

Use of whole = use of 10 + 6 + 24, or 40 times A's stock for 1 mo.

Therefore A should have $\frac{10}{40}$, or $\frac{1}{4}$; B $\frac{6}{40}$, or $\frac{3}{20}$; and C $\frac{24}{40}$, or $\frac{3}{5}$, of the gain, which will give the same answer as before.

2. Charles French, Francis Baker, and Otis Atherton traded in company, under the name of Charles French & Co. French put in \$1000 for 20 mo., Baker put in \$800 for 16 mo., and Atherton put in \$500 for 20 mo. They gained \$1500. How many dollars of the gain ought each to receive?

3. George Jackson, William Leach, and Albert Buffington traded in company. Jackson put in $144 for 6 mo., Leach put in $72 for 7 mo., and Buffington put in $216 for 6 mo. 20 da. They gained $114. What was each man's share of the gain?

4. A, B, C, and D hired a pasture together, in which A pastured 4 cows 13 weeks, B pastured 5 cows 16 weeks, C pastured 8 cows $10\frac{1}{2}$ weeks, and D pastured 4 cows 16 weeks. The rent of the pasture was $102. How many dollars ought each man to pay?

5. Samuel Austin, Jacob Brown, and Moses Sumner formed a partnership for 2 years, under the name of Samuel Austin & Co. Austin at first paid in to the stock $1000, but after 8 mo. had elapsed he paid in $500 more. Brown at first paid in $1250, and 16 mo. afterwards he paid in $250 more. Sumner at first paid in $1500, but at the end of 16 mo. he took out $500. They gained $3600. What was each man's share of the gain?

Solution.

$$\left.\begin{array}{l}\text{Interest of \$1000 for 8 mo.} = \$\ 40\\ \text{Interest of \$1500 for 16 mo.} = \$120\end{array}\right\} = \$160 = \text{int. Austin's stock.}$$

$$\left.\begin{array}{l}\text{Interest of \$1250 for 16 mo.} = \$100\\ \text{Interest of \$1500 for 8 mo.} = \$\ 60\end{array}\right\} = \$160 = \text{int. Brown's stock.}$$

$$\left.\begin{array}{l}\text{Interest of \$1500 for 16 mo.} = \$120\\ \text{Interest of \$1000 for 8 mo.} = \$\ 40\end{array}\right\} = \$160 = \text{int. Sumner's stock.}$$

By this, it appears that the interests of their respective stocks, for the time they were in trade, were alike. Hence, the gain should be divided equally, and each partner should have $\frac{1}{3}$ of $3600, which is $1200.

NOTE. — Other solutions similar in character to those given to the first example might have been added, but as the pupil can readily discover them, they have been omitted.

6. Joseph Southwick, Francis Lowe, and Henry Taft formed a partnership for 3 years, under the name of Southwick, Lowe, & Taft. When they commenced business, each partner put in $3000; but at the end of the first year Southwick put in $3000 more, and Lowe withdrew $1500. At the end of the second year, Southwick withdrew $2000, and

Lowe put in \$4000, and Taft put in \$2000. When the partnership expired, they found that they had gained \$9000. What was each partner's share of the gain?

7. S. Gamwell, C. Grover, R. Wheelock, and W. Godding formed a partnership, under the title of Gamwell, Grover, & Co. Gamwell at first put in \$8000, but at the end of 6 mo. he withdrew \$2000, and at the end of 12 mo. he withdrew \$1000 more. Grover at first put in \$6000, but at the end of 10 mo. he put in \$3000 more. Wheelock put in \$7000. Godding at first put in \$10,000; at the end of 6 mo. he withdrew \$2000, and at the end of 14 mo. he put in \$4000. At the end of 2 years they found that they had gained \$12,000. What was each man's share of the gain?

SECTION XVI.

POWERS AND ROOTS

211. *Definitions.*

(*a.*) The product of a number taken any number of times as a factor is called a POWER of the number. — See **105**, (*d.*) (*e.*) (*f.*) and Note.

(*b.*) A ROOT of any number is such a number as, taken some number of times as a factor, will produce the given number.

(*c.*) If the root must be taken twice as a factor to produce the number, it is the SQUARE ROOT, or the SECOND ROOT; if three times, it is the CUBE ROOT, or the THIRD ROOT; if four times, it is the FOURTH ROOT; &c.

Thus, 2 is the square root of 4, the third root of 8, the fourth root of 16, &c., because $2^2 = 4$, $2^3 = 8$, $2^4 = 16$; &c.

(*d.*) The character $\surd$, called the RADICAL SIGN, is used

to indicate that the root of the number over which it is placed is to be extracted.

(*e.*) The DEGREE of the root is indicated by a small figure, called an INDEX, which is placed a little above and at the left of the sign. When no index is written, the square root is required.

Thus, $\sqrt{4}$, or $\sqrt[2]{4}$, means the square root of 4.

$\sqrt[5]{243}$ means the fifth root of 243.

$\sqrt[4]{7^3}$ means the fourth root of the 3d power of 7.

(*f.*) We may also indicate that a root is to be extracted, by using a fractional exponent.

Thus, $9^{\frac{1}{2}} = \sqrt{9}$; $(125)^{\frac{1}{3}} = \sqrt[3]{125}$; $27^{\frac{2}{3}} = \sqrt[3]{27^2}$; &c.

(*g.*) The process of finding the powers of numbers is called INVOLUTION, and the process of finding their roots is called EVOLUTION, or the EXTRACTING OF ROOTS.

212. *Relation which the Denominations of a Square bear to those of its Root.*

(*a.*) TABLE OF SQUARES.

$1^2 = 1$	$4^2 = 16$	$7^2 = 49$
$2^2 = 4$	$5^2 = 25$	$8^2 = 64$
$3^2 = 9$	$6^2 = 36$	$9^2 = 81$

$10^2 = 100$	$10{,}000^2 = 1{,}000{,}000{,}000$
$100^2 = 10{,}000$	$100{,}000^2 = 10{,}000{,}000{,}000$
$1{,}000^2 = 1{,}000{,}000$	$1{,}000{,}000^2 = 1{,}000{,}000{,}000{,}000$

(*b.*) The above table shows that, — First. There are below 100 but 9 entire numbers which are perfect squares.

Second. The entire part of the square root of any number below 100 will be less than 10, and therefore contain but 1 figure; of any number between 100 and 10,000 will lie between 10 and 100, and therefore contain 2 figures; between 10,000 and 1,000,000 will lie between 100 and 1000, and therefore will contain 3 figures; &c.

1. How many figures are there in the entire part of the square root of 865698?

Answer. — Since 865698 lies between 10,000 and 1,000,000, its root

must lie between the roots of those numbers, i. e., between 100 and 1000, and must therefore contain 3 figures in its entire part.

How many figures are there in the entire part of the root of —

2. 69748769?	4. 12496743297?
3. 486497950068?	5. 5847695329?

213. *Division into Periods.*

(*a.*) As the square of 10 is 100, of 100 is 10,000, &c., it follows that the square of any number of tens will be some number of hundreds; of any number of hundreds will be some number of ten thousands, &c.; or, in other words, that the square of tens will give units of no denomination below hundreds; the square of hundreds will give units of no denomination below ten-thousands; &c.

(*b.*) Hence, the two right hand figures of any number will contain no part of the square of the denominations of the root above units; the four right hand figures will contain no part of the square of those above tens, &c.

(*c.*) Therefore, if we should begin at the right of any number, and separate it into periods of two figures each, the number of periods would be the same as the number of figures in its square root. The square of the highest denomination of the root would be found in the left hand period; the square of the two highest denominations would be found in the two left hand periods; &c.

1. Separate 8478695 into periods, and explain their uses.

Answer. $\dot{8}4\dot{7}8\dot{6}9\dot{5}$. The left hand period, 8, contains all of the square of the thousands of the root; the two left hand periods, 847, the square of the thousands and hundreds; &c.

Separate each of the following numbers into periods, and explain their uses: —

2. 5794865.	4. 375486792.
3. 89475948.	5. 32500675.

214. *Method of forming a Square.*

(*a.*) To find a law of universal application in squaring or extracting the square roots of numbers, we will use the letter a to represent any number whatever and b to represent any other number.

(*b.*) Then will $a + b$ represent the sum, and $(a + b)^2$, or $(a + b) \times (a + b)$ the square of the sum of any two numbers whatever.

(*c.*) Performing the multiplication, we have a times $a = a^2$; a times $b = a \times b$, or, as it may be written, $a\,b$; b times $a = a$ times $b = a \times b$, or $a\,b$; b times $b = b^2$.

(*d.*) Writing the work, as below, and adding the partial products, we have, —

$$\begin{array}{rl} & a + b \\ & a + b \\ \hline a \times (a + b) = & a^2 + a \text{ times } b \qquad = a^2 + a\,b \\ b \times (a + b) = & \qquad + a \text{ times } b + b^2 = \qquad a\,b + b^2 \\ \hline (a + b) \times (a + b) = & a^2 + 2 \text{ times } a\,b + b^2 = a^2 + 2\,a\,b + b^2 \end{array}$$

(*e.*) Hence, $(a + b)^2 = a^2 + 2\,a\,b + b^2$, or, since a^2 equals the square of the first number, and $2\,a\,b$ equals twice the product of the first number by the second, and b^2 equals the square of the second:

The square of the sum of any two numbers equals the square of the first, plus twice the product of the first by the second, plus the square of the second.

Illustrations.

$(7 + 5)^2 = 7^2 + 2 \times 7 \times 5 + 5^2 = 49 + 70 + 25 = 144 = 12^2$
$(8 + 4)^2 = 8^2 + 2 \times 8 \times 4 + 4^2 = 64 + 64 + 16 = 144 = 12^2$
$(20 + 3)^2 = 20^2 + 2 \times 3 \times 20 + 3^2 = 400 + 120 + 9 = 529 = 23^2$

(*f.*) But $a^2 + 2\,a\,b + b^2$ can be put into another form; for $2\,a\,b + b^2 = 2\,a$ times $b + b$ times b, $= (2\,a + b)$ times b, or $(2\,a + b) \times b$, or, by omitting the sign $\times$, as may be done without ambiguity, $(2\,a + b)\,b$.

Hence, $(a + b)^2 = a^2 + 2\,a\,b + b^2 = a^2 + (2\,a + b)\,b$.

(*g.*) But a^2 means the square of the first number; $2\,a + b$ means the sum of twice the first number, plus the second; and $(2\,a + b)\,b$ the sum of twice the first, plus the second, multiplied by the second.

(*h.*) *Hence, the square of the sum of two numbers is also equal to the square of the first number, plus the product obtained by multiplying the sum of twice the first number plus the second, by the second.*

Illustrations.

$(7 + 5)^2 = 7^2 + (14 + 5)\,5 = 49 + 95 = 144 = 12^2.$
$(8 + 4)^2 = 8^2 + (16 + 4)\,4 = 64 + 80 = 144 = 12^2.$
$(40 + 8)^2 = 40^2 + (80 + 8)\,8 = 1600 + 704 = 2304 = 48^2.$

(*i.*) Now, as any number above ten is composed of tens and units, its square will be composed of the square of the tens, plus the product of twice the tens plus the units multiplied by the units.

(*j.*) If there are more than ten tens in the number, the part which is

composed of tens may be considered as made up of hundreds and tens, and its square will equal the square of the hundreds, plus the product of twice the hundreds, plus the tens, multiplied by the tens.

(*k.*) Proceeding in this way, we shall at last reach the part which is expressed by one or two figures, and composed of only the two highest denominations of the given number. The square of this part will be the square of the highest denomination, plus the product of twice the highest denomination, plus the next lower, multiplied by the next lower Thus, —

$$(4837)^2 = (4830 + 7)^2 = 4830^2 + (2 \times 4830 + 7\ ) \times 7$$
$$(4830)^2 = (4800 + 30)^2 = 4800^2 + (2 \times 4800 + 30) \times 30$$
$$(4800)^2 = (4000 + 800)^2 = 4000^2 + (2 \times 4000 + 800) \times 800$$

215. *Method of extracting the Square Root.*

What is the value of $\sqrt{925444}$?

Solution. — (*a.*) Since this number lies between 10,000 and 1,000,000, its root must lie between 100 and 1000, and must therefore be composed of hundreds, tens, and units. Dividing it into periods of two figures each, it will take the form $9\dot{2}54\dot{4}4$.

(*b.*) If, now, we let **a** represent the hundreds of the root, and **b** the tens, the whole of $\mathbf{a}^2$ will be found in the left hand period, i. e., in the ten-thousands, and the whole of $(\mathbf{a} + \mathbf{b})^2$ in the two left hand periods, i. e., in 9254 hundreds.

(*c.*) The greatest square in 92 is 81, the root of which is 9. Therefore, $9 = \mathbf{a} =$ the hundreds figure of the root. Subtracting $\mathbf{a}^2, = 81$ ten-thousands, from 92 ten-thousands, leaves 11 ten-thousands, to which adding the 54 hundreds gives 1154, which must contain $(2\ \mathbf{a} + \mathbf{b})\ \mathbf{b}$.

(*d.*) Now, as we know **a**, we can find 2 **a**, and make use of it as a *trial divisor* to find **b**. But **a** being hundreds and **b** tens, 2 **a b** must be thousands, and no part of it will be found to the right of the thousands.

(*e*) Hence, in dividing, we may disregard the right hand figure of 1154, and see how many times the trial divisor, 18, is contained in 115. The quotient is 6, which is probably **b**, the tens figure of the root. If this is correct, $(2\ \mathbf{a} + \mathbf{b})$, or the *true divisor*, must be equal to 186, and $(2\ \mathbf{a} + \mathbf{b})\ \mathbf{b}$ must be equal to 6 times 186, or 1116. This last product, being less than 1154, shows that the work is correct. We subtract, and to the remainder, 38 hundreds, add the right hand period, 44 units, which gives 3844 for a new dividend.

(*f.*) Now, if we let $\mathbf{a}'$ represent the part of the root already found, i. e., the 96 tens, and $\mathbf{b}'$ the units, $\mathbf{a}' + \mathbf{b}'$ will represent the required root, and $(\mathbf{a}' + \mathbf{b}')^2 = \mathbf{a}'^2 + (2\ \mathbf{a}' + \mathbf{b}')\ \mathbf{b}'$ the given number.

(*g.*) But we have already subtracted $\mathbf{a}'^2$; the remainder, 3844, must contain the $(2\ \mathbf{a} + \mathbf{b}')\ \mathbf{b}'$.

(*h.*) We now make 2 a′, or 192, a trial divisor; and since 2 a′ b′ must be tens, we omit the right hand figure of 3844, and see how many times 2 a′, or 192, is contained in 384.

(*i.*) This gives $b' = 2$, = the probable units figure. If it be correct, the true divisor, $2\,a' + b'$, must equal 1922, and $(2\,a' + b')\,b'$ must be 2×1922. This, being equal to 3844, shows that the given number is a perfect square, and 962 is its root.

(*j.*) If there had been another figure in the root, we might have represented the part of the root already found by a″, or by some other letter, and the next figure by b″, or by some other letter, and have proceeded as before.

(*k.*) The numerical work would be written thus: —

$$
\begin{array}{rl}
 & \dot{9}2\dot{5}\dot{4}\dot{4}\dot{4}\ (\ 962 = \text{Root.} \\
 & 81 = a^2 \\
\hline
2\,a + b = 186\ ) & 1154 \\
 & 1116 = (\,2\,a + b)\,b \\
\hline
2\,a' + b' = 1922\ ) & 3844 \\
 & 3844 = (\,2\,a' + b'\,)\,b' \\
\hline
\end{array}
$$

216. *Rule for Square Root, with Problems.*

As a similar process can always be followed, we may describe the method of extracting the square root of a number thus: —

First. *Divide the given number into periods of two figures, beginning with the units.*

Second. *Find the greatest square in the left hand period, and place its root as the highest denomination of the required root.*

Third. *Subtract the square thus found from the left hand period, and to the remainder bring down the next period, calling the result a dividend.*

Fourth. *Double the part of the root already found for a trial divisor.*

Fifth. *See how many times this trial divisor is contained in all of the dividend, excepting the right hand figure, and write the quotient as the next figure of the root, and also place it at the right of the trial divisor, to form a true divisor.*

Sixth. *Multiply this true divisor by the root figure last found, and subtract the product from the dividend.*

Seventh. *Bring down the next period to the right of the remainder, to form the next dividend.*

Eighth. *Double the part of the root already found for a trial divisor, and proceed as indicated in the 5th, 6th, 7th, and 8th of these paragraphs.*

What is the square root of each of the following numbers: —

1. 7056.	5. 4137156.
2. 9025.	6. 22610025.
3. 3104644.	7. 4260096.
4. 349281.	8. 38580769440964.

217. *Square Root of Fractions.*

(*a.*) Since the square of a fraction equals the square of its numerator divided by the square of its denominator, the square root of a fraction must equal the square root of its numerator divided by the square root of its denominator.

Thus, $\sqrt{\frac{4}{9}} = \frac{2}{3}$, for $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$. $\sqrt{.25} = .5$, for $(.5)^2 = .25$.

(*b.*) If the numerator and denominator of a fraction are not perfect squares, we can only get the approximate value of its square root.

(*c.*) In such cases, if the *denominator* is not a perfect square, it will be well to multiply both terms by such number as will make it so. This number may be either the denominator, or the product of the prime numbers which are found as factors in the denominator, 1, 3, 5, or any odd number of times.

Thus, $\frac{7}{11} = \frac{7 \times 11}{11 \times 11} = \frac{77}{121}$. $\frac{5}{12} = \frac{5}{2^2 \times 3} = \frac{5 \times 3}{2^2 \times 3^2} = \frac{15}{36}$

$\frac{19}{24} = \frac{19}{2^3 \times 3} = \frac{19 \times 2 \times 3}{2^4 \times 3^2} = \frac{114}{144}$.

$.049 = .0490$, or $\frac{49}{1000} = \frac{49 \times 10}{1000 \times 10} = \frac{490}{10000}$.

(*d.*) What is the square root of $\frac{79}{84}$?

Solution. $\frac{79}{84} = \frac{79}{2^2 \times 3 \times 7} = \frac{79 \times 3 \times 7}{2^2 \times 3^2 \times 7^2} = \frac{1659}{2^2 \times 3^2 \times 7^2}$

Hence, $\sqrt{\frac{79}{84}} = \sqrt{\frac{1659}{2^2 \times 3^2 \times 7^2}} = \frac{\sqrt{1659}}{2 \times 3 \times 7} = \frac{40}{42}$, approximately.

This differs from the true root by less than $\frac{1}{42}$.

(*e.*) Should a greater degree of accuracy be required, both terms may be multiplied by such a square number as will make the denominator sufficiently large to secure the requisite degree of accuracy.

Thus, multiplying the numerator and denominator of $\frac{1659}{2^2 \times 3^2 \times 7^2}$

by 5^2, gives $\frac{41475}{2^2 \times 3^2 \times 7^2 \times 5^2}$, the approximate root of which is $\frac{203}{2 \times 3 \times 7 \times 5} = \frac{203}{210}$, which differs from the true root by less than $\frac{1}{210}$.

What is the square root of each of the following fractions, —

1. $\frac{441}{3724}$?	3. $\frac{6}{7}$?	5. $\frac{7}{8}$?
2. $\frac{1089}{2025}$?	4. $\frac{5}{6}$?	6. $\frac{3}{11}$?

218. *Square Root of Decimal Fractions.*

(*a.*) In order that a decimal fraction may have a perfect square for its denominator, it must have an even number of places in its numerator.

Thus, the denominators of .04, .25, .17, .6561, and .384736 are perfect squares, but the denominators of .4, 2.5, .017, .06561, .0384736, &c., are not.

(*b.*) If a decimal fraction, the root of which is required, does not have an even number of decimal places in its numerator, a zero must be annexed, to make the number even, so that the denominator in all cases may be a perfect square.

(*c.*) Since $\sqrt{.01} = \sqrt{\frac{1}{100}} = \frac{1}{10} = .1$, $\sqrt{.0001} = \sqrt{\frac{1}{10000}} = \frac{1}{100} = .01$, &c., it follows that there will be as many decimal places in the fractional part of the root as there are times two decimal places in the fractional part of the power. Hence, we can carry out the root to as many places as we choose, by annexing two zeros to the power for every additional figure which we wish to obtain in the root.

Required $\sqrt{.4}$

Solution. — Since $\sqrt{.4} = \sqrt{.40} = \sqrt{.4000} = \sqrt{.400000}$, we may have the following written work: —

```
           .40 ( .6324
           .36
          -----
  1.23 )   400
           369
          -----
 1.262 )   3100
           2524
          ------
1.2644 )   57600
           50576
          ------
            7024
```

The exact root of such a number as the above cannot be found, for a number which does not end with a zero cannot have zero for the right hand figure of its square.

What is the square root of —

1. .0625 ?	3. .8281 ?	5. 56.25 ?
2. 5.625 ?	4. 1.6 ?	6. .16 ?

219. *Cube Root. — Relation of Cube to Root.*

(*a.*) To find a method of extracting the cube root of a number, we must make some preliminary investigations similar to those of **213** and **214**.

TABLE OF ROOTS AND CUBES.

$1^3 = 1$	$4^3 = 64$	$7^3 = 343$
$2^3 = 8$	$5^3 = 125$	$8^3 = 512$
$3^3 = 27$	$6^3 = 216$	$9^3 = 729$
$10^3 = 1{,}000$	$1{,}000^3 = 1{,}000{,}000{,}000$	
$100^3 = 1{,}000{,}000$	$10{,}000^3 = 1{,}000{,}000{,}000{,}000$	

(*b.*) The above table shows that, —

First. There are below 1000 but 9 entire numbers which are perfect cubes.

Second. The entire part of the cube root of any number below 1000 will be less than 10, and therefore will contain but one figure; of any number between 1000 and 1,000,000 will lie between 10 and 100, and therefore will contain but two figures; &c.

How many figures are there in the entire part of the root of 47986754?

Answer. — Since 47986754 lies between 1,000,000 and 1,000,000,000, its root must lie between the roots of those numbers, i. e., between 100 and 1000, and hence must contain 3 figures in its entire part.

220. *Division into Periods.*

(*a.*) As the cube of 10 is 1000, of 100 is 1,000,000, &c., it follows that the cube of any number of tens will be some number of thousands, of any number of hundreds will be some number of millions, &c.; or, in other words, that the cube of tens will give units of no denomination below thousands, the cube of hundreds will give units of no denomination below millions, &c.

(*b.*) Hence, the three right hand figures of any number will contain no part of the cube of the denominations above units, the six right hand figures will contain no part of the cube of those above tens, &c.

(*c.*) Therefore, if we should begin at the right of any number, and separate it into periods of three figures each, the number of periods would be the same as the number of figures in its cube root. The cube of the highest denomination would be found in the left hand period the cube of the two highest denominations would be found in the two left hand periods, &c.

1. Separate 9876585925 into periods, and explain their uses.

Answer. $\dot{9}87\dot{6}58\dot{5}92\dot{5}$. The left hand period, 9, contains all of the cube of the thousands of the root; the two left hand periods, $\dot{9}876$, the cube of the thousand and hundreds; &c.

Separate each of the following numbers into periods, and explain their uses: —

2. 42783794.	4. 5847643759427.
3. 584376423.	5. 6972842903612.

221. *Method of forming a Cube.*

(*a.*) To find a law of universal application, we will use the letter a to represent any number whatever, and b to represent any other number

(*b.*) Then will $a + b$ represent the sum, and $(a + b)^3$, or $(a + b) \times (a + b) \times (a + b)$, the cube of any numbers whatever.

(*c.*) But $(a + b)^3 = (a + b)^2 \times (a + b) = (a^2 + 2\,a\,b + b^2) \times (a + b)$. Now, multiplying $a^2 + 2\,a\,b + b^2$ by a gives $a^3 + 2\,a^2\,b + a\,b^2$, and multiplying it by b gives $a^2\,b + 2\,a\,b^2 + b^3$. Adding these results together gives $a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3$.

(*d.*) This work may be written out thus: —

$$
\begin{array}{ll}
a^2 + 2\,a\,b + b^2 = (a + b)^2 & \\
\qquad a + b & \\
\hline
a^3 + 2\,a^2\,b + a\,b^2 & = a \times (a^2 + 2\,a\,b + b^2) \\
\qquad a^2\,b + 2\,a\,b^2 + b^3 & = b \times (a^2 + 2\,a\,b + b^2) \\
\hline
a^3 + 3a^2\,b + 3\,a\,b^2 + b^3 & = (a + b) \times (a^2 + 2\,a\,b + b^2) = (a + b)^3
\end{array}
$$

(*e.*) But $3\,a^2\,b + 3\,a\,b^2 + b^3 = 3\,a^2$ times $b + 3\,a\,b$ times $b + b^2$ times $b = (3\,a^2 + 3\,a\,b + b^2)$ times $b, = (3\,a^2 + 3\,a\,b + b^2)\,b$.

Again: $3\,a\,b + b^2 = 3\,a$ times $b + b$ times $b = (3\,a + b)$ times $b = (3\,a + b)\,b$.

Hence, $a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3 = a^3 + [3\,a^2 + (3\,a + b)\,b] \times b$;

i. e., *the cube of every number composed of two parts is equal to the cube of the first part; plus the product obtained by multiplying the second part by the sum of three times the square of the first part, plus the sum of three times the first part plus the second multiplied by the second.*

222. *To extract the Cube Root.*

What is the cube root of 259694072?

Solution. — (*a.*) Since the number lies between 1,000,000 and 1,000,000,000, its root must lie between 100 and 1000, and hence must contain three figures. Separating it into periods of three figures each, it will take the form 259694072.

(*b.*) If we now let **a** represent the hundreds of the root and **b** the tens, it is evident that the whole of a^3 will be found in the left hand period, and of $(a + b)^3$ in the two left hand periods.

(*c.*) The greatest cube in 259 is 216, the root of which is 6. Therefore, $6 = a, =$ the hundreds figure of the root. Subtracting $a^3, = 216$ millions, from 259 millions leaves 43 millions, to which adding the next period gives 43694 thousands. We may regard this as a dividend, and it must contain $[3\,a^2 + (3\,a + b) \times b]\,b$, i. e., the remaining part of $(a + b)^3$.

(*d.*) Now, as we know **a**, we can find $3\,a^2$, and make use of it as a trial divisor to find **b**. But **a** being 6 hundreds, $3\,a^2$ must equal 108 ten-thousands, and as **b** is tens, $3\,a\,b$ must be hundred-thousands. Hence, we may disregard the two right hand figures of the dividend, and see how many times the trial divisor, 108, is contained in 436.

(*e.*) The quotient being 4, we write it as the probable tens figure of the root, and have next to complete the true divisor, $3\,a^2 + (3a + b)\,b$. But $(3\,a + b) = 18$ hundreds $+ 4$ tens $= 184$ tens, and $(3\,a + b)\,b$ $= 184$ tens multiplied by 4 tens $= 736$ hundreds. Hence, the true divisor $3\,a^2 + (3\,a + b)\,b = 108$ ten-thousands $+ 736$ hundreds $=$ 11536 hundreds.

(*f.*) Multiplying this by 4 gives $[3\,a^2 + (3\,a + b)\,b]\,b = 46144$ thousands, which, being greater than 43694 thousands, shows that there are not as many as 4 tens in the root, and that **b** is less than 4.

(*g.*) Assuming $b = 3$, and proceeding as before, we find that the true divisor $3\,a^2 + (3\,a + b)\,b = 108$ ten thousands $+ 549$ hundreds $= 11349$ hundreds; and $[3\,a^2 + (3\,a + b)\,b]\,b = 34047$ thousands, which, being less than 43694 thousands, shows that 3 is the true tens figure of the root. Subtracting 34047 thousands leaves 9647 thousands, to which adding the next period, 072 units, gives 9647072 for a new dividend, which must contain the remaining part of the power.

(*h.*) If we now let a' represent the part of the root already found,

i. e., the 63 tens, and b the units, we shall have $259694072 = a'^3 + [3a'^2 + (3a' + b')b']b'$, and, as we have already subtracted a'^3, 9647072 will contain $[3a'^2 + (3a' + b')b']b'$.

(*i.*) But $3a'^2 = 3$ times the square of 63 tens $= 11907$ hundreds, which may, as before, be made use of as a trial divisor to find b'.

As $3a'^2$ is hundreds and b' is units, $3a'^2b'$ must be hundreds; hence, no part of $3a'^2b'$ can be found to the right of hundreds, and we may disregard the two right hand figures of the dividend, and see how many times the trial divisor, 11907, is contained in 96470.

(*j.*) The quotient being 8, we write 8 as the probable units figure of the root, and complete the true divisor, $3a^2 + (3a + b)b$. $3a + b = 189$ tens $+ 8$ units $= 1898$ units, and $(3a + b)b = 1898 \times 8 = 15184$. Therefore, $3a^2 + (3a + b)b = 11907$ hundreds $+ 15184$ units $= 1205884 =$ the true divisor.

(*k.*) Multiplying this by 8 gives 9647072, which shows that 8 is the true value of b', and 638 is the root required.

Proof. — See if $638^3 = 259694072$.

(*l.*) Had the root contained another figure, we might have taken a'' to represent the part already found, and b'' to represent the next figure, when we should have, $(a'' + b'')^3 = a''^3 + [3a''^2 + (3a'' + b'')b'']b''$ equal the number.

(*m.*) Much of the labor of finding the trial divisor, $3a'^2$, might have been avoided. For as $a' = a + b$, $3a'^2$ must equal 3 times the square of $a + b$, or 3 times $(a^2 + 2ab + b^2) = 3a^2 + 6ab + 3b^2$.

(*n.*) But the previous trial divisor, $3a^2 + (3a + b)b = 3a^2 + 3ab + b^2$, and the number which stands above it equals $(3a + b)b = 3ab + b^2$; hence the sum of these equals $3a^2 + 6ab + 2b^2$, which only lacks b^2 of being equal to $3a^2 + 6ab + 3b^2$, or to $3a'^2$. Hence, by squaring b, and adding it to this result, we have $3a' = 3a^2 + 6ab + 3b^2$.

(*o.*) The work may be written thus: —

$$
\begin{array}{rrl}
 & & \underline{a,\ b,\ b'} \\
 & 259694072 & (\ 6\ 3\ 8 \\
 & 216000000 & 630 = a' \\
3a^2 = 1080000\ ) & 43694000 & \\
(3a + b)\,b = 183 \times 3 = 54900 & & \\
[3a^2 + (3a + b)\,b] = 1134900 & 34047000 = & [3a^2 + (3a + b)\,b]\,b \\
b^2 = 900 & & \\
3a^2 = 1190700\ ) & 9647072 & \\
(3a' + b')\,b'\,b' = 1898 \times 8 = 15184 & & \\
[3a'^2 + (3a' + b')\,b'] = 1205884 & 9647072 = & [3a'^2 + (3a' + b')\,b]\,b
\end{array}
$$

(*p*.) By keeping in mind the denominations, so as to render the zeros unnecessary, we should have the following form: —

$$\begin{array}{rrl}
 & & \dot{2}59\dot{6}9\dot{4}07\dot{2} \ (\ 638 \\
 & & 216 = a^3 \\ \hline
\text{Trial divisor} = 3\,a^2 = & 108\ ) & 43694 = \text{Divid.} \\
(3\ a + b)\ b = 183 \times 3 = & 549 & \\ \hline
\textbf{True divisor}, 3\,a^2 + (3\,a + b)\,b, = & 11349 & 34047 = [3\,a^2 + \\
 & b^2 = 9 & \quad (3\,a + b)\,b]\,b \\ \hline
\text{Trial divisor} = 3\,a'^2 = & 11907\ ) & 9647072 = \text{Divid.} \\
(\,a' + b')\,b' = & 15184 & \\ \hline
\textbf{True div.} = 3\,a'^2 + (3\,a' + b')\,b' = & 1205884 & 9647072 = [3\,a'^2 + \\
 & & \quad (3\,a' + b')\,b']\,b'
\end{array}$$

223. *Rule for the Cube Root.*

As a similar process will always apply, we may describe the method of extracting the cube root of a number thus: —

First. *Divide the given number into periods of three figures each, beginning with the units.*

Second. *Find the greatest cube in the left hand period, and place its root as the first figure of the required root.*

Third. *Subtract this cube from the left hand period, and to the remainder bring down the next period, calling the result a dividend.*

Fourth. *Find three times the square of the part of the root already found, and make it a trial divisor.*

Fifth. *See how many times the trial divisor is contained in the dividend, excepting the two right hand figures, and write the quotient as the next figure of the root.*

Sixth. *To three times the part of the root previously found, annex the last root figure, multiply the result by the last figure, and placing the product under the trial divisor, two places to the right, add it to the trial divisor. This will give the true divisor.*

Seventh. *Multiply the true divisor by the last root figure, placing the product under the dividend.*

Eighth. *Subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

Ninth. *Add the square of the last quotient figure to the last true divisor and the number standing over it. The sum will equal three times the square of the root already found, and will be the second trial divisor.*

Tenth. *Now proceed as directed from the fifth forward.*

What is the cube root of each of the following numbers? —

1. 830584.	5. 432081216.
2. 262144.	6. 27054036008.
3. 676836152.	7. 30225545875.
4. 183250432.	8. 6804992375.

224. *Cube Root of Fractions.*

(*a.*) The cube root of a fraction equals the cube root of its numerator, divided by the cube root of its denominator; and if both terms of the fraction are not perfect cubes, only its approximate root can be obtained.

(*b*) If in such cases the denominator is not a perfect cube, it will be well to multiply both terms by the square of the denominator, or by such other number as will make it so. Thus, —

$$\sqrt[3]{\frac{2}{3}}=\sqrt[3]{\frac{2\times 3^2}{3\times 3^2}}=\sqrt[3]{\frac{18}{27}}=\frac{\sqrt[3]{18}}{3}$$

$$\sqrt[3]{\frac{3}{4}}=\sqrt[3]{\frac{3\times 2}{4\times 2}}=\sqrt[3]{\frac{6}{8}}=\frac{\sqrt[3]{6}}{2}$$

1. What is the cube root of $\frac{5}{24}$?

Solution. $\frac{5}{24}=\frac{5}{2^3\times 3}=\frac{5\times 3^2}{2^3\times 3^3}=\frac{45}{216}.$

Hence, $\sqrt[3]{\frac{5}{24}}=\sqrt[3]{\frac{45}{216}}=\frac{\sqrt[3]{45}}{6}=\frac{3}{6}$, approximately.

This differs from the true root by less than $\frac{1}{6}$.

(*c.*) If a greater degree of accuracy is required, both terms may be multiplied by some perfect cube before extracting the root. Thus, —

$$\sqrt[3]{\frac{45}{216}}=\sqrt[3]{\frac{45\times 2^3}{216\times 2^3}}=\sqrt[3]{\frac{360}{1728}}=\frac{\sqrt[3]{360}}{12}=\frac{7}{12}\text{, approximately,}$$

which differs from the true root by less than $\frac{1}{12}$.

What is the cube root of —

2. $\frac{512}{729}$?	4. $\frac{59319}{103823}$?	6. $\frac{6}{7}$?
3. $\frac{7}{12}$?	5. $\frac{19}{24}$?	7. $\frac{37}{36}$?

225. *Cube Root of Decimal Fractions.*

(*a.*) In order that a decimal fraction may have a perfect cube for its denominator, it must contain 3, 6, 9, or some multiple of three places in its numerator.

Thus, the denominators of .008, .027, .512, .003, and .375067 are each perfect cubes, while the denominators of .8, .08, .0027, and .56789 are not.

(*c.*) If a decimal fraction, the root of which is required, does not contain 3, 6, 9, or some exact multiple of 3 decimal places, zeros must be annexed, so that the denominator may be in all cases a perfect cube.

(*d.*) Since $\sqrt[3]{.001} = \sqrt[3]{\frac{1}{100}} = \frac{1}{10} = .1$, and $\sqrt[3]{.000001} = \sqrt[3]{\frac{1}{1000000}} = \frac{1}{100} = .01$, &c., it follows that there will be as many decimal places in the fractional part of the root, as there are times three decimal places in the fractional part of the power. Hence, we carry out the root to as many decimal places as we choose, by annexing three zeros to the power for each additional figure we wish to obtain in the root.

What is the cube root to four places of decimals of —

1. .37.	3. 1.76.	5. .427.
2. .6735.	4. 29.78.	6. .0007.

226. *Rule for extracting a Root of any degree.*

A root of any degree may be found as follows: —

First. *Divide the given number into periods of as many figures each as there are units in the index of the required root.*

Second. *Find the greatest power of the required degree in the left hand period, and place its root as the first figure of the required root.*

Third. *Subtract the power from the left hand figure, and to the remainder bring down the first figure of the next period for a dividend.*

Fourth. *Raise the part of the root already found to a power one degree less than the given power, and multiply the result by the index of the required root, calling the result a trial divisor.*

Fifth. *Divide the dividend by the trial divisor, and the quotient will probably be the next figure of the root. To ascertain whether it is, place it in the root, and raise the number thus found to the required power. If the result equals the first two periods of the given number, or is less, the root figure is correct; but if, as will often be the case, it is greater, the root figure is too large.*

Sixth. *Having found the true root figure, find the remainder, and form a trial divisor, &c., as before.*

SECTION XVII.

MENSURATION.

227. *Polygons.*

(*a.*) A TRIANGLE is a figure having three sides and three angles.

(*b.*) A RIGHT-ANGLED TRIANGLE is a triangle having a right angle.

(*c.*) For definitions of the ANGLE, RECTANGLE, AND SQUARE, see pages 33 and 34.

(*d.*) Lines are PARALLEL when they lie in the same direction; as, for instance, the lines forming the sign of equality.

(*e.*) A PARALLELOGRAM is a four-sided figure, having its opposite sides parallel.

(*f.*) A TRAPEZOID is a four-sided figure having two of its sides parallel.

(*g.*) A POLYGON is a figure bounded on all sides by straight lines.

Fig 1.

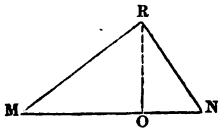


Fig. 2.

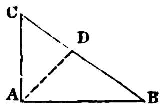


Fig. 3.

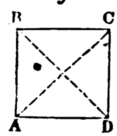


Fig. 4.

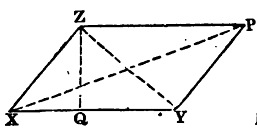


Fig. 5.

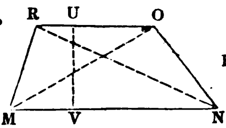
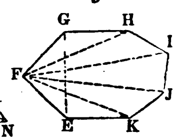


Fig. 6.



(*h.*) Figures 1 and 2 represent triangles; figure 2 represents a right-angled triangle; figure 3 a square; figure 4 a parallelogram; figure 5 a trapezoid; and all represent polygons.

(*i.*) Similar figures are those which have the same shape, i. e., which have the angles of the one equal to the corresponding angles of the other, and the sides about the equal angles proportional.

(*j.*) The BASE of a figure is the side on which it is supposed to stand. Any side of a figure may be regarded as its base. The side opposite the base of a rectangle or parallelogram is often called its upper base.

Thus, in figure 3, A D is the lower, and B C the upper base.

(*k.*) The altitude of a triangle is the perpendicular distance from the side assumed as its base to the vertex of the opposite angle

Thus, in figure 1, when M N is taken as the base, the distance O R is the altitude; in figure 2 when A B is the base, A C is the altitude; when B C is the base, A D is its altitude; and when A C is the base, A B is the altitude.

(*l.*) The ALTITUDE of a rectangle, a parallelogram, or a trapezoid is the perpendicular distance between its parallel bases.

Thus, A B is the altitude of figure 3, z q is the altitude of figure 4, and u v of figure 5.

(*m.*) A DIAGONAL of a polygon is a line drawn from the vertex of two angles lying opposite to each other.

Thus, B D and B C are diagonals of figure 3, z y and x p are diagonals of figure 4, F K, F J, G E, &c., are diagonals of figure 6.

(*n.*) The AREA of a square or of a rectangle equals its length multiplied by its breadth. — See **40,** (*c.*), Note, and **163,** Note.

(*o.*) The area of a triangle equals half the product of its base by its altitude.

(*p.*) The area of a parallelogram equals the product of its base by its perpendicular height.

(*q.*) The area of a trapezoid equals half the product of its altitude by the sum of its parallel bases.

(*r.*) The area of an irregular polygon can be found by dividing it into triangles.

(*s.*) The areas of different triangles, squares, and parallelograms are to each other as the product of their bases by their altitudes.

(*t.*) The areas of similar polygons are to each other as the squares of their like dimensions.

(*u.*) The circumference of a circle equals very nearly 3.1416* times its diameter.

(*v.*) The circumferences of circles are to each other as their diameters or radii.

(*w.*) The area of a circle equals half the product of its circumference by its radius, or quarter the product of its circumference by its diameter.

(*x.*) The area of a circle also equals the square of its radius multiplied by 3.1416.

(*y.*) The areas of circles are to each other as the squares of their diameters or radii.

---

* More accurately, 3.1415926535589; but the above is sufficiently exact for most purposes. Indeed, $3\frac{1}{7}$ is sometimes used where no great degree of accuracy is required.

228. *Problems.*

GENERAL DIRECTION. — Draw figures to correspond to the conditions of each problem.

1. What is the area of a triangle of which the base is 8 ft. and the altitude 6 ft.?

2. What is the area of a parallelogram of which the base is 9 ft. and the altitude 6 ft.?

3. What is the area of a trapezoid of which the upper base is 6 ft., the lower 10 ft., and the altitude 9 ft.?

4. What is the circumference of a circle of which the diameter is 16 ft.?

5. What is the diameter of a circle of which the circumference is 25 ft.?

6. What is the area of a circle of which the radius is 4 ft.?

7. What is the area of a circle of which the circumference is 22 ft.?

8. What is the radius of a circle of which the area is 527 ft.?

9. How many rods long is the side of a field which contains 40 acres?

10. How many rods in diameter is a circular field which contains 8 acres?

11. What must be the length of the side of a square field which is equal in area to a circular field 100 rods in circumference?

12. A rectangular field containing 100 square rods is twice as long as it is wide. What is its length?

13. What must be the diameter of a circular field which contains 4 times as much surface as a similar field 32 rods in diameter?

14. The diameter of one circular field is twice that of another. How do their areas compare?

15. How many square feet in a board 15 feet long, 2 feet wide at one end, and 1 foot wide at the other, allowing that the sides taper regularly?

229. *Properties of the Right-angled Triangle, with Problems.*

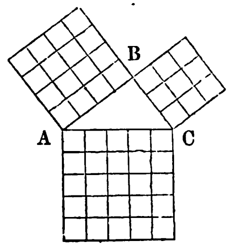
SQUARE OF THE HYPOTHENUSE.

(*a.*) The side opposite the right angle of a right-angled triangle is called the hypothenuse.

In figure 2 the side A C is the hypothenuse.

(*b.*) The square of the hypothenuse equals the sum of the squares of the other two sides.

(*c.*) The annexed figure will illustrate the meaning of this. Its truth can be rigidly demonstrated by geometry.



A B C represents a right-angled triangle right-angled at B. Let A B be 4 ft., and B C 3 ft. Then will A C be 5 ft., and $AB^2 + BC^2 = AC^2$, or $16 + 9 = 25$.

1. What is the hypothenuse of a right-angled triangle of which one side is 6 ft. and the other 11 ft.?

Suggestion. — The hypothenuse equals $\sqrt{36 + 121} = \sqrt{157}$ ft.

2. What is the third side of a right-angled triangle of which the hypothenuse is 12 ft. and the given side 9 ft.?

Suggestion. $\sqrt{144 - 81} = \sqrt{63} =$ *Ans.*

3. What is the distance from one corner of a floor to the opposite corner, if the floor is 24 ft. long and 18 ft. wide?

4. What is the diagonal of a square 30 ft. on a side?

5. A boy, flying his kite, found that he had let out 845 yards of string, and that the distance from where he stood to a point directly under the kite was 676 yards. How high was the kite?

6. A certain window is 20 ft. from the ground. How long must a ladder be which, having its foot 15 ft. from the bottom of the building, will just reach the window?

7. How long is a side of the greatest square which can be inscribed in a circle 3 feet in diameter?

NOTE. — The diagonals of a square bisect each other at right angles

8. A tower, 60 feet high, stands on a mound 30 feet above a horizontal plane. On this plane, and directly south of the tower, is a spring, and directly east from the spring, and at a distance of 160 feet from it, on the same plane, stands a large oak tree. Now, allowing that the distance in a direct line from the top of the tower to the spring is 150 feet, what is the distance from the top of the tower to the foot of the oak?

9. Two men started from the same place, and travelled, one north at the rate of 4 miles per hour, and the other east at the rate of 3 miles per hour. After travelling 7 hours, they turned, and travelled directly towards each other at the same rate as before, till they met. How many miles did each travel?

10. There is a rectangular field 100 rods long and 80 rods wide, the sides of which run north and south. A man started from the south-west corner, and travelled due north along the western boundary of the field for 60 rods, when he travelled across the field in a straight line to the north-east corner. How much farther did he travel than he would if he had gone in a straight line all the way?

230. *Solids.*

(*a.*) A SPHERE is a solid bounded by a curved surface, every part of which is equally distant from a point within, called the centre.

(*b.*) A line drawn from the centre to the surface is called a RADIUS, and a line drawn from any point in the surface through the centre to the opposite point is called a DIAMETER.

(*c.*) A PRISM is a solid having its several faces parallelograms, and its bases two equal and parallel polygons.

(*d.*) A CUBE (see **41**, *b.*) is a kind of prism.

(*e.*) A CYLINDER is such a solid as would be formed by revolving a rectangle about one of its sides. It has also been defined to be "a round body with circular ends."

(*f.*) A PYRAMID is a solid body bounded laterally by triangles, of which the vertices meet at a common point, and the bases terminate in the sides of a polygon, which forms the base of the pyramid.

(*g.*) A CONE is a solid which has a circular base, and tapers regularly to a point called the vertex.

(*h.*) A FRUSTUM of a cone or pyramid is a part cut off by a plane parallel to the plane of its base.

(*i.*) Similar solids have the same shape, i. e., the angles of one of them equal the corresponding angles of the other, and the sides about the equal angles are proportional.

All spheres are similar. Two cones, or two cylinders, are similar when their altitudes are to each other as the radii or diameters of their bases.

Fig. 1.

Fig. 2.

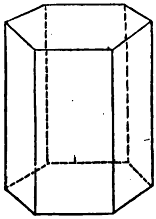


Fig. 3.

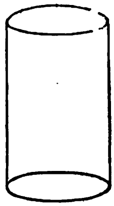


Fig. 4.

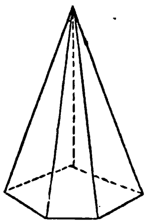


Fig. 5.

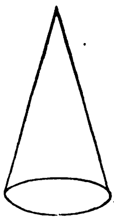
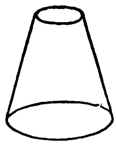


Fig. 6.



(*j.*) Figure 1 represents a sphere; figure 2 a prism; figure 3 a cylinder; figure 4 a pyramid; figure 5 a cone; figure 6 a frustum of a cone.

(*k.*) The SURFACE of a sphere equals the square of its diameter multiplied by 3.1416.*

(*l.*) The surfaces of spheres are to each other as the squares of their radii or diameters.

(*m.*) The SOLIDITY, or SOLID CONTENTS, of a sphere equals the product of the surface multiplied by $\frac{1}{3}$ of the radius, or by $\frac{1}{6}$ of the diameter, or it equals $\frac{1}{6}$ of the cube of the diameter multiplied by 3.1416.*

(*n.*) The solidities of spheres are to each other as the cubes of their radii or diameters.

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* See foot note, page 343.

(*o.*) The solidities of similar solids are to each other as the cubes of their like dimensions.

(*p.*) The solidity of a prism equals the area of its base multiplied by its altitude.

(*q.*) The solidity of a cylinder is equal to the area of its base multiplied by its altitude.

(*r.*) The convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude.

(*s.*) The solidity of a cone or of a pyramid equals the area of its base multiplied by $\frac{1}{3}$ of its altitude.

(*t.*) The solidity of a frustum of a cone, or of a pyramid, equals $\frac{1}{3}$ of the product of its altitude multiplied by the sum of its upper base, plus its lower base, plus the mean proportional between the two bases.

NOTE. — The mean proportional of two numbers is the square root of their product. Thus, the mean proportional of 4 and 9 $= \sqrt{4 \times 9} = 6$.

231. *Problems.*

1. What is the solidity of a sphere the diameter of which is 3 feet?

2. What is the surface of a sphere the radius of which is 1 foot?

3. What is the diameter of a sphere of which the solidity is 10 feet?

4. What is the circumference of a sphere the solidity of which is 12 feet?

5. What is the diameter of a sphere of which the surface is 6 feet?

6. What is the solidity of a prism of which the altitude is 9 feet, and the base contains 10 square feet?

7. What is the solidity of a cylinder of which the altitude is 6 feet and the radius of the base 2 feet?

8. What is the convex surface of a cylinder of which the diameter of the base is 5 feet and the altitude 4 feet?

9. What is the solidity of a cone of which the altitude is 9 feet and the circumference of the base 10 feet?

10. What must be the diameter of a sphere which contains 8 times as many cubic feet as one 3 feet in diameter?

SECTION XVIII.

PROGRESSIONS.

232. *Arithmetical Progression.*

(*a.*) A SERIES OF NUMBERS IN ARITHMETICAL PROGRESSION, or an ARITHMETICAL SERIES, is a series of numbers each of which differs from the preceding by the same number.

(*b.*) Such a series would be obtained by continually adding the same number to, or subtracting it from, any given number.

Thus we should have —

By adding 2's to 1, ... 1, 3, 5, 7, 9, 11, &c.
By adding 7's to 3, ... 3, 10, 17, 24, 31, 38, 45, &c.
By subtracting 4's from 29, . . . 25, 21, 17, 13, 9, &c.
By subtracting 3's from 56, . . . 53, 50, 47, 44, 41, &c.

(*c.*) If the series is formed by addition, it is called an INCREASING SERIES; if by subtraction, it is called a DECREASING SERIES.

(*d.*) The numbers composing a series are called the TERMS of the series.

(*e.*) The difference between the consecutive terms of any series is called the COMMON DIFFERENCE, and is always the number by the addition or subtraction of which the series is formed.

(*f.*) Since the terms of a series are formed by continual additions or subtractions of the same number, it follows that the second term of any series equals the first, plus or minus the common difference; that the third equals the first, plus or minus twice the common difference; that the fourth term equals the first, plus or minus three times the common difference; &c.

(*g.*) *Hence, any term of an arithmetical series is equal to the first term, plus or minus the common difference taken one less times than there are terms in the series ending with the required term.*

(*h.*) *Moreover, if the first term of an increasing arithmetical series be subtracted from the last, or if the last term of a decreasing series be subtracted from the first, the remainder will be the product of the common difference multiplied by one less than the number of terms.*

233. *Problems.*

1. What is the 10th term of the increasing series of which the first term is 3 and the common difference 8?

2. What is the 25th term of the decreasing series of which the first term is 85 and the common difference 2?

3. What is the common difference of the series of which the 1st term is 7 and the 13th term 43?

4. How many terms are there in the series of which the 1st term is 8, the last term 85, and the common difference 7?

5. What is the common difference of the series of which 596 is the 1st term and 491 the 22d?

6. How many terms are there in the series of which 12 is the first term, 4 the last, and $\frac{1}{8}$ the common difference?

234. *To find the Sum of a Series.*

(*a.*) If we should invert any series, we should have a new one, which would differ from the former only in the order of its terms, the one being an increasing while the other is a decreasing series. The first term of one series would equal the last of the other, and each term of one series would be as much greater than its preceding term as each of the other is less than its preceding term. Hence, if we should write the two series under each other, and add together the corresponding terms in the order in which they stand, the successive sums would equal each other, and each would equal the sum of the first and last terms of the original series.

(*b.*) Moreover, there would be as many such sums as there are terms in the series. Hence, the sum of the two series, or (since they are equal) twice the sum of either of them, is equal to the product obtained by multiplying the first and last terms by the number of terms.

Thus, by inverting the series 3, 7, 9, &c., to 35, we have, —

3	7	11	15	19	23	27	31	35	= given series.
35	31	27	23	19	15	11	7	3	= same series inverted.
38	38	38	38	38	38	38	38	38	= the sums of the successive terms.

(*c.*) Adding these last results together would give the sum of the two series, or twice the sum of either, which would manifestly be equal to 9 times 38, or 9 times the sum of the first and last terms. Dividing this by 2 would give the sum of one of the series.

(*d.*) *Hence, the sum of a series in arithmetical progression equals half the product obtained by multiplying the sum of the first and last terms by the number of terms.*

235. *Problems.*

1. What is the sum of the series of which 9 is the 1st term and 94 the 20th?

Answer. $\frac{(9+94) \times 20}{2} = 103 \times 10 = 1030.$

2. What is the sum of the series of which 427 is the 1st term and 187 the 81st?

3. What is the sum of the series of which 4 is the 1st term and 9 is the 6th? What is the common difference?

4. What is the sum of the series of which the 1st term is 7, the common difference 9, and the number of terms 15?

5. How many terms are there in a series of which the sum is 648, the 1st term 3, and the last term 78? What is the common difference?

6. What is the 1st term and common difference of a series of which the last term is 164, the number of terms 12, and the sum 2100?

7. Form the series of which the sum is 153, the 1st term 1, and the last term 17?

236. *Geometrical Progression.*

(*a.*) A series of numbers in geometrical progression, or a geometrical series, is a series of numbers each of which bears the same ratio to the one which follows it.

(*b.*) Such a series would be obtained by continually multiplying or dividing by the same number.

Thus, beginning with 2 and multiplying by 3, we should have 2, 6, 18, 54, 162, 486, &c.

By beginning with 3072 and multiplying by $\frac{1}{2}$, we have 3072, 1536, 768, 384, 192, 96, 48, &c.

(*c.*) The numbers comprising the series are called the TERMS OF THE SERIES.

(*d.*) The ratio of each term to that which follows it is called the COMMON RATIO, and is always the number by which we multiplied to produce the series.

(*e.*) If it be an increasing series, the common ratio will equal a whole number or an improper fraction; but if it be a decreasing series, the common ratio will equal a proper fraction.

(*f.*) From the method of forming such series, it is obvious that the second term must equal the first multiplied by the common ratio; that the third term must equal the first multiplied by the second power of the common ratio; &c.

(*g.*) *Hence, any term of a geometrical series must equal the product of the first term multiplied by the common ratio raised to a power one degree less than the number of the term.*

(*h.*) *Moreover, if the last term of a geometrical series be divided by the first, the quotient will be the common difference raised to a power one degree less than the number of the term.*

237. *Problems.*

1. What is the 7th term of the series of which 125 is the 1st term, and 2 the common ratio?

2. What is the 5th term of the series of which 1 is the 1st term and $\frac{1}{2}$ the common ratio?

3. What is the 9th term of the series of which 4096 is the 1st term and $\frac{1}{2}$ the common ratio?

4. What is the common ratio of the series of which 1 is the 1st and 81 the 5th term?

5. Construct the series of which 1 is the 1st and 6561 is the 9th term.

6. Construct a series of 8 terms, having 1 for the 1st term and $\frac{2}{3}$ for the common ratio.

238. *To find the Sum of a Geometrical Series.*

(*a.*) If each term of a geometrical series should be multiplied by the common ratio, a new series would be formed, of which the first term would equal the second term of the former series, the second term would equal the third of the former, &c.; the last term but one of the new series would equal the last of the given series. Hence, the first term of the original series would have no corresponding term in the derived

series, and the last term of the derived series would have no corresponding term in the original series.

Thus, by multiplying each term of the series 2, 6, 18, &c., to 1458 by the common ratio, we have —

2 6 18 54 162 486 1458 = given series.

6 18 54 162 486 1458 4374 = derived series = 3 times given series.

Again. Multiplying each term of the series 32, 8, 2, $\frac{1}{2}$, &c., to $\frac{1}{128}$ by the common ratio $\frac{1}{4}$, we have —

$$32 \quad 8 \quad 2 \quad \frac{1}{2} \quad \frac{1}{8} \quad \frac{1}{32} \quad \frac{1}{128} = \text{given series.}$$

$$8 \quad 2 \quad \frac{1}{2} \quad \frac{1}{8} \quad \frac{1}{32} \quad \frac{1}{128} \quad \frac{1}{512} = \text{derived series} = \frac{1}{4} \text{ of given series.}$$

(*b.*) Now, as the given series equals once itself, and the derived series equals the common ratio times the given series, it follows that the difference between the given and derived series will equal the product of the given series multiplied by the difference between 1 and the common ratio.

(*c.*) But, as has been shown, the difference between the series equals the difference between the first term of the given series and the last term of the derived series.

(*d.*) *Hence, to find the sum of a geometrical series, we may multiply the last term by the common ratio, and divide the difference between the product and the first term by the difference between* 1 *and the common ratio.*

239. *Problems.*

1. What is the sum of the series of which 2 is the 1st term, 1458 the last, and 3 the common ratio?

Solution. 3 times 1458 = 4374, from which subtracting 2 leaves 4372. Dividing this by 3 — 1, or 2, gives 2186 for the sum of the series.

2. What is the sum of the series of which 32 is the 1st term, $\frac{1}{128}$ the last, and $\frac{1}{4}$ the common ratio?

Solution. $\frac{1}{128} \times \frac{1}{4} = \frac{1}{1024}$, which taken from 32 leaves $31\frac{1023}{1024}$. This equals $1 - \frac{1}{4}$, or $\frac{3}{4}$, times the required sum. Hence, the sum of the series equals $31\frac{1023}{1024} \div \frac{3}{4} = 42\frac{511}{768}$.

3. What is the sum of the series of which $\frac{2}{27}$ is the 1st term, 486 the last, and 3 the common ratio?

4. What is the sum of a series of 11 terms of which 16 is the 1st term and $\frac{1}{2}$ the common ratio?

5. What is the sum of a series of 5 terms of which 5 is the 1st term and 3125 the last term?

240. *Infinite Decreasing Series.*

(*a*) As in a decreasing series each term is smaller than the preceding, it follows that if the series be carried far enough, the terms will become so small that they may be disregarded without affecting sensibly the sum of the series.

(*b*.) An infinite decreasing series will always be of this character, and hence its sum will equal the quotient obtained by dividing the first term by the difference between 1 and the common ratio.

1. What is the sum of the infinite decreasing series of which 4 is the 1st term and $\frac{1}{6}$ the common ratio?

Answer. $4 \div (1 - \frac{1}{6}) = 4 \div \frac{5}{6} = 4\frac{4}{5}$.

What is the sum of the infinite decreasing series —

2. Of which 1 is the 1st term and $\frac{1}{2}$ the common ratio?
3. Of which 3 is the 1st term and $\frac{1}{7}$ the common ratio?
4. Of which 2 is the 1st term and $\frac{2}{3}$ the common ratio?
5. Of which .37 is the 1st term and .01 the common ratio?
6. Of which .597 is the 1st term and .001 the common ratio?
7. Of which .2794 is the 1st term and .0001 the common ratio?

SECTION XIX.

241. CIRCULATING DECIMALS.

(*a*.) A circulating or repeating decimal is one which will never terminate, but in which the same figure, or succession of figures, will always follow each other in the same order.

Examples. .9999, &c.; .323232, &c.; .5174351743, &c.; .0200602006, &c.

(*b.*) Circulating decimals are equivalent to vulgar fractions, the exact decimal value of which cannot be found. — See **143**, (*a.*)

(*c.*) That such vulgar fractions must give rise to repeating decimals may be shown thus: —

Since the remainder after any division must be less than the divisor, we shall at some stage of the division, explained in **143**, find a remainder equal to a former remainder, and from this point the quotients and remainders will succeed each other in the same order as before.

(*d.*) A repeating decimal is indicated by placing a dot over the repeating figure, or over the first and last figures of the repeating period.

Thus, $.\dot{7} = .7777$, &c.; $.\dot{1}\dot{9} = .191919$, &c.,
$.45\dot{3}14\dot{2} = .4531423142$, &c.

(*e.*) The repeating part of a decimal will begin as soon as all the factors of 10 have been cancelled from the denominator of the vulgar fraction which produces it, the vulgar fraction being in all cases reduced to its lowest terms. — See **143**, (*b.*)

1. With which place will the repetend of the decimal value of $\frac{5}{24}$ begin?

Answer. — Since 12 contains the third power of 2, which is a factor of 10, the repetend will commence after 3 places have been obtained, i. e., with the fourth place.

2. With which place will the repetend of $\frac{5}{7}$ commence?

Answer. — Since 24 contains no factor of 10, the repetend will commence with the first place.

With which place will the repetend of each of the following fractions commence?

3. $\frac{8}{9}$.	5. $\frac{13}{14}$.	7. $\frac{10}{13}$.
4. $\frac{11}{75}$.	6. $\frac{149}{275}$.	8. $\frac{8}{17}$.

(*f.*) An expression which contains only the figures of the repetend is called a SINGLE REPETEND; one which contains other figures is called a MIXED REPETEND.

Thus, $.\dot{4}2\dot{7}$ is a simple repetend, and $.53\dot{4}2\dot{7}$ is a mixed repetend.

(*g.*) Two repetends are similar when they begin at the same decimal place, and conterminous when they end at the same decimal place.

Thus, $.\dot{5}2\dot{3}$ and $.\dot{9}$, or $.2\dot{4}7\dot{3}$, and $.4\dot{1}\dot{7}$ are similar; $.\dot{4}2\dot{7}$ and $.43\dot{6}$, or $.\dot{4}27\dot{9}$, and $03\dot{7}\dot{2}$ are conterminous.

(*h.*) The repeating period may be considered as beginning at any figure, provided that it is made to include the entire combination which is repeated.

Thus, $.\dot{4}1\dot{7} = .4\dot{1}7\dot{4} = .41\dot{7}4\dot{1} = .417\dot{4}1\dot{7}$; for each developed would give .417417417417, &c.

(*i.*) A repeating decimal is really an infinite decreasing series in geometrical progression, of which the first term is the first repeating period, and the common ratio is the decimal fraction, having 1 for a numerator and the power of 10, whose exponent contains as many units as there are places in the repeating period, for its denominator. (See *last three problems in* **239.**)

Thus, $.\dot{4}\dot{8}$ = a series of which .48 is the 1st term and .01 the com mon ratio. Hence, $.\dot{4}\dot{8} = .48 \div (1 - .01) = .48 \div .99 = \frac{48}{99}$.

Again. $.\dot{4}7\dot{9}$ = a series of which .479 is the 1st term and .001 the common ratio. Hence, $.\dot{4}7\dot{9} = .479 \div (1 - .001) = .479 \div .999 = \frac{479}{999}$.

So $.\dot{3}29\dot{8} = .3298 \div (1 - .0001) = .3298 \div .9999 = \frac{3298}{9999}$.

(*j.*) We should reach the same result by observing that —

$\frac{1}{9} = .111$, &c. $= .\dot{1}$; $\frac{1}{99} = .010101$, &c. $= .\dot{0}\dot{1}$;

$\frac{1}{999} = .001001$, &c. $= .\dot{0}0\dot{1}$; $\frac{1}{9999} = .00010001$, &c. $= .\dot{0}00\dot{1}$; &c.

Now, $.\dot{5} = 5 \times .\dot{1} = \frac{5}{9}$; $.\dot{8} = 8 \times .\dot{1} = \frac{8}{9}$;

$.\dot{2}\dot{7} = 27 \times .\dot{0}\dot{1} = \frac{27}{99}$; $.\dot{4}\dot{9} = 49 \times .\dot{0}\dot{1} = \frac{49}{99}$;

$.\dot{1}4\dot{7} = 147 \times .\dot{0}0\dot{1} = \frac{147}{999}$; $.\dot{0}4\dot{5} = 45 \times .\dot{0}0\dot{1} = \frac{45}{999}$, &c.;

$.4\dot{6}5\dot{7} = .4\frac{657}{999}$; $.32\dot{8}92\dot{3} = .32\frac{8923}{9999}$.

(*k.*) Hence, it follows that every repeating decimal is equivalent to a vulgar fraction, of which the numerator is expressed by the repeating period, and the denominator by as many 9's as there are figures in the repeating period.

What is the value of —

1. $.\dot{8}\dot{7}$?	3. $.\dot{3}00\dot{6}$?	5. $.\dot{5}25\dot{2}$?
2. $.\dot{4}86\dot{4}$?	4. $.2\dot{7}9\dot{4}$?	6. $.2\dot{3}798\dot{2}$?

(*l.*) In multiplying a repeating decimal by any multiple or power of ten, care must be taken to fill the places left vacant by the change of the point by the figures of the repeating period, and to observe what figures would have to be added to any period on account of the multiplication of the preceding period.

Thus, $.\dot{9} \times 1000 = 99.\dot{9}$ $\quad .\dot{3}4\dot{5} \times 10 = 3.\dot{4}5\dot{3}$

$2.\dot{7}4\dot{3} \times 30 = 2.743743, \&c., \times 30 = 82.\dot{3}1\dot{2}$

What is the product of —

1. $.\dot{4} \times 100$?
2. $27.1\dot{4}3\dot{6} \times 4000$?
3. $.\dot{8}463\dot{5} \times 200000$?
4. $.\dot{3}0\dot{7} \times 10000$?
5. $28.\dot{4}\dot{3} \times 1000$?
6. $.1\dot{3}7\dot{4} \times 8000$?

(*m.*) Repeating decimals, like other fractions, are of the same denomination only when they are fractional parts of the same unit, and have a common denominator.

(*n.*) This is the case only when they are similar and conterminous. To make circulating decimals similar and conterminous, then, is to reduce them to the same denomination, which may be done by carrying them out so far that repeating periods of the same number of figures beginning and ending at the same place may be formed in each.

1. Make $.35\dot{8}\dot{7}$ and $.\dot{4}2\dot{3}$ similar and conterminous.

Answer. $.35\dot{8}7878\dot{7}$ and $42\dot{3}4234\dot{2}$, i. e., $.35\frac{878787}{999999}$ and $.42\frac{342342}{999999}$.

Make the repetends in each of the following cases similar and conterminous: —

2. $.\dot{2}\dot{4}$ and $.\dot{3}7\dot{5}$.
3. $.\dot{4}6\dot{7}$ and $.\dot{5}\dot{2}$.
4. $.1\dot{7}8\dot{4}$ and $.3\dot{2}\dot{8}$.
5. $.51\dot{8}\dot{2}$ and $4.\dot{1}\dot{6}$.
6. $.\dot{6}15\dot{3}$ and $.4\dot{1}3\dot{7}$.
7. $94\dot{2}8\dot{7}$ and $\dot{5}128\dot{1}$.

(*o.*) In adding repetends, or in multiplying them, care must be taken to observe what figures should be added to the written periods, on account of the addition or multiplication of the next lower periods.

(*p.*) Thus, in adding $.\dot{2}4\dot{7} + .\dot{6}3\dot{9} + .\dot{5}8\dot{7}$, we may observe that since the sum of the left hand figures is 13, there must be 1 to bring from the next period below. Adding this with the period gives $1.\dot{4}7\dot{4}$.

1. What is the product of $.2\dot{3}\dot{9} \times 8$?

Answer. — Observing that 8 times 39 gives 3 of the next higher denomination, we have $8 \times 3.2\dot{3}\dot{9} = 1.9\dot{1}\dot{5}$.

What is the value of —

2. $.\dot{4}\dot{7} + .\dot{3}2\dot{5} + .\dot{1}\dot{7} + .\dot{3}2\dot{7}$?
3. $.6\dot{2}7\dot{9} + .328\dot{4}56\dot{8}$?
4. $.\dot{5}7\dot{9} \times 7$?
5. $69.43\dot{2}74\dot{6} \times 37$?
6. $.\dot{5}18\dot{3} + 4.\dot{6}\dot{1} + 3.\dot{8}\dot{5} + .\dot{2}1\dot{7}$?
7. $52.17\dot{6}9\dot{7} + 1.\dot{3}846\dot{3}$?
8. $29.\dot{4}7\dot{6} \times 12$?
9. $43.\dot{0}0\dot{6} \times 8$?

Circulating decimals are multiplied and divided by each other as vulgar fractions are.

What is the value of—

1. $.\dot{6}7\dot{9} \times .\dot{4}$?
2. $.\dot{2}7\dot{9} \times .5\dot{2}8\dot{6}$?
3. $.\dot{6}\dot{8} \div .\dot{1}\dot{7}$?
4. $57\dot{6}\dot{3} \div .\dot{0}\dot{2}$?
5. $42.\dot{7}\dot{6} \times .\dot{4}3\dot{7}$?
6. $23.84 \times 27.\dot{9}\dot{6}$?
7. $2.\dot{9}7\dot{4} \div .\dot{3}00\dot{7}$?
8. $1.7\dot{2}\dot{8} \div .0\dot{1}4\dot{4}$?

SECTION XX.

242. *Miscellaneous Examples.*

1. How many pounds of iron are equal in weight to 100 pounds of gold?

2. $4\frac{2}{3}$ times a certain number added to 1 equals $\frac{3}{7}$ of the quotient obtained by dividing $6\frac{2}{9}$ by $1\frac{3}{5}$. What is the number?

3. How many seconds are there from $9\frac{3}{4}$ o'clock, A. M., of June 4, 1855, to $2\frac{1}{4}$ o'clock, P. M., of Aug. 7, 1855?

4. John can do a piece of work in 5 days, and James can do it in 6 days. How many days will it take both to do it?

5. How many hogsheads, of 63 gallons each, will a cylindrical cistern, 8 feet in diameter and 10 feet high, hold?

6. What is the least common multiple and the greatest common divisor of 74333, 313131, 171717, 146853, and 53095?

7. What is the value of $\dfrac{5\frac{1}{2}}{4\frac{2}{3}} + \dfrac{6\frac{4}{5}}{7\frac{1}{15}}$?

8. Mr. Taylor has invested $\frac{1}{5}$ of his money in railroad stock, $\frac{2}{9}$ of it in bank stock, $\frac{1}{2}$ of it in real estate, and the rest in trade. Moreover, what he has invested in railroad stock is $20,000 more than he has invested in trade. How much money has he, and how much has he invested in each way?

9. Moses E. Fuller bought 18 shares of bank stock at an advance of 8 per cent on their par value of $100 per share. Six months afterwards, and at the end of every subsequent 6 months, he received a dividend of $4\frac{1}{2}$ per cent. At the end of 2 yr. 3 mo. he sold the stock at a premium of 12 per cent. Money being worth 6 per cent per year, compound interest, how much did he gain by the speculation?

10. Mr. Goodwin and Mr. Brown commence trade with equal sums of money. Mr. Goodwin gained $2000, and Mr. Brown lost 10 per cent

of his stock, when it was found that Mr. Goodwin had just twice as much as Mr. Brown. What was the original stock of each?

11. A young man having contracted a debt equal to $\frac{2}{3}$ of his income, found that, by saving $\frac{7}{15}$ of his income annually, he could in 5 years pay up his debt and have $50 left. What was his income?

12. The hour and minute hands of a watch are together at 12 o'clock When will they next be together?

13. At what time between 12 and 1 will the hour and minute hands of a watch point in opposite directions?

14. What was due on the following account, July 1, 1855, interest being 6 per cent per year?

Dr. Wm. Barnes, in account with James Shedd. Cr.

1855.				1855.			
Jan. 17.	To Sund., 6 mo.	$673	42	Feb. 1.	By Sund., 4 mo.	$237	80
Jan. 28.	To Sund., 4 mo.	542	31	Mar. 5.	By Sund., 3 mo.	492	50
Feb. 13.	To Sund., 6 mo.	237	23	Mar. 21.	By Sund., 6 mo.	873	27
Apr. 22.	To Sund., 3 mo.	720	60	Apr. 25.	By Sund., 3 mo.	594	82
May 10.	To Sund., 2 mo.	54	20	May 27.	By Sund., 4 mo.	376	15
June 23.	To Sund., 3 mo.	133	60	June 2.	By Sund., 2 mo.	142	60
				June 20.	By Sund., 6 mo.	225	00

15. What is the value of a pile of wood 40 ft. long, 4 ft. wide, and 5 ft. high, at $5.30 per cord?

16. A owes B $144, due in 6 mo. 20 da., and B owes A $324, due in 1 yr. 4 mo. and 20 da. If A should pay half of his debt now, and the other half when, by the conditions, the whole debt was due, when ought B to pay the whole of his?

17. A owes B $600 dollars, due in 9 mo., and B owes A $900, due in 15 mo. If A does not pay his debt till B's would otherwise have become due, when ought B, in justice, to pay his debt to A?

18. A man travelled 100 miles in two days. $\frac{1}{3}$ of the distance he travelled the first day, added to $\frac{1}{4}$ the distance he travelled the second day, equals $\frac{1}{2}$ the distance he travelled the first day. How far did he travel each day?

19. A set out from Providence to go to Boston, a distance of 42 miles, and B at the same time left Boston for Providence. At the end of six hours they met, when it appeared that A had travelled $1\frac{1}{2}$ miles per hour more than B. How far had each travelled?

20. A dishonest silversmith bought a bar of gold at $192 per lb., and sold it for $16 per ounce, weighing it in both cases by avoirdupois weight. How much did he gain by the fraud, allowing that the true weight of the bar was 5 pounds, and that gold was worth $16 per ounce?

21. A certain room is 16 ft. long, 15 ft. wide, and 12 ft. high. What is the distance from the right hand upper corner to the left hand lower corner?

22. A vessel of war left port with provisions enough to last 600 men 18 months At the end of three months she captured and sunk an enemy's vessel, and took on board her crew of 150 men. Two months after she captured another vessel, on board which she placed the prisoners taken from the former prize, and 100 men besides. Four months after she captured another vessel, on board which she placed 50 men. When she returned to port again she had provisions enough to last the crew which was left on board of her 1 month. How long was she absent from port?

23. A prize of $31,000 is to be distributed among three officers and eight sailors, so that each officer shall have $2\frac{1}{2}$ times as much as a sailor. What will be the share of each?

24. Take any number whatever, multiply it by 3, add 7, subtract the original number, multiply by 2, add 6, divide by 4, add 27, subtract the original number, and the result is 32. Why is this?

25. A merchant sold 50 bushels of wheat for Mr. Randall, and 60 bushels for Mr. Palmer, receiving $150 for the lot. Now, allowing that Mr. Randall's was worth 20 per cent more per bushel than Mr. Palmer's, how ought the money to be divided?

26. I bought 25.000 feet of boards at $2.25 per thousand, and sold $\frac{1}{2}$ of them for what $\frac{2}{3}$ of them cost. What per cent did I gain on the part sold?

27. I bought 63 kegs of nails, each keg containing 100 pounds, at $4\frac{1}{2}$ cents per pound, and sold $\frac{2}{3}$ of them for what $\frac{1}{2}$ of them cost. What per cent did I lose on the part sold?

28. I sold $\frac{1}{2}$ of a lot of land for the cost of $\frac{3}{4}$ of the lot, and the remainder for $\frac{1}{2}$ of what I sold the first part for. What per cent of its cost did I gain on the entire lot?

29. A certain sum is to be divided among three persons in such a way that the first has 2 dollars as often as the second has 3 and the third 6. It turns out that the third has 12 dollars more than both the others. What was the sum divided, and the share of each person?

30. A hare starts 30 yards before a greyhound, but is not seen by him till she has been up 20 seconds. If the hare runs at the rate of 8 miles per hour, and the hound at the rate of 10 miles per hour, how long will the chase continue, and how far must each run from his place of starting?

31. How many per cent in advance of the cost must a merchant ask for goods, that, after allowing for a loss of 6 per cent of his sales by bad debts, an average credit of 6 months, and expenses equal to 8 per cent

of the cost of the goods, he may make a gain of 10 per cent of the original cost, money being worth 6 per cent per year?

32. At a certain time between 12 and 1 the minute hand lacked as much of being at the 1 mark as the hour hand was beyond the 12 mark. What time was it?

33. A gentleman bought a horse, chaise, and harness for $450. The horse cost $3\frac{1}{2}$ times as much as the harness, and the chaise cost $50 more than the horse. What was the cost of each?

34. George Rice has 80 yards of broadcloth, which he will sell for cash at $4 per yard, but for which he will ask $4.50 per yard in exchange for other commodities. Edward Tyler has silk for which he charges $1.25 per yard, cash. How much ought he to charge per yard for it, in exchange for Rice's broadcloth? If, however, he should pay $100 cash, and the balance in silk, how many yards of silk ought he to give?

35. My agent in New Orleans has sold for me a lot of goods for $3375, on a credit of 6 months, and got the note discounted at a bank. If he charges 4 per cent for services in selling, and 3 per cent for guarantying the note at the bank, how much ought he to remit to me?

36. July 1, 1851, I got my note for $1000, payable in 3 months, discounted at a bank, and immediately invested the money received on it in land. Oct. 7, 1851, I sold the land at an advance of 12 per cent, receiving $\frac{1}{2}$ of the sales in cash, and a note for the other half, payable July 1, 1852, without grace, and to be on interest at 7 per cent after Jan. 1, 1852. I lent the cash at 6 per cent interest. When my note at the bank became due, I renewed it for the same time as before, and at the proper time renewed it again; and when this last note became due, I renewed it for such time that the new note would become due July 1, 1852. Allowing that I paid 6 per cent interest on the money borrowed at the bank, and that I made a complete settlement July 1, 1852, what was the amount of my gains?

37. April 16, 1850, I bought of Mr. Curry 498 cords of wood at $3.50 per cord, giving in payment my note payable on demand with interest. Oct. 5, 1850, I sold 232 cords of it at $3.75 per cord, cash, and immediately lent the money received for it on interest. Oct. 17, 1850, I sold the remainder at $4.07 per cord, to be paid Jan. 1, 1851, and to be on interest after Nov. 1, 1850. Jan. 1, 1851, I collected the money due me, and paid that due to Mr. Curry. How much did I gain or lose by the transactions?

38. How much will it cost to plaster the walls and ceiling of a room 16 ft. 3′ long, 14 ft. 2′ wide, and 12 ft. high, allowing for 2 doors, each 7 ft. high and 3 ft. 6′ wide, four windows 5 ft. high and 2 ft. 8′ wide, a fireplace and mantel 4 ft. 3′ high and 5 ft. wide, and a mopboard 6 inches wide, provided that it costs 11 cents per yard.

31

39. An importer sold cloth to a wholesale dealer, and gained 10 per cent of what it cost him. The wholesale dealer sold it to a retail dealer at an advance of 10 per cent on what it cost him. The retail dealer sold it at an advance of 20 per cent on what it cost him. Now, allowing that the retail dealer received $726 for the cloth, how much did it cost the importer?

40. What must be the diameter of a globe which contains 27 times as many cubic inches as a globe 2 inches in diameter?

41. A man spent $\frac{1}{4}$ and $\frac{1}{5}$ of his money, and then earned $36, when he had $88 more than $\frac{1}{3}$ of what he had at first. How much money had he at first?

42. George says that if he should earn as much more money as he now has, $\frac{1}{2}$ as much more, and $37\frac{1}{2}$ dollars, he should have $555. How much money has he?

43. Four men bought a grindstone 4 feet in diameter, paying equal sums, and they agreed that the first should grind off his share, then the second, and so on to the last. What was the thickness of the portion ground off by each?

44. June 1, 1852, I bought for cash 500 casks of oil, each cask containing 42 gallons, at $1.10 per gallon. Oct. 1, 1852, I sold it on 3 months' credit, at a price per gallon equal to 125 per cent of its cost per gallon, deducting 5 per cent of the whole quantity of oil for leakage. I immediately got the note received for the oil discounted at a bank. Allowing that I paid 10 cents per cask for truckage, and $25 for storage and other expenses, and that money was worth 6 per cent per year, did I gain or lose, and how many dollars?

45. A, B, and C trade in company, and gain $100, of which A has $12.50, B has $25, and C has $62.50. C put in $21 more of the original stock than A and B together. What was the original stock?

46. Reuben Aldrich and George Guild bought cloth together, Aldrich paying $6 more than $\frac{1}{2}$ of its cost. They sold the cloth at such rate that Aldrich's share was $39, and Guild's share was $36. What did the cloth cost?

47. Charles, John, and James were talking of their money. Charles has 50 dollars. James says that if Charles should give him his money, he should have twice as much as John; and John says that if Charles should give him his money, he should have three times as much as James. How much has each?

48. A speculator borrowed $2000, agreeing to pay interest at the rate of 9 per cent per year, and invested the money in land at $80 per acre. 3 mo. afterwards he sold $\frac{1}{2}$ the land for $900, and the rest at $100 per acre, and expended the proceeds for flour. 2 mo. 15 da. afterwards he sold $\frac{1}{4}$ of the flour for $600, and the remainder for what

he paid for the whole, and immediately paid the amount of the borrowed money. How much was his gain?

49. If $\frac{1}{3}$ of 12 were 6, what, in the same ratio, would $\frac{1}{5}$ of 50 be?

50. I own a square lot of land measuring 10 rods on a side. How deep a ditch 6 feet wide must I dig around it within its limits to raise its surface 1 foot?

51. A merchant sold a lot of flour at $8.40 per barrel, and thereby gained 20 per cent. He afterwards sold another lot of the same flour for $203, and thereby gained 16 per cent. How many barrels were there in the last lot?

52. What must be the diameter of a sphere to contain as many cubic inches as a cone 1 foot high and having a base of 1 foot in diameter?

53. Mr. Hicks invested a certain sum in flour, and Mr. Gardiner invested twice as much. It turned out that Mr. Hicks lost 10 per cent, and Mr. Gardiner gained 10 per cent, and that the difference between what Mr. Hicks received for his lot and what Mr. Gardiner received for his was $260. How much did each invest?

54. Jan. 1, 1852, I borrowed $954, agreeing to pay interest at the rate of 5 per cent, and immediately expended it for cloth at $3 per yard. Four days afterwards I sold the cloth at $3.50 per yard, to be paid June 17, 1852. On receipt of the money, I immediately expended it for cloth at $1 per yard. July 1, 1852, I sold the cloth at $1.12½ per yard, payable Sept. 22, 1852. As soon as this debt was paid, I put the money on interest at 6 per cent. Jan. 1, 1853, I collected the amount due me, and paid that which I owed. How much had I gained by the transactions?

55. Wishing to find the distance between two trees, which cannot be directly measured on account of a swamp, I measure due east 80 yards from the foot of one of them; then turning south, I measure 100 yards, when I find that I am just 40 yards to the east of the other tree. How far apart are the trees?

56. The eaves of a house are at the same height, and 30 feet apart. The ridge pole is 12 feet higher than the eaves, and just midway between them. The house is 40 feet long. How many shingles will it take to cover the roof, if each shingle covers a space 6 inches long and 4 inches broad?

57. Multiply any number by 4, add 6, divide by 2, add 7, divide by 2, subtract 5, add twice the original number, divide by the original number, subtract 1, multiply by the original number, add 3, multiply by 3, add 11, divide by 2, and the result will always be 10 more than 3 times the original number. Why is this?

58. A man spent $\frac{1}{2}$ of his money and $\frac{1}{2}$ a dollar more; then $\frac{1}{2}$ of what he had left, and $\frac{1}{2}$ of a dollar more; then $\frac{1}{2}$ of what he had left, and $\frac{1}{2}$

of a dollar more, when he had just 7 dollars. How much money had he at first?

59. A rectangular box is 3 times as long as it is wide, and twice as wide as it is high, and contains 96 cubic feet. How many square feet are there in its surface?

60. Obtained at a bank, on my note payable in 3 months, money enough to buy 20 acres of land at $100 per acre. One month afterwards I sold the land, receiving in payment a note on demand, with interest at 6 per cent per year. I collected the amount of this note the day my note became due at the bank, and found that it took $\frac{9}{10}$ of it to pay the latter. For how much per acre did I sell the land?

61. Obtained at a bank, on my note payable in 5 months, money enough to buy 20 acres of land at $100 per acre, and at the same time hired of a friend money enough to buy another lot of the same size and price as the first, giving in payment my note payable on demand, with interest at 6 per cent per year. When the note at the bank became due, I sold both lots for cash at the same price per acre, and found that the money received for 16 acres of it was sufficient to pay my note at the bank. How much did I gain by the transactions? How much more on the second lot than on the first?

62. Bernard Farwell and Francis Dana traded in company. Farwell's stock was $860, and Dana's $420. On dividing their profits, they found that Farwell's share was $4 more than twice Dana's. How many dollars did each gain?

63. A commission merchant received on consignment 100 bags of corn from A, 150 bags from B, and 75 bags from C, and putting them all into 1 lot sold them for $400. Now, allowing that A's lot is 10 per cent better than B's, and 15 per cent better than C's, what is the just share of each?

64. Is the reasoning process contained in the following solution true or false? If false, in what does its fallacy consist?

Question. — What is the effect of adding 3 to both numerator and denominator of a fraction?

Solution. — Since adding 3 to both numerator and denominator of a fraction gives for a result a fraction which expresses 3 more parts than the former, but of such kind that it will take 3 more of them to equal a unit, the addition has both increased and diminished the fraction, and has therefore not altered its value.

65. Show the fallacy, if any, in the following solution: —

If the numerator of a fraction is 4 greater than its denominator, what will be the effect of adding the same number to both numerator and denominator?

Solution. — Adding the same number to both numerator and denomi

nator of a fraction will not affect the difference between them. Hence, the resulting fraction in the case supposed must, like the original one, express 4 more fractional parts than it takes to equal a unit. Therefore, the resulting fraction equals the original one, and the supposed addition has not affected the value of the fraction.

66. Two men, A and B, hired a horse and carriage for $7, to go from Providence to Boston and back, the distance between the cities being 42 miles. At Attleboro', 12 miles from Providence, they took in C, agreeing to take him to Boston and back to Attleboro' for his proportionate share of the expense. At Walpole, 24 miles from Providence, they took in D, agreeing to take him to Boston and back to Walpole for his proportionate share of the expense. What ought each person to pay?

NOTE. — Arithmeticians do not agree as to the correct solution of such examples as the above; some contending that each person should pay in exact proportion to the number of miles he rode, and others that A and B should each pay $\frac{1}{2}$ the expense of the ride from Providence to Attleboro', $\frac{1}{3}$ the expense of the ride from Attleboro' to Walpole, and $\frac{1}{4}$ the expense of the ride from Walpole to Boston; that C should pay $\frac{1}{3}$ the expense of the ride from Attleboro' to Walpole, and $\frac{1}{4}$ the expense of the ride from Walpole to Boston; and that D should pay only $\frac{1}{4}$ of the expense of the ride from Walpole to Boston. Which is the just principle?

67. I sold $\frac{2}{3}$ of a lot of land for 20 per cent more than it cost, and the remainder for 20 per cent less than it cost. What per cent did I gain on the whole?

68. I sold $\frac{3}{4}$ of a cask of wine for $36, which was 25 per cent more than the part sold cost. I then sold the remainder at an advance of 20 per cent on its cost. What per cent of the cost of the cask did I gain?

69. I sent to my agent in Boston a lot of flour, which he sold for $6075, charging a commission of 2 per cent on the sales. He invested the remainder, after deducting his commission of $1\frac{1}{4}$ per cent on the purchase, in cloths, which he shipped to my agent in Savannah. The latter sold them at an advance of 25 per cent on the cost, charging a commission of 5 per cent on the sales, and invested the balance, after deducting a commission of 2 per cent on the purchase, in cotton. The cotton was shipped to my agent in Boston, who sold it at an advance of 20 per cent on its cost, charging a commission of $1\frac{2}{3}$ per cent. Allowing that the expenses of freight, insurance, &c., were $1000, what was my gain, supposing that the flour cost me $6075?

70. A traveller had to pass three toll gates. At the first gate he paid 5 cents less than half the money he had; at the second he paid 2 cents less than half of what he had left; and at the third he paid 1 cent more

than half of what he then had, after which he had only 4 cents left. How much money did he have at first?

71. Lyman Richards and John Dexter traded in company, Richards paying in \$9 less than $\frac{3}{5}$ of the whole stock. They gained \$200, of which Richard's share was \$117. What was the original stock of each?

72. A farmer had his sheep in three pastures. In the first pasture there were twice as many as in the second, and in the second twice as many as in the third. 40 jumped out of the first pasture into the second, and 32 jumped from the second into the third, when the number of sheep in each pasture was equal. How many were originally in each pasture?

73. A water tub holds 73 gallons. The pipe which conveys water to it admits 7 gallons in 5 minutes, and the tap discharges 20 gallons in 17 minutes. Now, suppose that both being carelessly left open, the water is turned on at 4 o'clock, and a servant discovers it at 6, and puts in the tap, at what time will the tub be full? *

74. A gentleman has two horses, and a carriage worth £100. Now, if the first horse be harnessed in the carriage, he and the carriage together will be worth three times as much as the second horse; but if the second horse be harnessed, he and the carriage will together be worth 7 times as much as the first. What is the value of each horse? *

75. There is a fish whose head is 10 feet long; his tail is as long as his head and half of his body, and his body is as long as his head and tail together. What is the whole length of the fish? *

76. "If 12 oxen will eat $3\frac{1}{3}$ acres of grass in 4 weeks, with all that grows during that time, and 21 oxen eat 10 acres in 9 weeks, with all that grows during that time, how many oxen would eat 24 acres, with the growth, in 18 weeks, the grass all the while growing uniformly?"

77. Jan. 1, 1851, my agent in Buffalo bought for me 1000 barrels of flour at \$4 per barrel, for which he charged a commission of 1 per cent. On the 3d of January, I sent him cash to pay for the flour and his commission. It cost me \$1 per barrel to have the flour transported to Boston, and I incurred other expenses upon it to the amount of \$20. Feb. 1, 1851, I sold the flour to J. Smith & Co., at an advance of 25 per cent on its entire cost, receiving in payment half cash, and their note payable in 6 months for the remainder. I had their note discounted at a bank; but before it became due they failed, so that when it became due, I, as indorser, was obliged to pay it. Jan. 1, 1852, I settled with J. Smith & Co., receiving 50 cents on each dollar they owed me. Allowing that money was worth 6 per cent per year, and that I paid the freight and other expenses of the flour on the 15th of January, what was the amount of my loss?

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* From Pike's Arithmetic, edition of 1788.

x = length of fish

10 = head, y = body

$\frac{20+y}{2}$ = tail

$$10+y+\frac{20+y}{2}=x$$

$$10+\frac{20+y}{2}=y$$

$$20+2y+20+y=2x$$

$$20+20+y=2y$$

$$-2x+3y=-40$$

$$-y=-40$$

$$y=40$$

$$-2x+120=-40$$

$$-2x=-160$$

$$x=80$$

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